The PINTEX Program

(Polarized Internal Target Experiments)

The PINTEX group has been studying proton-proton and proton-deuteron scattering and reactions between 100 and 500 MeV at the Indiana University Cyclotron Facility (IUCF). The experiments made use of electron-cooled polarized proton or deuteron beams, orbiting in the 'Indiana Cooler' storage ring, and of a polarized atomic-beam target of hydrogen or deuterium in the path of the stored beam.

The collaboration involved researchers from several midwestern universities, as well as a number of European institutions. The PINTEX program ended when the Indiana Cooler was shut down in August 2002.

On this web site you can find information on the experiments that were carried out, some of the numerical results that were obtained, a complete list of publications that resulted from the program, and some information and pictures on the technical aspects of the experiments.
You may click the small image at the top of this page, to get a closer look.

If you have any questions, press the 'more information' button and leave a message.
PINTEX Experiments

the three-nucleon force

CE80: spin correlation coefficients in p+d elastic scattering

The experiment used a vector and tensorpolarized deuteron target and a polarized stored proton beam. Data have been taken at incident proton energies of 135 and 200 MeV. The data are compared to Faddeev calculations with and without three body force.

CE64: spin observables in pd breakup

Using a polarized deuteron beam and a polarized hydrogen target, pd breakup was studied at 270 MeV deuteron energy (corresponds to 135 MeV protons). The analysis of the data for the so-called axial observables, that are parity-forbidden in elastic scattering constituted the thesis topic of T.J. Whitaker.

pion production with polarized collision partners

ce44, ce64: neutral pion production in pp collisions

The original goal of this experiment was to measure the spin-dependent total cross sections \( \Delta \sigma_T/\sigma_{tot} \) and \( \Delta \sigma_L/\sigma_{tot} \), using A polarized internal atomic hydrogen target and a stored, polarized beam. This is sufficient to determine certain constraints on the partial waves of the reaction.
It was soon realized, that the experiment performed so well that all possible polarization observables everywhere in phase space could be determined. A measurement at four bombarding energies between 325 and 400 MeV then determines all partial waves that are relevant in the first 100 MeV above threshold.
An interesting side-issue concerned the longitudinal analyzing power, measured either with the target or the beam longitudinally polarized. This observables is zero by parity conservation if there are only two particles in the final state. It turned out that for pion production with a three-body final state, the longitudinal analyzing power can be as large as the transverse one.

ce72, ce73: charged pion production in pp collisions

Concurrently with the observation of neutral pions, it was possible to also register charged pion production. These reaction channels were the domain of the Pittsburgh contingent, and Wilfried Daehnick worked on the analysis of these experiments until his untimely death.
p+p \rightarrow p+n+\pi^+ :
p+p \rightarrow d+\pi^+ :
cc79: pion production in pd collisions

This is a measurement of spin correlation coefficients in pd-->tpi+ between 220 and 270 MeV. The involvement of pion production with its large momentum transfer would test the nuclear interaction at short distances where three-nucleon effects should be largest.
Status: analysis (June 2004)

pp elastic scattering

A measurement of analyzing power\textcolor{red}{and} spin correlation coefficients in pp elastic scattering was the commissioning experiment for the PINTEX setup, demonstrating that an internal polarized atomic target enhanced by a storage cell is not only feasible, but makes measurements of spin correlation parameters with unprecedented accuracy possible. The first measurement (ce35) was limited to forward angles (4.5 - 17.5 degrees) and one energy (200 MeV).
data: pp scattering (ce35)
Later, full angular distributions and a measurement of the Azz correlation coefficient was added.
After mastering the problems associated with changing the energy of a stored polarized beam, the energy range from 200 to 450 MeV was mapped out (ce42).
data: pp scattering (ce42)
A new mathematical method, based on 'diagonal scaling' was invented to reduce the measured yields to the observables.

polarization of the stored beam

polarization calibration

The knowledge of the polarization of stored protons was based on a single, well-measured analyzing power in pp scattering. This calibration point was measured concurrently with p+12C scattering. At the calibration energy p+12C scattering exhibits a point in energy and angle where the analyzing power is exactly unity, which can be used as an absolute reference.
Exporting the calibration to all beam energies accessible in a ring is another benefit of the storage ring environment.

polarization lifetime

Usually, the lifetime of the polarization of the stored beam is much longer than the lifetime of the beam itself. Near a depolarizing resonance, however, polarization lifetime becomes measurable. This was studied for an intrinsic resonance (ce55), as well as for an induced (by an RF solenoid) resonance.
**lifetime of vector and tensor polarization**

Interestingly, the lifetime of the tensor and vector polarization of a stored deuteron beam is not the same. This was studied with a deuteron beam that had both kinds of polarization at the same time.


**polarization of the target**

**CE70: polarized molecules?**

The atoms in a polarized target recombine to some degree. Do the resulting molecules retain some polarization? (the answer is yes). This was studied in a dedicated experiment, nicknamed 'polmol'.


**spin exchange in dense, polarized deuterium gas**

In a dense deuterium target collisions between the target atoms leads to spin exchange which in turn may affect the polarization of the target. This effect is significant for tensor polarization and has been observed during ce80.


**storage cell in ring**

Test of a Windowless Storage Cell Target in a Proton Storage Ring


Performance of a Polarized-Hydrogen Storage Cell Target


Effect of a Polarized Hydrogen Target on the Polarization of a Stored Proton Beam


*Last modified: June 2004 by H.O. Meyer*
# Figures and data in numerical form

The clickable items on this page contain data tables and figures.

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_Last modified: January 27 2004 by H.O. Meyer_
List of Publications

| journal articles |
| conference proceedings |
| invited papers |
Collaboration Members

(All members of the collaboration are listed in alphabetical order; highlighted names are links to home pages; email addresses are "mailto" links)

sm = senior member
st = PhD student
pd = research associate

Jan Balewski (pd), Indiana University Cyclotron Facility (balewski@iucf.indiana.edu)

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Andy Dezarn (st), Indiana University Cyclotron Facility, now at

Jack Doskow (sm), Indiana University Cyclotron Facility (doskow@iucf.indiana.edu)

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John Hardie (pd), Indiana University Cyclotron Facility, now Ass. Prof., Dept. of Physics and Computer Science, Christopher Newport University, Newport News, VA.

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Arne Wellingshausen (pd), Indiana University Cyclotron Facility, now at ?

T.J Whitaker (st), Indiana University Cyclotron Facility, now at IU Medical Center, Oncology Dept., Indianapolis (tjwhitak@indiana.edu)

Tom Wise (sm), University of Wisconsin - Madison (wise@uwnuc0.physics.wisc.edu)

Mark Wolanski (pd), Indiana University Cyclotron Facility

Last modified: July 2004 by H.O. Meyer
some pictures of and scattered info about the PINTEX apparatus

A coherent treatment of the experimental setup and the functions of the various components can be found in: Facility for studying spin dependence in pion production near threshold T. Rinckel et al., Nucl. Instr. and Meth. A439 (2000) 117

click:
detector setup and components
target
silicon barrel
polarized internal target upgrade (March 2000)
The picture shows the event distribution in our wire chambers when measuring elastic proton-proton scattering. The wedge structure is caused by a coincidence requirement with a set of silicon detectors located at 45, 135, 225, and 315 degrees.
## ce80

**p+d elastic scattering, Indiana Cooler 5/6/2003, B.v.Przewoski et al.**

Measured angular distributions for 15 (13) of the 17 possible polarization observables in p+d elastic scattering at 135 and 200 MeV

theta is c.m. polar angle in degrees

For questions, contact przewoski@iucf.indiana.edu

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腹部: AD = Axx - Ayy; CD,y = Cxx,y - Cyy,y

For questions, contact przewoski@iucf.indiana.edu

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http://www.iucf.indiana.edu/experiments/PINTEX/data/ce80/ce80_results.html[10/16/2013 2:33:27 PM]
## PinTeX Results

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### Summary

- **Tp = 200 MeV**

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[http://www.iucf.indiana.edu/experiments/PINTEX/data/e80/e80_results.html](http://www.iucf.indiana.edu/experiments/PINTEX/data/e80/e80_results.html) [10/16/2013 2:33:27 PM]
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Spin Correlation Coefficients in pp-->dpi+ from 350 to 400 MeV

The spin correlation coefficient combinations $A_{xx}+A_{yy}$ and $A_{xx}-A_{yy}$, the spin correlation coefficients $A_{xz}$ and $A_{zz}$, and the analyzing power were measured for $\vec{p}\vec{p} \rightarrow d\pi^+$ between center-of-mass angles $25$ deg $\leq \theta \leq 65$ deg at beam energies of 350.5, 375.0 and 400.0 MeV. The experiment was carried out with a polarized internal target and a stored, polarized beam. Non-vertical beam polarization needed for the measurement of $A_{zz}$ was obtained by the use of solenoidal spin rotators. Near threshold, only a few partial waves contribute, and pion s- and p-waves dominate with a possible small admixture of d-waves. Certain combinations of the observables reported here are a direct measure of these d-waves. The d-wave contributions are found to be negligible even at 400.0 MeV. B.v. Przewoski et al., Phys. Rev. C61, 064604 (2000)

Results:

350 MeV

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375 MeV

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Last modified: December 21 1999
by B. v. Przewoski
## CE44 etc

**Neutral pion production in p+p -> p+p+pi0**


For questions, contact meyer@iucf.indiana.edu

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[http://www.iucf.indiana.edu/experiments/PINTEX/data/ce44/ce44_results.html][10/16/2013 2:33:28 PM]
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**Azz(th_q, Dph)**

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*Last modified: June 2004, H.O. Meyer*
Proton-Proton Spin Correlation Coefficients and Analyzing Powers at 197, 250, 280, 294, 310, 350, 399 and 449 MeV beam energy

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[PinTeX Results](http://www.iucf.indiana.edu/experiments/PINTEX/data/ce42/ce42_results.html) [10/16/2013 2:33:28 PM]
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*Last modified: June 2004, H.O. Meyer*
CE35
Proton-Proton Spin Correlation Coefficients and Analyzing Powers at 197.8 MeV beam energy


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**PINTEX Journal articles**

(most recent on top)


Last modified: June 2004 by H.O. Meyer
Conference Contributions by members of the PinTeX Collaboration

**Polarization Observables in pd-Breakup**, T.J. Whitaker et al.
DNP Fall Meeting, Oct. 9 – 12, 2002, East Lansing, MI

First Summerschool and workshop on Cosy Physics, Aug. 30 – Sept. 4, 2002, Juelich, Germany

**Spin Observables in pd-elastic scattering as a probe of the three-nucleon force**, P. Thoerngren-Engblom et al.
Int. Conf. on Nuclear Physics with Storage Rings (STORI02), June 16-20, 2002, Uppsala, Sweden

**Nuclear Polarization of Molecular Hydrogen**, F. Rathmann et al.
Spring meeting of the German Physical Society, March 11-15, 2002, Muenster, Germany

**Complete Set of Polarization Observables, in pp>pppi0**, H.O. Meyer et al.
DNP meeting of the APS, October 17-20, Maui, Hawaii

**Vector and Tensor Polarized Internal Gas Target**, J. Balewski et al.
DNP meeting of the APS, October 17-20, Maui, Hawaii

**Spin parameters at 375 and 400 MeV with neutron and proton detection in pp -> pn pi+**, Swapan K. Saha et al.
April Meeting of the American Physical Society, Long Beach, 29 April - 2 May 2000

**Study of pp -> pn pi+ with longitudinally polarized beam and target from 325 to 400 MeV**, W.W. Daehnick et al.
April Meeting of the American Physical Society, Long Beach, 29 April - 2 May 2000

**Observation of a large longitudinal analyzing power in a nuclear reaction**, H.O. Meyer et al.
April Meeting of the American Physical Society, Long Beach, 29 April - 2 May 2000

**Polarization of H2 from recombination of polarized H-atoms** T. Wise et al.
April Meeting of the American Physical Society, Long Beach, 29 April - 2 May 2000

**Background studies with internal storage cell targets** B. v. Przewoski et al.
Fall Meeting of the Division of Nuclear Physics of the American Physical Society, Asilomar, 21-23 October 1999

**Polarization of molecular hydrogen** A. Wellinghausen et al.
Fall Meeting of the Division of Nuclear Physics of the American Physical Society, Asilomar, 21-23 October 1999

**Spin Correlation Coefficients in pp-->pnPi+ from 325 to 400 MeV**, Swapan K. Saha et al.
4th Int. Conf. On Nuclear Physics at Storage Rings (STORI99), Bloomington, IN, 12-16 September 1999

**Partial wave contributions in the reaction pp-->pppi0 near threshold** P. Thörngren-Engblom, et al.,
PANIC, Uppsala, Sweden, June 10-16 1999.

**Amplitude decomposition of the pp-->pppi0 Reaction** J.T. Balewski et al.,
Spring Meeting of the American Physical Society, Atlanta, March 1999.
Contributions


Longitudinal Spin Correlation Coefficient Azz in pp elastic scattering at 200 MeV B. Lorentz et al.

Polarization Observables in Pion Production P. Thörngren-Engblom, et al.

Analyzing Power and Spin Parameters at 350, 375, and 400 MeV in pp -> pn pi+ S.K. Saha et al.

Spin Correlation Coefficients at 350, 375, and 400 MeV in pp -> d pi+ B. v. Przewoski et al.


Measurements of Spin Correlation Coefficients in pd Elastic Scattering with an Internal Target F. Sperisen et al.

Analyzing Power in pp -> pn pi+ S.K. Saha et al.

Spin-Dependent total Cross Section of the Reaction pp -> pp pi0 T. Rinckel et al.

Measurements of Spin Correlation Coefficients in pp Elastic Scattering between 200 and 450 MeV B. Lorentz et al.

Spring Meeting of the American Physical Society, Columbus, OH, 18-21 Apr 1998

Measurements of Spin Correlation Coefficients in pp Elastic Scattering between 200 and 450 MeV: Analysis, Results and Comparison to Theory F. Rathmann et al., Spring Meeting of the Deutsche Physikalische Gesellschaft, Bochum, 16-20 March 1998


Measurements of Spin Correlation Coefficients in pp Elastic Scattering between 200 and 450 MeV: Analysis, Results and Comparison to Theory F. Rathmann et al.

Spin-Dependent Cross Sections in Pion Production, B. v.Przewoski et al.

Production of a 200 MeV Longitudinally Polarized Beam in a Storage Ring, B. Lorentz et al.

Spring Meeting of the American Physical Society, Washington, DC, April 1997


Calibration of the polarization of a beam of arbitrary energy in a storage ring B.v.Przewoski contributed talk at the 12th international symposium on high energy spin physics, Amsterdam, The Netherlands, Sep 10-14, 1996


A Measurement of Spin Correlation Coefficients in pp Elastic Scattering at Energies between 200 and 450 MeV at the Indiana Cooler B. von Przewoski for the PINTEX Collaboration APS Div. of Nucl. Phys. Fall meeting, Bloomington, IN , 10/26-28/1995

Towards Longitudinal Beam Polarizationin the Indiana Cooler F. Sperisen (for the PINTEX collaboration). Fall Meeting of the Division of Nuclear Physics, American Physical Society, Oct. 25-28, 1995, Bloomington, IN


Technique for making Spin Correlation Measurements at the IUCF Cooler T. Rinckel, W.A. Dezarn, J. Doskow,
Contributions


Last modified: May 18 2000 by B. v. Przewoski
Invited talks by members of the PinTeX Collaboration

Spin and The Three-Nucleon Force Effects, B. von Przewoski, 15th Int. Spin Physics Symposium, Sept 9-14, Brookhaven Nat. Lab, Upton, NY

Polarization Experiments with Storage Rings, H.O. Meyer at STORI02, Uppsala, Sweden, June 16 – 20, 2002

Storage Ring Experiments (Including Polarization), lecture series, Paul V. Pancellla, 1st Summer School and Workshop on COSY Physics, Forschungszentrum Juelich, August 2002. Search for a Three-Nucleon Force, H.O. Meyer, 8th Int. Conf. on Mesons and Light Nuclei, July 2-6, 2001, Prague, Czech Republic


Pion Production as a Test of Nucleon-Nucleon Interaction Models B. v.Przewoski, April meeting of the American Physical Society, Long Beach, Apr 29 - May 2 2000.

Recent Nuclear Physics Measurements with Polarized Beams and Targets at IUCF, Paul V. Pancellla colloquium Ball State University, Muncie IN, January 20, 2000.


Pion production at IUCF with polarized beam on polarized target, a window on single partial waves. B. v.Przewoski, Seminar at TJNAF, April 2, 1999.

From nucleon-nucleon to three nucleon forces, pp and pd scattering at IUCF with polarized beam and polarized target B. v.Przewoski, Seminar at Christopher Newport University, April 1, 1999.

Mesonenerzeugung an der Schwelle Hans Meyer, Spring meeting of the Deutsche Physikalische Gesellschaft, Freiburg i.B., Germany, Mar 24 1999.


Experimental possibilities with stored polarized deuteron beams or polarized internal deuteron targets Hans


**Polarization experiments in NN -> NNpi as a window on single partial waves** B. v.Przewoski, Spring Meeting of the American Physical Society, Columbus, OH, 18-21 Apr 1998.


**Recent results from IUCF** B. v.Przewoski, "Photons probing dynamics in simple systems (NN and pA Bremsstrahlung)" workshop ECT Trento, Italy, Oct 6-11 1997


**Experiments with Stored Beams at IUCF**, Hans Meyer colloquium, Western Michigan University, Kalamazoo, MI, April 1 1997.


**Polarization Experiments at Proton Storage Rings** W. Haeberli, 3rd Int. Conf. on Nuclear Physics at Storage Rings STORI96, Bernkastel-Kues, Germany, September 30 - October 4, 1996, Nucl. Phys. A626 (1997)


**Lecture series** as Guest Lecturer of the Graduierten Kolleg, H.O. Meyer, Physics with Cooled Light Ions, Storage Rings for Cooled Light Ions, Polarization, Experiments with Internal Targets, University of Bonn, Germany, 16 - 25 May, 1996

**Summary Report on Round-Table Discussion - Polarized Internal Targets** W.Haeberli, World Scientific (1996) p.429

**Polarized-Target Experiments in Ion Storage Rings** W. Haeberli, World Scientific (1996) p. 408
New proton-proton scattering data from IUCF B. v.Przewoski "Nucleon-nucleon interaction at intermediate energies", workshop Bad Honnef, Germany, Sep 16-18 1996


Axial Charge in Pion Production and Absorption H.O. Meyer, fall meeting of the Division of Nuclear Physics of the APS, Bloomington, Indiana, October 27 (1995)


Nuclear Physics with Polarized, Internal Targets at the IUCF Cooler B. v.Przewoski, Fall meeting of the American Physical Society, Williamsburg, VA, 29-29 October, 1994

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Last modified: May 18 2000 by B. v. Przewoski
Experimental Setup

The PinTeX experiments take place in the *Cooler* ring of the Indiana University Cyclotron Facility (I.U.C.F.). The Cooler ring is a 500 MeV accelerator capable of providing nearly monochromatic high intensity proton and deuteron beams. These properties make the Cooler ideal for threshold experiments.

The following selections provide further information about the construction and layout of Pintex experiments, concentrating on the detector system and target feed.

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*Last modified: January 27 2000 by B. von Przewoski*
The Wisconsin ABS Target

PinTeX experiments use a polarized proton target developed at the University of Wisconsin - Madison. This target, known as the ABS (Atomic Beam Source) provides a polarized internal proton target with user selectable polarization axis. The next three selections describe the behavior of the target during several Pintex experimental runs.

Polarization Direction Change  
Target Thickness  
Target Polarization

In conjunction with a strong holding field the target is used for a measurement of the polarization of hydrogen molecules after recombination. The experiment is commonly referred to as POLMOL. View the PolMol target.

Last modified: January 28 2000 by B. v.Przewoski
Silicon Barrel Pictures 16-May-2000
CE80 Silicon Barrel Pictures

Upgrade of the PINTEX ABS, March 6, 2000

assemble of the ABS

scattering chamber with the alignment fixture
Axial Observables in $d\bar{p}$ Breakup and the Three-Nucleon Force

H. O. Meyer, T. J. Whitaker, R. E. Pollock, B. von Przewoski, T. Rinckel, and J. Doskow

Indiana University Cyclotron Facility, Bloomington, Indiana 47405, USA

J. Kuroś–Żołnierczuk

Nuclear Theory Center, Indiana University, Bloomington, Indiana 47405, USA

P. Thöngren-Engblom

Uppsala University, Uppsala, Sweden

P. V. Pancella

Western Michigan University, Kalamazoo, Michigan 49008, USA

T. Wise

University of Wisconsin, Madison, Wisconsin 53706, USA

B. Lorentz and F. Rathmann

Institut für Kernphysik, Forschungszentrum Jülich, Jülich, Germany

(Received 16 April 2004; published 10 September 2004)

We have measured three axial polarization observables in $d\bar{p}$ breakup with a polarized 270 MeV deuteron beam on a polarized proton target. Axial observables are zero by parity conservation in elastic scattering but can be easily observed in the breakup channel at the present energy. Based on a symmetry argument, the sensitivity of these observables to the three-nucleon force might be enhanced. Calculations without three-nucleon force are in fair agreement with our measurement, indicating that the expected sensitivity of axial observables to the three-nucleon force is not confirmed. Including a three-nucleon force in the calculation does not improve the agreement with the data.

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The interaction between nucleons, like any exchange force, must on some level include contributions beyond a superposition of pair interactions. Indeed, it is believed that a three-nucleon force (3NF) is required to explain the binding energies of $^3$H and $^3$He [1]. On the other hand, an unambiguous manifestation of a 3NF has not yet been established in $pd$ or $nd$ scattering or breakup observables. An empirical characterization of the 3NF is thus still in the future.

In the past, studies of the 3NF in $pd$ or $nd$ reactions have often been based on the premise that state-of-the-art Faddeev calculations with modern nucleon-nucleon ($NN$) potentials describe so well how nature without a 3NF would present itself that the remaining, small discrepancies with cross section and polarization data must be attributed to the 3NF. In order to select observables that are most sensitive to the 3NF one would have to rely on calculations with and without a theoretical three-nucleon potential. However, the currently available theoretical 3NFs do not lead to a better agreement with the data, which include cross sections and many polarization observables at energies up to the pion threshold (see, e.g., [2]). It would thus be preferable to have a more fundamental criterion to identify 3NF-sensitive observables.

Knutson [3] has pointed out that so-called “axial” observables may have an enhanced sensitivity to the 3NF. By definition, these polarization observables are antisymmetric under space inversion. If parity is conserved, such observables vanish when the momenta of the beam and of all reaction products lie in a common plane, as is the case for elastic scattering, but not, in general, for the breakup reaction. Knutson’s argument is based on quantum numbers and small energies and states that axial observables are likely to be sensitive to a certain kind of spin operator that can occur in three-nucleon potentials, but not in the interaction between nucleon pairs. Thus, axial observables are linked, in principle, to the 3NF, and may exhibit enhanced 3NF sensitivity.

Whether the 3N spin operators contribute significantly to axial observables must be established by experiment. An attempt to do this is described in Ref. [4], which reports the measurement of the longitudinal proton analyzing power $A_L$ in $\bar{d}p$ breakup at a proton bombarding energy of $T_p = 9$ MeV. The reported analyzing powers are zero within the experimental uncertainty (typically ±0.01), and consistent with a corresponding Faddeev calculation.
Here, we present the measurement of three of the five axial observables that can be observed in the $d\bar{p}$ breakup using a polarized deuteron beam on a polarized proton target. The beam energy is $T_d = 270$ MeV, which is equivalent to $T_p = 135$ MeV proton bombarding energy.

In the following we briefly describe the experiment; more details can be found in Ref. [5]. The deuteron beam from a polarized ion source was accumulated and accelerated to 90 MeV in an injector synchrotron and transferred to the Cooler storage ring, where it was electron cooled and accelerated to the final energy. After taking data for 110 s, the remaining beam was discarded and the cycle was repeated. For a new cycle, the beam polarization was changed to the next of five polarization states, including (i) positive vector ($Q_z \sim +0.8$), (ii) negative vector ($Q_z \sim -0.6$), (iii) pure tensor ($Q_{tzz} \sim +0.8$), (iv) pure negative tensor polarization ($Q_{tzz} \sim -1.6$), and (v) unpolarized beam. States (i) and (ii) contained a tensor polarization admixture ($Q_{tzz} \sim +0.7$). The quoted values of the tensor and vector polarizations, $Q_{t}$ and $Q_{tzz}$, are with respect to the spin alignment axis and represent approximate average values. The relevant polarization moments follow from the orientation of the spin alignment axis, which throughout the experiment was vertical.

The target was produced by a source of polarized H atoms [6]; the atomic beam was aimed through a fill tube into a 25 cm long, 12 mm diameter, thin-walled (25 μm) aluminum storage cell through which the stored beam passed. The thickness of the extended target was a few times $10^{13}$ atoms/cm$^2$. The cell position could be adjusted remotely with respect to the stored beam to minimize reactions in the cell wall. Magnetic fields in the region of the target cell were used to produce one of six polarization directions, i.e., vertical, sideways, or longitudinal, each with both directions. The average target polarization was $P \sim 0.6$.

The azimuthally symmetric detector covered a forward cone of about a 45° half angle and was capable of measuring the direction and energy of charged particles. In the beam direction, it contained a thin scintillator $(F)$, two pairs of wire chambers, a 15 cm thick scintillator array $(K)$ divided into quadrants, and a 10 cm thick scintillator array $(E)$ divided into octants. All detectors had a hole in the center to accommodate the stored beam. All particles of interest were stopped in $K$ and $E$. Protons were distinguished from deuterons, based on stopped energy and energy loss in $F$. All events with a response in at least two segments of $K$ were written to disk. Breakup events (two protons in coincidence) were selected by conditions on the reconstructed tracks, matching of the tracks with the scintillator segments, particle identification, and a match of the reconstructed mass of the unobserved particle with the actual neutron mass.

Concurrently with the breakup data, elastic scattering events, with a coplanar proton and deuteron in coincidence, were also registered, covering the polar angle range $76° < \theta_{cm} < 140°$. These events are crucial since they provide a sample of particles of known energy, needed to deduce an absolute energy calibration of the $K$ and $E$ detectors, and because they also provide the beam and target polarizations by comparison to previously measured $pd$ scattering polarization observables [2,7-9].

To describe the beam and target polarization moments and the kinematics of an event, we define a fixed Cartesian frame with the $z$ axis in the beam direction, the $y$ axis upwards, and the $x$ axis to the left, completing a right-handed frame. Polar angles $\theta$ are measured from the $z$ axis, and azimuths $\phi$ from the $x$ axis, clockwise if viewed in the beam direction. Since for breakup events, the energy and direction of both protons in the final state are known, the experiment is kinematically complete; i.e., all five kinematic variables are determined. To further process these events, we deduce the center-of-mass momentum vector $\tilde{q}$ of the neutron and $\tilde{p}$, the relative momentum of the two protons. Since the two detected particles are identical, we arbitrarily define $\tilde{p}$ such that it points into the forward direction ($p_z > 0$). The five independent kinematic variables are then $\theta_p$, $\theta_q$, $\phi_p$, $\phi_q$, and $|\tilde{p}|$.

Axial polarization observables are invariant under rotations around the $z$ axis and can depend only on the difference $\Delta \phi = \phi_p - \phi_q$ between the two azimuths. This reduces the number of kinematic variables to four. Retaining only terms with rotational symmetry around the $z$ axis, and only terms that can be measured with a vertical deuteron spin alignment axis, the differential cross section $\sigma$ for a $d\bar{p}$ reaction with polarized collision partners (see, e.g., [10]) becomes

$$\sigma = \sigma_0\left[ 1 + \frac{1}{3} q_{zz} A_{zz} + \frac{1}{3} (q_{xz} p_x + q_{yz} p_y) (C_{xx} + C_{yy}) + p_x A_{y} + \frac{1}{3} (q_{xz} p_x - q_{yz} p_y) (C_{xy} - C_{yx}) + \frac{1}{3} q_{zz} p_z C_{zz,z} \right]. \tag{1}$$

In this expression, the $p_m$ $(m = x, y, z)$ are the components of the proton polarization (magnitude $P$), the $q_m$ are the components of the deuteron vector polarization (magnitude $Q_z$), and $q_{zz}$ is the only tensor moment that plays a role. For a vertical spin alignment axis, $q_{zz} = -\frac{1}{2} Q_{tzz}$. The observables (here in a Cartesian basis) include the proton analyzing power $A_z$, the tensor analyzing power $A_{zz}$, two combinations of vector correlation coefficients, and the tensor correlation coefficient $C_{zz,z}$; all of these are a function of $\theta_p$, $\theta_q$, $|\tilde{p}|$, and $\Delta \phi$. The terms on the first line of Eq. (1) are related to “normal” observables, unconstrained by parity conservation, while the remainder contains axial observables, which change sign under a reflection on the $x$ axis, i.e., $O(\Delta \phi) = O(-\Delta \phi)$ [3]. The two remaining axial observables $A_t$ and $C_{zz,z} - C_{yz,y}$
cannot be measured with vertical deuteron spin alignment.

By combining the yields measured with the appropriate combinations of the five beam and six target polarization states, individual terms in Eq. (1) are singled out. The data are evaluated as a function of $x/0.0030$. The other three kinematic variables are ignored; thus their full range within the detector acceptance is included.

Figure 1 shows the longitudinal proton analyzing power $A_z$ as a function of $\Delta \phi$. The solid and dashed curves are based on the CD-Bonn and the AV18 $NN$ interaction, respectively. When the TM' three-nucleon potential is combined with the CD-Bonn interaction, the dotted curve results.

average has to be weighted by the cross section and the probability that the detector registers an event at a given point $\xi$ in phase space. To do this is often difficult: for the present experiment, for instance, the acceptance angle of the detector depends on the location of the event along the extended target, there is a lower limit for the energy of protons that reach the trigger detector, the joints between detector segments may locally reduce the efficiency, and so on.

In order to take instrumental constraints into account correctly, we have developed a new method [11], which is
applicable for any kinematically complete experiment. This method is based on the notion that the local density of measured events at some phase space point \( \xi \) represents the proper weight that should be used when averaging the theoretical expectation. It can then be shown that the theoretical expectation, properly averaged over a given phase space region \( \eta \), which could, for instance, consist of a \( \Delta \phi \) bin with unconstrained remaining variables, is simply given by

\[
\langle O^{th}(\eta) \rangle = \frac{1}{N(\eta)} \sum_{i=1}^{N(\eta)} O^{th}(\xi_i),
\]

where \( \xi_i \) represents the four kinematic parameters of the \( i \)th event and \( N(\eta) \) is the number of events in region \( \eta \). The uncertainty due to the stochastic sampling, and corrections due to polarization effects that are not averaged out by summing over all events, are discussed in Ref. [11] but are not important in this context. To evaluate Eq. (2), one needs to calculate the observable \( O^{th}(\xi) \) for each event. In order to do this efficiently, we use a uniform four-dimensional grid that spans all of phase space. At each grid point the observable value is calculated from the Faddeev amplitudes. The value for \( O^{th}(\xi) \) is then obtained from this table of numbers by interpolation. The solid line in Figs. 1–3 shows the expectation from a Faddeev calculation [12] based on the CD-Bonn nucleon-nucleon potential [13]. In order to explore the dependence on the choice of the \( NN \) interaction, the dashed line represents a calculation with the AV18 potential [14]. As can be seen, the two calculations are nearly identical. The effect of including a theoretical 3NF (in this case, the TM' 3NF [15]) is illustrated by the dotted line.

For \( A \), the collected statistics is sufficient to explore the dependence of this observable on any of the kinematic variables over which we have averaged so far. This dependence turns out to be generally flat. Moreover, the effect of including the 3NF in the calculation is still small and shows no significant variation as a function of these variables (for more detail, see Ref. [16]). Thus, we can dismiss the worry that interesting information may be lost by averaging over phase space.

From our results and the calculations presented in Figs. 1–3, we conclude the following:

(i) We have observed nonzero values for the longitudinal analyzing power in \( pd \) breakup, as well as for two additional observables that are also forbidden in reactions with a two-body final state. This is the first experimental verification that axial observables in \( pd \) breakup can differ from zero and, in fact, be quite large. This may indicate that at the present energy axial observables are dominated by 2N contributions, which in turn would dilute the sensitivity to the 3NF, discussed by Knutson [3].

(ii) If axial observables were indeed especially sensitive to the 3NF, we would expect that calculations without a 3NF would differ significantly from the data. Instead, we find that these calculations already provide a fairly good description of the measurements. The remaining discrepancies are quite similar to those found in polarization observables in elastic scattering at the same energy. Thus, we conclude that the sensitivity of the axial observables reported here to the 3NF is not enhanced as we had hoped.

(iii) The difference between the measured \( A \), and the calculation without a 3NF is reduced by the inclusion of the TM' 3NF at some angles but not at others. For the other two observables the TM' 3NF does nothing or moves the calculation in the wrong direction. Based on the present data, we conclude that either the difference between the data and the calculation without a 3NF is not due to the 3NF, or the TM' potential is not a valid description of the 3NF.

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Analyzing powers and spin correlation coefficients for $p+d$ elastic scattering at 135 and 200 MeV

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The proton and deuteron analyzing powers and ten of the possible 12 spin correlation coefficients have been measured for $p+d$ elastic scattering at proton bombarding energies of 135 and 200 MeV. The results are compared with Faddeev calculations using two different NN potentials. The qualitative features of the extensive data set on the spin dependence in $p+d$ elastic scattering over a wide range of angles presented here are remarkably well explained by two-nucleon force predictions without inclusion of a three-nucleon force. The remaining discrepancies are, in general, not alleviated when theoretical three-nucleon forces are included in the calculations.

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I. INTRODUCTION

During the last several years, proton-deuteron scattering has been studied in a number of experiments at intermediate-energy facilities, including RIKEN [1–3], the KVI [4,5], and IUCF [6,7]. The declared purpose of all of these experiments was the search for evidence of a three-nucleon force.

This considerable experimental activity was stimulated by the availability of parameter-free and computationally exact predictions of scattering observables in the three-nucleon system, derived from a given nucleon-nucleon (NN) potential. These “Faddeev” calculations, carried out mainly by the Bochum-Cracow group [8], are now available at intermediate energies, owing to advances in computing power that made the inclusion of a sufficient number of partial waves possible. However, pion production (above ~200 MeV proton energy) is not included in these calculations.

It is commonly argued that discrepancies between data and calculations are a manifestation of physics that is omitted in these calculations, and that the most obvious contender is the three-nucleon force (3NF). Bombarding energies above 100 MeV are of interest because 3NF effects are expected to grow with increasing energy, and because the Coulomb interaction is of minor importance, making it feasible to compare the calculations (which are really for $n+d$ scattering) to $p+d$ scattering data.

It is also possible to include model representations of the 3NF in the Faddeev calculations. If this were to lead to a systematic improvement of the agreement with the data, one would have uncovered evidence for a 3NF.

Polarization observables contain sums of interfering pairs of amplitudes and are potentially more sensitive than the cross section to contributions from a small effect such as the 3NF. In order to test the present (and any future) models of a 3NF, it is crucial to have measured as many polarization observables as possible. The experiments cited above cover the cross section, the proton analyzing power, the four deuteron analyzing powers, and in one case [2], polarization transfer coefficients. Only a single spin correlation coefficient measurement (beam and target polarized) has been reported [7]. In this paper, we report the measurement of ten of the 12 possible spin correlation coefficients, in addition to the five analyzing powers. The measurement was carried out at 135 and 200 MeV proton bombarding energy, and used a polarized proton beam and a vector- and tensor-polarized deuteron target.

The paper is organized as follows. In Sec. II we define the measured observables and derive the spin-dependent scattering cross section. In Secs. III and IV we describe the equipment and the measurement. In Sec. V we explain how the observables were deduced from the data and present the results. In Sec. VI we describe the calculations and present models of the 3NF and compare them to the measurements. This is followed by our conclusions in Sec. VII.

II. OBSERVABLES

A. Coordinate frames and definition of observables

The following discussion is limited to the tools that are needed to analyze the data of this experiment; details of
the polarization formalism and its foundation can be found, e.g., in Ohlsen’s discussion of spin correlation experiments involving particles with spin 1/2 and 1 [9]. For the treatment of spin-1 polarization, two different bases are in common use and various normalization conventions can be found in the literature. Here, we are using the Cartesian basis (as opposed to the spherical tensor basis) because it is more intuitive when dealing with spin correlation coefficients. For normalization we follow the Madison Convention [10]. The production and description of polarized beams is also well explained in Ref. [11].

We define as the “scattering frame” a Cartesian coordinate system \((X, Y, Z)\) with the \(Z\) axis along the momentum of the incident proton, \(\vec{p}_{\text{inc}}\), the \(Y\) axis in the direction of \(\vec{p}_{\text{inc}} \times \vec{p}_{\text{out}}\) where \(\vec{p}_{\text{out}}\) is the momentum of the scattered proton, and the \(X\) axis completing a right-handed coordinate frame. The differential cross section \(\sigma\) for elastic scattering of polarized protons from polarized deuterons, in units of the unpolarized differential cross section \(\sigma_0\), is given by Eq. 6.8 of Ref. [9] as follows:

\[
\sigma/\sigma_0 = 1 + Q_x A_y^0 + 3/2 P_y A_z^0 + 2/3 P_{ZZ} A_{xz} \\
+ 1/3 (P_{XX} A_{xx} + P_{YY} A_{yy} + P_{ZZ} A_{zz}) \\
+ 3/2 (P_{XY} Q_{xx} C_{xx,x} + P_{XZ} Q_{xx,z} + P_{YY} Q_{yy,y} \\
+ P_{YZ} Q_{yy,z} + P_{XY} Q_{yy,y} + P_{ZZ} Q_{zz,z} + P_{ZZ} Q_{zz,z}) \\
+ 2/7 (P_{XX} Q_{xx,x} + P_{XY} Q_{xx,y} + P_{ZZ} Q_{zz,z} + P_{ZZ} Q_{zz,z}) \\
+ P_{XY} Q_{xy,x} + P_{XY} Q_{xy,y} + P_{XY} Q_{yy,y} + P_{XY} Q_{yy,y} + P_{YY} Q_{yy,y} + P_{YY} Q_{yy,y} + P_{ZZ} Q_{zz,z} + P_{ZZ} Q_{zz,z}).
\]

Using indices \((I, K = X, Y, Z)\) the \(Q_I\) are the components of the proton polarization in the scattering frame, the \(P_I\) are the components of the deuteron vector polarization and the \(P_{IK}\) are the Cartesian moments of the deuteron tensor polarization. The observables, defined by this equation, include the proton analyzing power \(A_I^0\), the deuteron analyzing power \(A_I^0\), the tensor analyzing powers \(A_{ik}\), the vector spin correlation coefficients \(C_{ik}\), and the tensor spin correlation coefficients \(C_{ik,m}\). These observables are functions of the scattering angle \(\theta\).

In the derivation of Eq. (1), the constraints of parity conservation have been taken into account. We note that the Cartesian basis is over-complete, and that the following three relations between the terms of Eq. (1) hold:

\[
P_{XX} + P_{YY} + P_{ZZ} = A_{xx} + A_{yy} + A_{zz} = C_{xx,x} + C_{yy,y} + C_{zz,z} = 0.
\]

Defining \(A_\Delta \equiv A_{xx} - A_{yy}\) and \(C_{\Delta,y} \equiv C_{xx,y} - C_{yy,y}\), we use the relations of Eq. (2) to eliminate \(A_{xx} + A_{yy}\) and \(C_{xx,x} + C_{yy,y}\). There are then 17 spin observables in all, namely the proton and deuteron vector analyzing power, three tensor analyzing powers, five vector correlation coefficients and seven tensor correlation coefficients.

The task of extracting spin observables from the data requires the measurement of the azimuthal dependence of the cross section, calling for a cylindrically symmetric detector. In such a detector, the azimuth \(\varphi\) of the scattering plane around the beam axis is a measured quantity that varies from event to event. To describe this, we define a second Cartesian frame \((x, y, z)\) that is fixed in space with the \(x\) axis pointing to the left, the \(y\) axis upwards and the \(z\) axis in the beam direction. The azimuth \(\varphi\) of the outgoing proton (i.e., the orientation of the scattering plane) is measured clockwise from the positive \(x\)-axis, looking in the beam direction. Thus, the scattering frame is obtained by rotating the fixed frame by \(\varphi\) around the \(z\) (or \(Z\)) axis.

The polarization of the \((\text{spin-1/2})\) proton beam is specified in the fixed frame by a three-component vector with magnitude \(Q\) and direction \(\hat{Q} = (\beta_Q, \Phi_Q)\), where \(\beta_Q\) is the polar angle (with respect to the \(z\) axis) and \(\Phi_Q\) is the azimuth. The polarization components in the scattering frame are then given by

\[
Q_x = Q \sin \beta_Q \cos (\Phi_Q - \varphi),
\]

\[
Q_y = Q \sin \beta_Q \sin (\Phi_Q - \varphi),
\]

\[
Q_z = Q \cos \beta_Q.
\]

The description of the polarization of the deuteron target is more complicated. For an ensemble of spin-1 particles prepared by an atomic beam source there exists an axis of rotational symmetry \(\hat{S}\), called “spin alignment axis”. Let us denote by \(m_+\), \(m_0\) and \(m_-\) the fractional populations of the three magnetic substates with projection +1, 0, and −1 with respect to a quantization axis in the direction of \(\hat{S}\). The vector polarization of the ensemble is then given by \(P_z = m_+ - m_-\) and the tensor polarization by \(P_{\Delta} = 1 - 3m_0\). In order to characterize the polarization of the deuteron target, the orientation of the spin alignment axis, \(\hat{S} = (\beta_P, \Phi_P)\), must be known, in addition to the values of \(P_z\), \(P_{\Delta}\). The spin alignment axis is associated with the expectation value of the magnetic moment (either parallel or antiparallel) and thus can be controlled by the guide field at the target as explained in Sec. III C. The components of the vector polarization are analogous to the proton case,

\[
P_x = P_z \sin \beta_P \cos (\Phi_P - \varphi),
\]

\[
P_y = P_z \sin \beta_P \sin (\Phi_P - \varphi),
\]

\[
P_z = P_z \cos \beta_P,
\]

while the tensor moments are given by [9,11]

\[
P_{xy} = \frac{3}{2} P_{\Delta} \sin^2 \beta_P \sin (2(\Phi_P - \varphi)),
\]

\[
P_{yz} = \frac{3}{2} P_{\Delta} \sin \beta_P \cos \beta_P \sin (\Phi_P - \varphi),
\]

\[
P_{xz} = \frac{3}{2} P_{\Delta} \sin \beta_P \cos \beta_P \cos (\Phi_P - \varphi),
\]

\[
P_{zz} = \frac{1}{2} P_{\Delta} (3 \cos^2 \beta_P - 1).
\]

**B. Polarized cross section**

We start from Eq. (1), eliminate the dependent variables using Eq. (2) and insert Eqs. (3)–(5). This leads to an equation for \(\sigma/\sigma_0\) that contains the values for beam and target polarization, \(Q\), \(P_z\), \(P_{\Delta}\), the orientations of beam polarization vector \(\hat{Q}\) and of the target spin alignment axis \(\hat{S}\), the observables, and the azimuth \(\varphi\) of the scattering plane. At this stage it is practical to evaluate the cross section for those specific orientations \(\hat{Q}(\beta_Q, \Phi_Q)\) and \(\hat{S}(\beta_P, \Phi_P)\) that are actually used in this experiment.
We used different scenarios for beam and target polarization. In scenario V90 (see Sec. IV A2) the beam polarization was vertical (along the y axis), thus \( \beta_P = \pi/2 \), and \( \Phi_Q = \pi/2 \). For a sideways deuteron spin alignment axis \( \hat{S} \), we have \( \beta_P = \pi/2 \), and \( \Phi_P = 0 \). Eq. (1) then reduces to

\[
\frac{\sigma}{\sigma_0} = 1 + QA_y^p \cos \varphi - \frac{1}{2} P_t A^p_y \sin \varphi
\]

\[
- \frac{1}{4} P_{t,t} \left[ A_{zz} - A_\Delta \cos 2\varphi \right] + \frac{1}{2} P_t Q \left[ C_{zz,y} + \left( C_{xy,x} - \frac{1}{2} C_{\Delta,y} \right) \right] \cos \varphi
\]

\[
- \left( \frac{1}{2} C_{yy,x} + \frac{1}{2} C_{\Delta,y} \right) \cos 3\varphi
\]

In deriving this equation, when products and powers of trigonometric functions of \( \varphi \) occur, they are transformed to expressions containing only members of the orthogonal set \( \cos(k_\varphi) (k_\varphi = 0, 1, 2, \ldots) \) and \( \sin(k_\varphi) (k_\varphi = 1, 2, \ldots) \). On the other hand, for a vertical deuteron spin alignment axis we have \( \beta_P = \pi/2 \), and \( \Phi_P = \pi/2 \), and we obtain

\[
\frac{\sigma}{\sigma_0} = 1 + QA_y^p \cos \varphi + \frac{1}{2} P_t A^p_y \cos \varphi
\]

\[
- \frac{1}{4} P_{t,t} \left[ A_{zz} + A_\Delta \cos 2\varphi \right]
\]

\[
+ \frac{1}{2} P_t Q \left[ C_{zz,y} - \left( C_{xy,x} - \frac{1}{2} C_{\Delta,y} \right) \right] \cos \varphi
\]

\[
+ \left( \frac{1}{2} C_{yy,x} + \frac{1}{2} C_{\Delta,y} \right) \cos 3\varphi
\]

and choosing the deuteron spin alignment axis along the beam direction \( \beta_P = 0 \), leads to

\[
\frac{\sigma}{\sigma_0} = 1 + QA_y^p \cos \varphi + \frac{1}{2} P_t A^p_y \cos \varphi
\]

\[
+ \frac{1}{2} P_t Q C_{zz,y} \sin \varphi + \frac{1}{2} P_{t,t} Q C_{\Delta,y} \cos \varphi
\]

During the course of the experiment, the values of \( Q \), \( P_t \) and \( P_{t,t} \) can be made positive, negative or zero. This is used to separate terms with vector and tensor polarization, and terms that contain only the beam or the target polarization (analyzing powers), or both (spin correlation coefficients). The remaining decomposition makes use of the known azimuthal dependence of the cross section. It should be pointed out that the actual results of the experiment are the factors associated with the trigonometric functions in Eqs. (6)–(8). In some cases these are linear combinations of spin observables. Inspecting Eqs. (6)–(8) one sees that these combinations can be extracted by using the following observables: \( A_y^p, A_y^d, A_\Delta, A_{zz}, C_{xx,x}, C_{yy,y}, C_{zz,y}, C_{xy,x}, \) and \( C_{\Delta,y} \).

Other choices of the polarization directions (see Sec. IV A1) are treated in an analogous fashion. The resulting spin-dependent cross sections are given in the Appendix.

III. EXPERIMENTAL EQUIPMENT

A. Overview

This experiment makes use of a stored, polarized proton beam in the Indiana Cooler. The experiment is located in the A-region of the Cooler where the dispersion almost vanishes and the horizontal and vertical betatron functions are small [12], favoring the use of a narrow target cell. The target setup (Fig. 1a–1d) consists of an atomic beam source [13,14] that injects polarized deuterium atoms into a storage cell. The

![FIG. 1. Top view of the target and detector setup. The stored beam travels from right to left. Shown are the atomic beam source and the target cell (a–d), the detector system (e, j–m), and the guide field (i, g) and compensating (h) coils. An additional 6.4 mm thick scintillator detector (n) is not used in this experiment. Also shown are two beam position monitors (f).](image)

proton and the deuteron from elastic scattering are detected in coincidence by a detector system consisting of scintillators, wire chambers (j–m) and recoil detector array (e) surrounding the target cell.

B. Polarized proton beam

1. Beam properties

Protons are produced by a polarized ion source, accumulated in the injector synchrotron and then injected into the Cooler. About ten transfers at 1 Hz result in a typical stored current of about 500 \( \mu A \). The experiment was carried out at 135 and 200 MeV (the actual beam energies are known to \( +/−0.1 \) MeV and have been measured from the orbit frequency and ring circumference to be 135.0 and 203.3 MeV). The beam polarization is typically 0.75; its sign is reversed for every fill of the Cooler. Prior to each fill, the ring is completely emptied by resetting the main magnets. The betatron tunes of the Cooler are adjusted to avoid any depolarizing resonances; the polarization lifetime is then much longer than the beam lifetime.

2. Longitudinal beam polarization

In the absence of nonvertical fields, the stable spin direction in a circular accelerator is vertical. In order to obtain longitudinal beam polarization at the target, two "spin rotators" (longitudinal magnetic fields) are used [15]. One rotator is introduced by operating all solenoids in the cooling region with the same sign. These include the main solenoid that confines the electron beam and two solenoids, immediately upstream and downstream, which are normally used to compensate for the cooling solenoid field. Between the target and the cooling region, the beam is bent by 120°. The other rotator consists
of a superconducting solenoid halfway between the target and the cooling region (for details, refer to Ref. [15]). Data with longitudinal beam polarization were taken only at 135 MeV. At this energy, a longitudinal field integral of 0.56 T m for both rotators results in nearly longitudinal polarization with a small (about 0.08) vertical component.

Of the injected beam polarization, only the component that is parallel to the stable spin direction at the injection point is preserved. When the spin rotators are used, the stable spin direction at injection is tilted by about 45° towards the beam direction, i.e., no longer vertical. Thus, an additional solenoid was used in the transfer beam line between injector synchrotron and the Cooler to match the two directions.

C. Polarized deuteron target

1. Overview

The internal, polarized deuteron target is generated by injecting polarized atoms from an atomic beam source (ABS) into a storage cell. The target is placed in a weak guide field generated by a set of Helmholtz-like coils (Fig. 1(g), 1(i)). A set of similar coils with opposite field (h) practically eliminates a correlated position shift of the stored beam.

In the ABS, atoms from an 18 MHz dissociator (a) emerge through an aluminum nozzle that is kept at liquid nitrogen temperature. The atoms then pass through two stages, each consisting of a set of sextupole magnets (b) followed by a medium field transition unit (c). In the sextupole magnets the atoms are separated according to their electron polarization. In the first medium-field transition unit (MF1), transitions between hyperfine states are induced. After passing through the second set of sextupole magnets, which rejects one of the three hyperfine states present in the beam, another transition between hyperfine states may be induced in the second medium field transition unit (MF2).

For previous operation with hydrogen, the ABS had been equipped with a single, fixed-gradient medium-field transition unit located after the first set of sextupole magnets. Operation of the ABS in this configuration is extensively described elsewhere [14]. Here, we concentrate on the description of two new medium-field transition units (c) that were added for operation of the source with deuterium and were used for the first time by this experiment.

2. Medium-field transitions

A medium-field transition operates in magnetic fields of $0.1B_s$ to $0.2B_s$, where $B_s$ is the hyperfine interaction field of 50.7 mT for hydrogen and 11.7 mT for deuterium. In addition to a uniform (offset) field, a field gradient along the beam direction is required to satisfy the condition of adiabatic passage.

Multiple transitions can be made by adjusting the offset field so that the beam passes in sequence through field regions where the populations of different pairs of hyperfine states are interchanged at a given, fixed RF frequency [16].

In order to enable remote change between different operating modes of the target, two new transition units with variable gradient and variable offset field were installed. The linearity of the gradient field over the transition region as well as the homogeneity of the offset field were measured prior to installation of the units in the ABS.

For deuterium the gradient field is set to +0.2 mT/cm. The RF coil of each MF unit consists of a 70 mm long, 12-turn solenoid with 34 mm diameter, made from 1.6 mm diameter wire. For deuterium, the coils are operated at 60.5 MHz, and for hydrogen at 30 MHz. The transition units are water-cooled. The currents in the offset and gradient coils are remotely controlled. This makes it possible to quickly change between vector, positive tensor, and negative tensor polarization, while data are being acquired. Hall probes are used to monitor the field in the transition units.

3. Operation of the atomic beam source

After the first set of sextupoles the atomic beam consists of states $1+2+3$, where the states are labeled in order of decreasing energy in a nonzero magnetic field [17]. Up to three transitions are made sequentially in MF1. The gradient field is kept constant while the offset field is changed for different spin states. For a small offset field no transition is made in MF1. When the offset field is increased, the atoms undergo an $3 \rightarrow 4$ transition.

When the field is further increased the atoms pass through the $3 \rightarrow 4$ transition followed by the $2 \rightarrow 3$ transition. If the offset field is increased even further, the atoms undergo the $3 \rightarrow 4, 2 \rightarrow 3$, and $1 \rightarrow 2$ transitions sequentially. The second set of sextupoles eliminates state 4, so that one is left with states $1 + 2 + 3, 1 + 2, 1 + 3$ or $2 + 3$ depending on whether none, one, two, or three transitions are made in MF1. The corresponding maximum nuclear polarizations of the atomic beam, before entering MF2, are $(P_\zeta, P_{\zeta\zeta}) = (+1/3, -1/3), (P_\zeta, P_{\zeta\zeta}) = (+2/3, 0), (P_\zeta, P_{\zeta\zeta}) = (+1/3, 0)$ and $(P_\zeta, P_{\zeta\zeta}) = (0, -1)$. MF2 is only needed to produce positive tensor polarization. Then, its parameters are set such that atoms in states 1 and 3 with polarizations $(P_\zeta, P_{\zeta\zeta}) = (+1/3, 0)$ undergo the $3 \rightarrow 4$ transition. Consequently, after passing through MF2 the atomic beam contains states 1 and 4 with polarization $(P_\zeta, P_{\zeta\zeta}) = (0, +1)$.

4. Target cell

The target cell (Fig. 1(d)) is a 27 cm long tube of 12 mm diameter made from 0.05 mm thick aluminum, through which the stored beam travels, very similar to a design used earlier [18]. The cell is coated with Teflon in order to minimize depolarization by wall collisions [19]. The atomic beam from the ABS enters through a feed tube attached to side of the cell. The length of the cell between the feed tube and the downstream end is 12.5 cm; the upstream part is 14.5 cm long. The cell is supported at the intake of the feed tube (away from the beam), minimizing obstructions in the path of the scattered particles. Routinely, the target thickness is about $10^{11}$ atoms/cm$^2$.

The target cell is centered within an array of Helmholtz-like coils that provide horizontal, vertical and longitudinal guide fields of about 0.3 mT for alignment of the target polarization [13,20]. Certain polarization observables require that the angle
of the spin alignment axis is at $\beta = 45^\circ$ with respect to the beam. This is achieved by simultaneously exciting either the vertical and longitudinal coils, or the horizontal and longitudinal coils.

5. Spin exchange

The measured values for both, vector and tensor, target polarizations were about 0.45. This means that the tensor polarization is less than half and the vector polarization only about 70% of the theoretical maxima (1.0 and 2/3, respectively). Some decrease from the maximum values can be expected from wall depolarization, incomplete rejection of unwanted states by the sextupoles and an inefficiency of the transition units.

However, in a dedicated measurement [21] we also found that the tensor polarization decreases with increasing target thickness, while, at the same time, the vector polarization shows no such dependence. This behavior is consistent with the loss of polarization due to spin exchange between the deuterium atoms in the cell. A model calculation of the effect of spin exchange [22] explains the observed tensor polarization in a weak magnetic field as a function of target density.

D. Unpolarized target

The procedure to calibrate the beam polarization (see Sec. VB), calls for an unpolarized, mixed hydrogen and deuterium target. To this aim, an $^{1}\text{H}_{2}/^{2}\text{H}_{2}$ gas mixture is prepared by filling an empty cylinder with approximately equal parts of hydrogen and deuterium (one does not have to know the exact mixing ratio for the calibration). The gas mixture is admitted to the cell through a thin (1 mm diameter) Teflon hose, connected to a nipple at the center of the cell at a rate comparable to the flux of atoms from the ABS.

E. Detector system

1. Overview

The outgoing proton and deuteron from $p + d$ elastic scattering are detected in coincidence. The detector setup is shown in Fig. 1. Most of the components of the detector have been used previously and are described in detail in Ref. [18].

2. Forward detector

The forward going particle is detected in a stack consisting of a $\Delta E$ ("F") detector (Fig. 1(j)), two wire chambers (k,l) with two wire planes each, and a stopping ("K") detector (m).

The F-detector is made from organic scintillator material, segmented into an upper and a lower half. Its initial thickness of 1.5 mm has been increased to 6.4 mm during the course of the experiment. The thicker detector improves the mass resolution for particle identification. The two wire chambers are positioned 22.4 cm and 30.2 cm from the target center and have a wire spacing of 3.2 mm and 6.4 mm, respectively. The K-detector is made from 15.2 cm thick scintillator, segmented into four quadrants. The forward detector system covers the laboratory polar angles between $10^\circ$ and $45^\circ$.

3. Recoil detector

The recoil particle is detected in a so-called silicon barrel (Fig. 1(e)) that consists of an array of 18 silicon strip detectors [23] surrounding the target cell. Figure 2 shows the silicon barrel with the target cell in its center. The strips are oriented in such a way that they measure the azimuth of the recoil with a resolution of $2^\circ$. The silicon detectors yield an energy measurement from the back plane and a logic signal for each strip on the front plane. Energy and time are read out for each individual detector, but the strips at the same azimuth for a group of three detectors along the beam are electrically connected to reduce the number of electronics channels. The detector with the hit is identified from the energy signal. The silicon detectors are calibrated periodically using an array of six low-level ($n\text{Ci}$) $^{241}\text{Am}$ sources, mounted at the upstream end of the silicon barrel. Each source is positioned to illuminate one of the six sides of the barrel.

The active area of each detector is $4 \times 6 \text{ cm}^2$. The downstream ring consists of six 500 $\mu\text{m}$ thick detectors while all other detectors are 1000 $\mu\text{m}$ thick. The detectors are operated at full depletion and cooled to about $0^\circ\text{C}$.

It has been found that exposure to atomic deuterium or hydrogen has a detrimental effect on silicon detectors. Even a short exposure (30 min) to ambient atomic deuterium causes an increase in leakage current that renders the detectors useless for data acquisition. To prevent atomic deuterium that is leaking from pinholes in the cell from reaching the detectors, the target cell is placed in a bag made from thin Kapton. In addition, copper recombination baffles are placed around the feed tube and at the ends of the barrel. On a copper surface, atoms recombine into harmless molecular deuterium. In this way, the effect of atomic deuterium can be reduced to manageable proportions. Fortunately, the effect of atomic deuterium on the detectors is reversible. Thus, while no longer exposed to atomic deuterium, i.e., between runs, the detectors recovered.
IV. MEASUREMENT

A. Cycle time scenarios

1. Definitions, parameters varied

A “cycle” is the time between fills of the Cooler with beam. Proton beam of opposite polarization is injected for alternating cycles. After the fill, the experiment is enabled for data taking. The operating parameters (guide fields and transition units) of the target are varied during the cycle in order to acquire data with different target polarizations, but with the same stored beam. This is invaluable in minimizing systematic errors.

The guide field that determines the spin alignment axis of the deuteron target is changed in 2 s intervals. The normal sequence includes the six directions left (+x), right (−x), down (−y), up (+y), along (+z) and opposite (−z) to the beam axis. We call this a “sub-cycle”. Note that a sign change of the guide field affects the vector, but not the tensor polarization.

Vector or tensor polarization of the target is selected by enabling different sets of transitions (Sec. III C3) by remotely changing the offset field in the transition units, while keeping the gradient field constant. To overcome the effects of hysteresis, the transition units are de-gaussed before any change. This is accomplished by applying a 2 Hz alternating current with exponentially decreasing amplitude to all transition-unit coils. De-gaussing takes about 5 s (see Fig. 3). In the following we describe the three cycle-time scenarios used in this experiment.

2. Scenario V90

In scenario V90 the beam polarization is vertical. The target guide field is along the \(x\) or \(y\) axis (\(\beta_P = 90^\circ\)), or the \(z\) axis (\(\beta_P = 0^\circ\)). Within each cycle, the state of the atomic beam source is set to positive tensor polarization for two normal subcycles, to vector polarization for two subcycles, and finally to negative tensor polarization for three subcycles. Negative tensor is measured longer to approximately compensate for the loss in intensity due to the finite beam lifetime. Note, that both signs of vector polarization are available because the guide field changes sign during the subcycle.

Figure 3 shows three selected quantities measured during a V90 cycle. The top panel illustrates the beam current in the ring. The current in the offset field coil in transition unit MF1 is shown in the middle panel. One can see the three current plateaus (positive tensor, vector, negative tensor), each preceded by the de-gaussing of the coil. The event rate during data taking is depicted in the bottom panel. During de-gaussing, no transitions are made, admitting an additional sub-state to the target cell; thus, the target thickness and therefore the event rate increase during de-gaussing. A total of 5662 (7737) V90 cycles were acquired at 135 (200) MeV.

3. Scenario V45

The purpose of scenario V45 is to measure observables that require a deuteron spin alignment axis that is not along the axes of the coordinate frame. To this aim, a subcycle is used for the guide fields in which two sets of coils are energized simultaneously, the corresponding magnetic field directions adding vectorially. This special subcycle consists of the eight states (\(+x, +z\)), (\(+x, -z\)), (\(-x, +z\)), (\(-x, -z\)), (\(+y, +z\)), (\(+y, -z\)), (\(-y, +z\)), and (\(-y, -z\)). This corresponds to orientations of the deuteron spin alignment axis at angles \(\beta_P = 45^\circ\) or \(135^\circ\), either in the horizontal or the vertical plane. Again, these states are changed every 2 s. The atomic beam source is set in turn to positive polarization for two special subcycles and negative tensor polarization for three subcycles. Vector polarization is not used in scenario V45. The beam polarization is also vertical. A total of 2317 (1873) V45 cycles were acquired at 135 (200) MeV.

4. Scenario L90

The purpose of scenario L90 is to measure some observables that require longitudinal beam polarization (see Table I). During the whole cycle the target is vector-polarized, and a normal subcycle is used as in scenario V90. Scenario L90 is used only at 135 MeV (a series of power outages is responsible for the lack of data at the higher energy). A total of 1905 L90 cycles were acquired.

B. Event sorting

The goal of event sorting is to select \(p + d\) elastic scattering events using the signals generated by the detectors. The condition that triggers the readout of the entire detector is a coincidence between the upper half of the K-detector and the lower half of the silicon barrel, or vice versa.

For each event, the angles of the forward prong (\(10^\circ < \theta_{lab} < 45^\circ\), \(0^\circ < \varphi < 360^\circ\)) are determined from the wire chambers. Normally there is one hit in each of the four wire chamber planes, however, events with one plane missing or with two hits in one or two planes can be reconstructed and

FIG. 3. Stored beam current, the current in the MF1 offset coil, and the event rate during data taking during a scenario-V90 cycle. The cycle length is 140 s. The increases in event rate are due to the thicker target during the de-gaussing of the transition units.
are also used. The angular resolutions estimated from the wire spacing are $\delta \theta_{lab} = 2.2^\circ$ and $\delta \phi = 2.6^\circ$.

The gains of all scintillator tubes are corrected in software for shifts due to different guide fields in order to eliminate spin dependence of the detector performance. Also corrected are the position dependence of the light collection efficiency and the time response of the F- and the K-d Detectors. For more details, see Ref. [18].

The forward particle can be either a proton or a deuteron. At 135 MeV incident energy both particles stop in the K-detector, while at 200 MeV only the deuteron is stopped. Particle identification makes use of the correlation between the deposited energies in the F- and the K-detector (Fig.4A), as well as the correlation between F-K time-of-flight and the deposited energy in the K-detector (B). To further discriminate against background from breakup events, additional gates are placed on the correlation between the scattering angle and energy deposited in the K-detector (C), consistent with elastic scattering kinematics, and the correlation between energy deposited in the silicon detector and the scattering angle of the forward prong (D).

The silicon detectors measure the azimuth of the recoil with a resolution of $2^\circ$. Events where a single strip or a pair of adjacent strips fires are accepted in the analysis. This determines the azimuth of the recoil, and thus the difference $\Delta \phi$ between the two prongs. Elastic scattering events, being coplanar, are required to have $\Delta \phi$ between $175^\circ$ and $185^\circ$.

The center-of-mass-angle $\theta$, calculated from the forward lab angle, is sorted into $4^\circ$ wide bins, and the azimuth $\phi$ into $12^\circ$ bins. After applying all software conditions, two-dimensional ($\theta$ versus $\phi$) arrays of yields are generated for each spin state, including all combinations of two signs of the beam polarization, target vector, positive tensor or negative tensor, and six (scenarios V90, L90) or eight (scenario V45) guide field directions. A software gate on the cycle number versus cycle time is used to eliminate incomplete subcycles in order to reduce spin-dependent luminosity corrections.

FIG. 4. Identification of elastic scattering events at 135 MeV. Since the cross section for the two cases is very different, the contour values have been adjusted separately. Panels A–C show the energy in the F detector, the time-of-flight between the F and the K detector and the angle of the forward prong versus the energy deposited in the K detector (in arbitrary units). The forward angle versus the energy of the recoil is shown in panel D. The loci corresponding to a forward-going proton or deuteron are labeled accordingly.
C. Background

One expects that unwanted background events arise mainly from $p + d$ breakup. In order to assess the effect of background on the spin observables, we study the distribution of the difference $\Delta \varphi$ between the azimuths of the forward and recoil particle. Figure 5 shows this distribution after all other cuts have been applied. One sees that the coplanar peak at 180° from elastic scattering is superimposed on a wider distribution, which we associate with background. For good events, $\Delta \varphi$ is required to fall between 175° and 185°. In order to generate a background-enriched event sample, we instead select the wings with 50° < $\Delta \varphi$ < 150° and 210° < $\Delta \varphi$ < 310°, and repeat the process of event sorting with the same conditions as for good events, except for the coplanarity requirement. From the resulting yields we then deduce background-enriched observables.

The amount of background (5%–10%) under the $\Delta \varphi$ peak is determined from a smooth approximation of the wings (solid line in Fig. 5). Assuming that the observables associated with the background under the peak are the same as for the background in the wings, it is straightforward to calculate a background correction for the good data. This is done for all $\theta$ bins separately. We find that these corrections for all observables at all angles are smaller than the statistical errors in all cases, reflecting the fact that the observables from events in the peak or in the wings are very similar. Thus, it seems that the event conditions discriminate rather well against $p + d$ breakup, and that the events in the $\Delta \varphi$ wings are not background at all, but real events in the tail of the angular resolution.

We conclude that corrections due to background are negligible. This conclusion is supported by an analysis of the cross section, discussed in Sec. V D.

D. Corrections

1. Geometric corrections

The wire chambers define the coordinate frame of the experiment. Their positions have been surveyed optically prior to the experiment. The beam position, which may vary for different setups of the Cooler ring, can be extracted from the distribution of the event vertex positions. The original wire chamber coordinates are then offset such that the beam coincides with the $z$ axis. The magnitude of the offset was always less than 1.5 mm.

The scattering angle is determined from the intercept of the forward track with the two wire chambers. The distance between the chambers affects the absolute value of this angle. A small correction to the wire chamber positions is applied such that the zero transitions of the vector analyzing power at 135 MeV [3] at forward and backward angles are reproduced.

With the wire chamber offsets known, the positions of the silicon detectors are determined. For each silicon detector three parameters are adjusted, namely the $x$ and $y$ coordinates of the center of strip 1 and an angle of rotation about the strip direction. These parameters remained the same throughout the experiment, unless a detector was replaced. In addition, overall $x$ and $y$ offsets of the entire barrel are determined to account for shifts of the beam position (usually accompanying an energy change) by requiring that the difference in azimuth, $\Delta \varphi$, between the forward and the backward prongs peaks at 180°.

2. Spin-dependent deadtime

In the case of longitudinal beam polarization the trigger rate may depend on the alignment of beam and target spin, which may translate into a spin-dependent deadtime. When the deadtime of the acquisition system, determined from the ratio of triggers issued and processed, is sorted according to spin states, a small dependence of the deadtime on the relative alignment of beam and target spin is found. Correcting the measured yields accordingly results in a small offset (0.026) to $C_{z,z}$, which is measured only at 135 MeV. All other observables are unaffected by deadtime.

V. DATA ANALYSIS

A. Extraction of observables from spin-sorted yields

1. Spin-dependent yields

Throughout this experiment, the proton beam polarization is either vertical or longitudinal and its sign is alternated every cycle. In addition, the target polarization (vector or tensor, guide field direction) is varied, during the cycle, according to

FIG. 5. Distribution of $\Delta \varphi$, the difference between the azimuth of the forward and the recoil particle. The peak at 180° is due to (coplanar) elastic scattering. Gates used for real event and background identification are indicated by the solid and dashed lines, respectively. The effect of the background (solid line) on the data is discussed in Sec. IV C.
three different scenarios (Sec. IV A1). For each combination of the beam and target parameters, the event sorting (Sec. IV B) results in yields \( Y \) (or, number of events), stored in an array as a function of \( \theta \) (4\(^o\) bins) and \( \phi \) (12\(^o\) bins).

2. Extracting observables

We make the following assumptions:

(i) The magnitude of the target polarization does not depend on the direction of the guide field. This has been verified to a high degree of precision (±0.005) in previous measurements with this apparatus [13]. For guide fields of opposite sign, the vector polarization has opposite sign, but the tensor polarization stays the same.

(ii) The integrated luminosity in two target states of opposite sign of the target field is the same. A possible difference that arises from the decrease of the beam intensity by about 0.1% per second is negligible.

(iii) The ratio of the luminosities acquired with positive and negative tensor target polarization is the same for both signs of the beam polarization.

(iv) When the target is vector-polarized, the tensor polarization vanishes (verified during commissioning of the transition units). The converse admixture of vector polarization to a tensor target, is of no concern since in the analysis of tensor terms, vector terms cancel because of the changing sign of the guide field.

We do not assume that the magnitudes of opposite-sign beam polarization and of opposite-sign target tensor polarization are the same, or that data with equal integrated luminosity have been acquired with opposite sign of beam and target tensor polarization, since in the present experiment this is not strictly the case. However, we start with the concept of an ideal experiment, where these conditions would also be fulfilled, and introduce departures from an ideal experiment as corrections.

3. Asymmetries

We select four yields, \( Y_{++}, Y_{+-}, Y_{-+}, Y_{--} \), where the first sign refers to the sign of the beam polarization, and the second to the sign of the target polarization. This can be done either for the vector or the tensor target. From the four yields we form the following three ratios, henceforth called asymmetries:

\[
R_Q = \frac{(Y_{++} + Y_{+-}) - (Y_{-+} + Y_{--})}{Y_{++} + Y_{+-} + Y_{-+} + Y_{--}}, \quad (9)
\]

\[
R_P = \frac{(Y_{++} + Y_{+-}) - (Y_{-+} + Y_{--})}{Y_{++} + Y_{+-} + Y_{-+} + Y_{--}}, \quad (10)
\]

\[
R_{QP} = \frac{(Y_{++} + Y_{+-}) - (Y_{-+} + Y_{--})}{Y_{++} + Y_{+-} + Y_{-+} + Y_{--}}. \quad (11)
\]

For an ideal experiment, \( R_Q \) only depends on the beam polarization, \( R_P \) only on the target polarization, while the correlation asymmetry \( R_{QP} \) depends on both. In these ratios, the detector efficiency cancels, and thus azimuthal variations in efficiency disappear. Like the yields, the asymmetries \( R \) are functions of \( \theta \) and \( \phi \).

In scenario V90, there are the three guide-field directions, \( B_x, B_y, \) and \( B_z \) (sideways, vertical and longitudinal), and data are taken with a vector or a tensor target. Thus, there are 18 asymmetries. An example of the \( \varphi \)-dependences of these asymmetries is shown in Fig. 6. These \( \varphi \) distributions form the basis for the extraction of the observables.

It is straightforward to express the asymmetries in terms of the observables by inserting the expressions for the polarized cross section into Eqs. (9)–(11). The beam asymmetry is independent of the target state and given by

\[
R_Q = \frac{Q A_x^p \cos \varphi}{A_y^p \sin \varphi}. \quad (12)
\]

The target asymmetries \( R_P \) and the correlation asymmetries \( R_{QP} \) depend on the direction of the guide field (\( x, y, \) and \( z \), indicated by a superscript) and on whether the target is vector \( (P_\zeta^\nu) \) or tensor \( (P_\zeta^\tau) \) polarized. The target asymmetries are then given by

\[
R^\nu_{P_\zeta} = \frac{1}{2} P_\zeta A_y^p \sin \varphi, \quad (13)
\]

\[
R^\nu_{P_\zeta} = \frac{1}{2} P_\zeta A_x^p \cos \varphi, \quad (14)
\]

\[
R^\nu_{P_\zeta} = \frac{1}{4} P_\zeta Q A_{zz} - A_\Delta \cos 2\varphi, \quad (15)
\]

\[
R^\nu_{P_\zeta} = \frac{1}{4} P_\zeta Q A_{zz} + A_\Delta \cos 2\varphi, \quad (16)
\]

\[
R^\nu_{P_\zeta} = \frac{1}{2} P_\zeta A_{zz}, \quad (17)
\]

and the correlation asymmetries by

\[
R^\tau_{P_\zeta Q} = \frac{1}{3} P_\zeta Q (C_{x,x} - C_{y,y}) \sin 2\varphi, \quad (18)
\]

\[
R^\tau_{P_\zeta Q} = \frac{1}{3} P_\zeta Q [(C_{x,x} + C_{y,y}) - (C_{x,-} - C_{y,-}) \cos 2\varphi], \quad (19)
\]

\[
R^\tau_{P_\zeta Q} = \frac{1}{3} P_\zeta Q C_{x,z} \sin \varphi, \quad (20)
\]

\[
R^\tau_{P_\zeta Q} = -\frac{1}{4} P_\zeta Q [(C_{zz} + C_{zy} + \frac{1}{2} C_{y,y}) \cos \varphi - (C_{zy} + \frac{1}{2} C_{x,x}) \cos 3\varphi], \quad (21)
\]

\[
R^\tau_{P_\zeta Q} = \frac{1}{4} P_\zeta Q [(C_{zy} - (C_{x,x} - \frac{1}{2} C_{y,y}) \cos \varphi + (C_{yy} + \frac{1}{2} C_{x,x}) \cos 3\varphi], \quad (22)
\]

\[
R^\tau_{P_\zeta Q} = \frac{1}{2} P_\zeta Q C_{zy} \cos \varphi. \quad (23)
\]

Comparison of these expressions with Fig. 6 shows that the expected \( \varphi \)-dependences are borne out nicely by the data. The values for the observables times the respective polarizations (henceforth called “asymmetry terms”) are then extracted from the yields by fitting simple trigonometric functions [Eqs. (12)–(23)] to the \( \varphi \)-dependence (solid curves in Fig. 6). This procedure is carried out for each polar angle bin. The primary measured quantities are thus these asymmetry terms. Note, that in some cases asymmetry terms are linear combination of observables.

The statistical errors are derived from the errors \( \delta Y^2 = Y \) of the yields in Eqs. (9)–(11) by standard error propagation, neglecting covariance terms. This is justified since the yields are the result of separate experiments and taken at interleaved, but different times. The same is true for the \( R \)’s in Eqs. (13)–(23), which are obtained with different states of the polarized
source or the target field. The asymmetry terms follow from a fit to $\varphi$ distributions where each bin corresponds to a different part of the detector.

4. Departure from an ideal experiment

Alignment of the polarization directions. The coordinate axes of the experiment are defined by the wire chambers, while the beam polarization direction is given by the spin closed orbit and the target polarization direction by the guide fields. Discrepancies between these three frames are taken into account by a shift $\delta\varphi$ of the azimuth scale. This shift is easily determined by comparing the data with the predicted $\varphi$ dependence. For the target orientation we find $\delta\varphi = -3^\circ$, while the beam orientation is shifted by $\delta\varphi = -6^\circ$ at 135 MeV and by $\delta\varphi = -3.5^\circ$ at 200 MeV. The error in determining the $\varphi$ offsets is $\pm 0.5^\circ$. A small correction term is introduced in the analysis that takes into account that the orientations of target and beam are slightly different.

Differences in polarization and luminosity of states of opposite polarization. Beam polarization of opposite sign is produced with different transition units in the ion source and it is not guaranteed that the two polarizations have the same magnitude. The imbalance $q$ (the difference divided by the sum) varies from run to run and is typically 10%. Similarly, target tensor polarization of opposite sign uses different transitions in the ABS. The imbalance $p$ in this case is 1%–2%. The relative luminosities with beam of opposite sign may also differ, but when averaged over many cycles, the corresponding imbalance $\mu$ is typically small (1%). The largest departure from an ideal experiment arises from the difference in luminosity with the tensor target states of opposite sign, occurring at different times in the cycle. There is systematic imbalance.
η of about 18%, consistent with the beam lifetime. All four imperfection parameters, q, p, μ, and, η can be deduced from the data. Once they are known, the yield equations are worked out including new terms that depend on these parameters. Ignoring higher-order terms, this leads to a system of linear equations between the nonideal (measured) asymmetries and their corresponding ideal values. The latter are deduced and used in the analysis described in the preceding section.

5. Results from different scenarios

So far, we have described the method of analysis for scenario V90. The same principle is used to deduce observables from runs under scenarios V45 and L90 (the corresponding cross sections are given in the appendix). Scenario V45 (Sec. IV A3) uses a deuteron spin alignment axis bisecting the x- and z-axes, or the y- and z-axes, and scenario L90 (Sec. IV A4) employs longitudinal beam polarization. Because longitudinal polarization is accompanied by a small vertical component, this measurement is also sensitive to some of the terms measured in scenario V90, albeit with much larger error. The asymmetry terms obtained from the three scenarios are listed in Table I. This list includes 15 of the 17 observables that can be measured with a polarized beam and target. Missing are the tensor correlation coefficients \( C_{yz} \) and \( C_{xy} \), which would have required a dedicated run with guide fields as in scenario V45, but with longitudinal beam polarization.

Often angular distributions of the same asymmetry term (polarization times observable) are obtained from different scenarios. In addition, data have been collected during five cycles. It has been experimentally verified that the target polarization is constant during the ramp [25]. For the calibration export only the forward angles, where the cross section is large, are used. The data at both energies are not necessarily the same, these measurements may differ by an overall factor. We have checked that multiple measurements of the same asymmetry term (from different scenarios or from different runs), after normalization, are consistent with each other. We have also verified that the relative normalizations obtained from the analyzing powers are consistent with the (dependent) normalizations of the correlation coefficients. Multiple measurements of the same term are then averaged, resulting in an angular distribution for each asymmetry term.

2. Vertical beam polarization at 135 and 200 MeV

At both energies a set of data is obtained with an unpolarized target, obtained by bleeding an \(^1\text{H}_2/\text{H}_2\) gas mixture into the target cell (Sec. III D). The mixing ratio is adjusted to yield approximately the same number of \( p + d \) and \( p + p \) events. In addition to the normal sorting conditions for \( p + d \) scattering events, a second set of conditions is used to select \( p + p \) scattering events. Thus, the \( p + d \) analyzing power \( A^p_x \) and the \( p + p \) analyzing power \( A_x(pp) \) are measured simultaneously, with the same beam. The values of \( A_x(pp) \) at the appropriate angles are obtained from the SAID phase shift solution SP03 [24]. Therefore, for this data sample, the beam polarization and consequently the \( p + d \) analyzing power \( A^p_x \) are known. This establishes a calibrated standard that can be used to deduce the beam polarization \( Q \) from any data set that contains the asymmetry term \( QA^p_x \).

The statistical error that arises from normalizing the \( p + p \) data to the phase shift solution is 0.9% at 135 MeV and 2.3% at 200 MeV.

3. Deuteron target polarization at 135 MeV

The vector and tensor analyzing powers for \( p + d \) scattering have been measured recently at RIKEN [3] with 270 MeV deuterons, corresponding to a proton beam energy of 135 MeV. To obtain the RIKEN values at the angles measured in this experiment, we interpolate using a spline fit. The error of the interpolated values is taken as the average of the errors of the nearest-angle RIKEN points.

Scaling our asymmetry term \( P_\zeta A^d_y \) to the RIKEN vector analyzing power \( A^d_y \) yields the target vector polarization \( P_\zeta \). After scaling, the two angular distributions are consistent. The statistical error of the normalization factor is 1.5%.

Scaling our asymmetry terms \( P_\zeta A_\Lambda, P_\zeta A_2, \) and \( P_\zeta A_3 \) simultaneously to the corresponding RIKEN data yields the target tensor polarization \( P_\zeta \). After scaling, the angular distributions for all three observables are consistent, with the exception of \( A_\Lambda \) at backward angles, which we thus exclude from the scaling procedure. The statistical error of the normalization factor is 1.9%.

4. Deuteron target polarization at 200 MeV

In order to transport the target polarization calibration from 135 to 200 MeV, the Cooler is set up to accelerate the beam during an experimental cycle, a technique that has been described previously [25]. At the beginning of the cycle, unpolarized proton beam is injected at 135 MeV, and data are taken with a vector- and tensor-polarized target for about 100 s. The energy of the stored beam is then ramped to 200 MeV and data taking continues until the end of the cycle. This scenario is repeated for every cycle. It has been experimentally verified that the target polarization is constant during the ramp [25]. Since the target analyzing powers at 135 MeV are known [3], such a measurement calibrates the analyzing powers at 200 MeV.

For the calibration export only the forward angles, where the cross section is large, are used. The data at both energies
are then scaled by the (common) target polarizations until they agree with the standard established at the lower energy. The statistical error of this normalization factor is 1.6% for the vector, and 2.4% for the tensor normalization. This results in calibrated deuteron analyzing powers at 200 MeV. The asymmetry terms of the main measurement at 200 MeV (at forward angles) are then scaled to the new standard. The error of this normalization is 1.2% for the vector, and 2.0% for the tensor normalization.

The combined normalization errors due to the target polarization at 200 MeV are then 2.0% for the vector, and 3.1% for the tensor part.

5. Longitudinal beam polarization

Data with longitudinal beam polarization have been obtained only at 135 MeV, and only with a vector-polarized target (scenario L90, Sec. IV A4).

The longitudinal beam polarization is determined from $p+p$ elastic scattering. Since the longitudinal analyzing power vanishes, spin correlation coefficients must be used and a polarized target is necessary. To this effect, the ABS experiment are shown as solid symbols in Figs. 7 and 8. The errors shown are statistical only. The errors included in the data are reduced to single observables. The final results of this measurement are obtained only at 135 MeV, and only with a vector-polarized target.

The measured asymmetry terms $QC_{z,z}(pp)$ and $QC_{z,z}(pp)$ for $p+p$ scattering are then scaled simultaneously to the corresponding values of the SAID phase shift solution SP03 at the appropriate angles. The scaling error is 1.4%. This establishes the longitudinal beam polarization.

The $p+p$ data are bracketed in time by $p+d$ data runs immediately before and after. The measured asymmetry term $QA_{p}^{y}$ from the $p+d$ runs is the same within error, thus the beam polarizations $Q$ for the $p+p$ and the $p+d$ runs are also the same. The target vector polarization for scenario L90 is obtained as described in Sec. V B3, with a normalization error of 1.7%. The measured vector correlation coefficients can then be evaluated.

C. Results

The normalization procedure described in the preceding section removes the polarizations from the asymmetry terms. At this stage, the terms containing more than one observable are reduced to single observables. The final results of this experiment are shown as solid symbols in Figs. 7 and 8. They are also available in numerical form from the authors upon request. The errors shown are statistical only. The corresponding normalization uncertainties are summarized in Table II.

The open symbols in Figs. 7 and 8 mark previous polarization measurements in $p+d$ elastic scattering at or near the two energies of this experiment. A fairly large number of proton analyzing power data ($A_{p}^{y}$) have been measured; they include Ref. [5] ($T_{p} = 135$ MeV, $31^\circ < \theta < 170^\circ$), Ref. [26] ($T_{p} = 198$ MeV, $80^\circ < \theta < 170^\circ$), Ref. [27] ($T_{p} = 120$, $200$ MeV, $75^\circ < \theta < 99^\circ$), Ref. [6] ($T_{p} = 135, 199$ MeV, $\theta = 94^\circ$), and Ref. [4] ($T_{p} = 190$ MeV, $30^\circ < \theta < 115^\circ$).

At $T_{p} = 135$ MeV, a comprehensive set of all four deuteron analyzing powers ($A_{d}^{y}, A_{d}, A_{zz}$ and $A_{xz}$), measured with a 270 MeV polarized deuteron beam, is reported in Ref. [1] ($57^\circ < \theta < 138^\circ$) and Refs. [2] and [3] ($10^\circ < \theta < 66^\circ, 117^\circ < \theta < 178^\circ$). At $T_{p} = 200$ MeV, an older measurement of the deuteron analyzing powers ($35^\circ < \theta < 135^\circ$) exists [28]. Finally, the deuteron analyzing power ($A_{d}^{y}$) and the only previous spin correlation data ($C_{y,y}$) have been measured with an optically pumped target at the Indiana Cooler [7] ($T_{p} = 200$ MeV, $68^\circ < \theta < 113^\circ$).

Our data agree well with previous measurements, with the exception of the RIKEN measurement of $A_{d}$ at 135 MeV near $\theta \sim 155^\circ$. Note that the normalization of the present data is independent of earlier measurements with the exception of the deuteron analyzing powers at 135 MeV [3] that were used to determine the target polarizations for the 135 MeV measurement.

D. Cross section

It is difficult to obtain a reliable figure for the absolute luminosity with an extended internal target and a stored beam, and thus a normalization for a cross section measurement. Nevertheless, it is still possible to extract a relative cross section, i.e., its angular dependence except for an unknown normalization factor. Agreement with existing data would then demonstrate that we understand our detector acceptance as a function of angle, and that any contributions from background are indeed negligible (Sec. IV C).

To establish the detector efficiency, a Monte Carlo simulation is used, which contains a detailed account of all detector elements, including the silicon barrel, and describes the interaction of the reaction products with the detector setup to the best of our knowledge. Required input parameters include the detector positions, thicknesses and resolutions, the dimensions of the target cell and the target gas distribution. Also included is the loss of detected deuterons due to reactions in the forward detector (based on the parameterized total deuteron breakup cross section [29,30]). The simulation code produces output with the same format as that of the actual events recorded during data acquisition; therefore it can be analyzed with exactly the same software.

Elastic scattering events at random angles are processed by the Monte Carlo code and reconstructed with the same conditions as real events. The ratio between the number of reconstructed and generated events then constitutes the $\theta$-dependent detector efficiency $\epsilon(\theta)$. The relative cross section is obtained by multiplying the measured yields by $\epsilon^{-1}(\theta)$. As
a crosscheck, the relative $pp$ elastic scattering cross section can be determined from the data set obtained with the $^4\text{He}/^2\text{H}_2$ gas mixture. It agrees well with the shape of the cross section predicted from the SAID phase shift solution SP03.

Our data at 135 MeV are shown as solid dots in Fig. 9. Two existing measurements by Ermisch et al. [31] (open circles) and Sekiguchi et al. [3] (stars) are in serious disagreement with each other in shape and magnitude. The shape of our cross section, in particular its forward/backward ratio, agrees well with the Ermisch data set, and is not compatible with the Sekiguchi measurement. We have thus normalized our cross section to the Ermisch data. In the past
it has been argued that the minimum of the cross section is sensitive to three-nucleon forces [32]. For this reason, we also show in Fig. 9(a) Faddeev calculation based on the CDBonn NN potential before (solid line) and after (dashed line) the inclusion of the Tucson-Melbourne three-nucleon force.

At 200 MeV (not shown) we have normalized our cross section to the data of Rohdjess et al. [33] at $\theta = 26^\circ$. The Rohdjess cross section is linked to $p + p$ scattering by the use of an $^3$H$^3$H gas target. With this normalization, our data are consistent at all angles with an older cross section measurement at 198 MeV [27].

We thus find that the shape of our cross section agrees well with existing data, without any correction for a background contribution. This supports our conclusion of Sec. IV C that background can be neglected.
Bonn potential. Both potentials are charge dependent (i.e., not the same for $p+p$ and $n+p$), and the parameters of both have been adjusted by comparing to the Nijmegen $NN$ phase shift analysis [37] at energies below 350 MeV.

The Faddeev calculations include the $3N$ partial wave states with total angular momenta of the two-nucleon subsystems up to $j_{\text{max}} = 5$, resulting in up to 142 partial-wave states at each $3N$ system total angular momentum and parity. Convergence of observables for energies up to 200 MeV has been checked by comparing calculations with $j_{\text{max}} = 5$ and $j_{\text{max}} = 6$. Faddeev calculations ignore the Coulomb interaction. However, at our energies we expect Coulomb effects to be negligible, except perhaps at small angles. This is supported by experiment [38]. Thus, we assume that observables in $n+d$ and $p+d$ scattering are the same. On the other hand, Faddeev calculations are nonrelativistic and use nonrelativistic $NN$ interactions. With increasing energy, relativistic effects become more important and may be responsible for some of the discrepancies between calculations and the data.

B. Comparison of two-nucleon force predictions with the data

Our measured analyzing powers and spin correlation coefficients at 135 and 200 MeV are shown as solid circles in Figs. 7 and 8. Open symbols indicate the results of previous experiments (Sec. VC). The solid and dashed lines show calculations with the CD-Bonn and the AV18 $NN$ potential, respectively.

The ability of the calculations to account for the general behavior of all observables at both energies is quite impressive, especially since they were carried out before the data became available, and thus are true predictions. The difference between predictions of two potentials is generally small, as would be expected for $NN$ potentials that have been adjusted to reproduce the $NN$ database.

Discrepancies between the calculations and the data are mostly confined to backward angles but may be sizeable down to $\theta = 40^\circ$, especially in the tensor analyzing powers. Even though relatively small, these discrepancies are the focus of the present research, since they represent the physics that is missing in the 2$N$ Faddeev calculations. The favored candidate for this physics is a three-nucleon force (3NF).

C. Inclusion of a three-nucleon force

Most present day theoretical models of the 3NF are based on the exchange of two mesons with an intermediate nucleon excited state. There are two basic approaches. The first restricts the intermediate state to a $\Delta$ resonance and uses an additional, phenomenological, spin and isospin independent short-range part. An example is the Urbana IX force (UIX) [39]. The second approach is based on a parametrization of the $\pi$-$N$ off-shell scattering amplitude and contains any intermediate state. A representative of the latter is the Tucson-Melbourne (TM) force [40]. Recently, The TM force has been criticized on the basis of chiral symmetry and a modified force (TM') has been constructed that avoids these difficulties [41,42].
FIG. 10. (Color online) Difference between the present data at 135 MeV and the Faddeev calculation with the CD-Bonn potential. The effect of including the old or the new Tucson-Melbourne 3NFs is shown by the solid lines (TM) and the dashed lines (TM'). The dotted lines show the difference between calculations with the AV18 and the CD-Bonn potentials, both without a 3NF.

All three forces mentioned above have been adopted for insertion into Faddeev calculations [35], including angular momenta of the 3N system up 13/2 [8]. All theoretical 3NFs contain adjustable parameters that are determined experimentally. In particular, the overall strength of the 3NF potential is adjusted by varying the cutoff parameter \( \Lambda \) of the \( \pi-N \) form factor until the \(^3\)H binding energy is reproduced. The adjusted cut-off parameter depends on the NN potential used [43].

D. Comparison of 3NF predictions with the data

The differences between our measurements and the Faddeev calculation with the CD-Bonn potential are plotted in Figs. 10 and 11, i.e., the calculation is the zero line. The effect of including the old (TM) or the new (TM') Tucson-Melbourne 3NFs is shown by the solid lines and the dashed lines, respectively. A comparison of these curves with the data is justified if calculations with different NN potentials.
FIG. 11. (Color online) Difference between the present data at 200 MeV and the Faddeev calculation with the CD-Bonn potential. Otherwise, the caption of Fig. 10 applies.

agree with each other. To illustrate this, the difference between calculations with the AV18 and the CD-Bonn potentials, both without a 3NF, is shown as a dotted line. This difference is indeed generally small, but there are many cases where the variation between the two potentials competes in size with the 3NF effects.

As can be seen from Figs. 10 and 11, the two 3NFs agree with each other for some observables and in some angular regions (e.g., in $A_{1y}^d$), but in numerous cases the predictions with the TM and the TM' 3NF are quite different. Both sometimes improve the agreement with the data (e.g., in $A_{1y}^d$), but equally often this is not the case. Thus, neither 3NF is a successful
representation of the discrepancies between the $p + d$ spin observables and Faddeev calculations without a 3NF.

In Fig. 12 we investigate the systematics of the performance of various 3NFs and underlying $NN$ potentials. Each panel shows the measured observables versus the scattering angle, thus each pixel corresponds to one of our 868 data points. A pixel is colored black if the inclusion of a 3NF improves the agreement with the data and gray if it does not. The top four panels are for 135 MeV, the lower four for 200 MeV. The left and right columns are for the CD-Bonn and the AV182 force, respectively. The effect of three different 3N forces is shown.
panels are for 135 MeV, the lower four for 200 MeV. The left column is with the CD-Bonn NN potential (TM or TM’), the right with the AV18 (TM or UIX). It is interesting to note that there are no systematic differences between different regions in scattering angle, different 2N potentials, or different 3NFs.

In summary, there is no indication that any of the 3NFs studied here consistently alleviates the discrepancies between the data and 2N Faddeev calculations, and thus represents the physics that is responsible for these discrepancies.

VII. CONCLUSIONS

We have measured all analyzing powers, and all but two spin correlation coefficients for \( p + d \) elastic scattering at 135 and 200 MeV. The experiment was motivated by the availability of computationally exact Faddeev calculations of these observables. These calculations are based on a given, phenomenological 2N potential.

The Faddeev calculations shown in this paper were carried out prior to this experiment. We find that the 2N calculations predict the general features of all observables impressively well. In other words, the absolute differences between data and the two-nucleon force calculations are relatively small, mostly confined to backward angles but in some cases sizeable down to \( \theta = 40^\circ \). Statistically, the discrepancies are relatively large owing to the high precision of the data. If the 2N input to the calculation is sufficiently well defined, such that it uniquely describes how nature would behave if there were only 2N forces, the differences between these calculations and the data are a manifestation of additional physics. Our measurement then would provide a testing ground for the spin dependence of this missing physics.

Many believe that the prime candidate for the missing physics is a three-nucleon force. It is possible to include theoretical models of three-nucleon potentials in the Faddeev calculations. We have investigated the ability of three different three-nucleon forces to account for the discrepancies between data and 2N calculations. We find that for some observables at some angles the inclusion of a 3NF improves the agreement with the data, but often the agreement also gets worse. When there is an improvement, it does not depend systematically on the scattering angle, or the energy, or the choice of a particular 3NF. We thus conclude that existing 3NFs are not successful in explaining the discrepancy between the spin observables presented here and the corresponding 2N calculations. Thus, recent claims that local improvements of the calculation resulting from inclusion of a 3NF constitute evidence for such a 3NF must be met with caution. For example, in Ref. [7], that claim is based on a (fortuitous) choice of a single observable \( (C_{\gamma}, \gamma) \) in a limited angular range (the data of Ref. [7] are in agreement with the present measurement, but the conclusion is not).

We have also resolved a serious discrepancy between two recent measurements of the differential cross section at 135 MeV (Sec. V D).

Note added in proof. The published version of this paper is the originally accepted version as of 21 July 2004.

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APPENDIX

In Sec. II B we have discussed the derivation of the spin-dependent cross section for scenario V90. Here we give the corresponding expressions that apply in case of the other two scenarios used (Sec. IV A).

For scenario L90, the beam polarization \( Q \) is longitudinal (\( \beta_Q = 0 \)). For sideways spin alignment axis \( \hat{S} \) (\( \beta_P = \pi/2, \Phi_P = 0 \)), we then obtain

\[
\sigma/\sigma_0 = 1 - \frac{1}{2} P_z A_{\delta} \sin \phi - \frac{1}{4} P_{z\xi} \left[ A_{zz} - A_\Delta \cos 2\phi \right]
+ \frac{3}{4} P_z QC_{x,z} \cos \phi - \frac{1}{2} P_{z\xi} QC_{xy,z} \sin 2\phi,
\]

with a vertical spin alignment axis (\( \beta_P = \pi/2, \Phi_P = \pi/2 \)),

\[
\sigma/\sigma_0 = 1 + \frac{1}{2} P_z A_{\delta} \cos \phi - \frac{1}{4} P_{z\xi} \left[ A_{zz} + A_\Delta \cos 2\phi \right]
+ \frac{3}{4} P_z QC_{x,z} \sin \phi + \frac{1}{2} P_{z\xi} QC_{xy,z} \sin 2\phi,
\]

and with a longitudinal spin alignment axis (\( \beta_P = 0 \)),

\[
\sigma/\sigma_0 = 1 + \frac{1}{2} P_z A_{zz} + \frac{1}{2} P_z QC_{zz}.
\]

For scenario V45, the beam polarization was vertical, and the longitudinal and one of the transverse guide fields was energized simultaneously. The subcycle covered all eight possible orientations of the spin alignment axis. When combining the transverse with the longitudinal field, the following four spin alignment axis directions result

\[
(\beta_P, \Phi_P) = \begin{pmatrix}
\left( \frac{\pi}{4}, 0 \right), \left( \frac{\pi}{4}, 0 \right), \\
\left( \frac{\pi}{4}, \pi \right), \left( \frac{\pi}{4}, \pi \right)
\end{pmatrix}.
\]

The corresponding four cross sections are the same except for the signs of the terms. The signs in the following equation are shown as matrices that correspond to the directions of
When combining the vertical with the longitudinal guide field, the following four spin alignment axis directions result:

\[
\frac{\beta_p}{\Phi_p} = \begin{pmatrix}
\frac{3\pi}{4}, & \frac{\pi}{4} \\
\frac{3\pi}{4}, & \frac{3\pi}{4}
\end{pmatrix}
\]  

and the corresponding four cross sections are

\[
\sigma/\sigma_0 = 1 \begin{pmatrix}
++ & ++ & ++ & ++ \\
+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
Q A^p_x \cos \phi & \begin{pmatrix}
Q C_{xx} & -Q C_{yy}
\end{pmatrix} & \begin{pmatrix}
Q C_{zz} & -Q C_{zz}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
Q A^d_y \sin \phi
\end{pmatrix}
\]

\[
\times \begin{pmatrix}
++ & ++ & ++ & ++ \\
+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
++ & ++ & ++ & ++ \\
+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta z} A_{zz} & \begin{pmatrix}
P_{\Delta z} C_{zz}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
\begin{pmatrix}
P_{\Delta y} A_{yy}
\end{pmatrix}
\end{pmatrix}
\]

\[
\times \begin{pmatrix}
++ & ++ & ++ & ++ \\
+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
++ & ++ & ++ & ++ \\
+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta y} A_{yy} \sin \phi & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix} & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
-Q_{xx} & Q_{yy}
\end{pmatrix}
\]

\[
\times \begin{pmatrix}
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-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
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-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta y} A_{yy} \sin \phi & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix} & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
-Q_{xx} & Q_{yy}
\end{pmatrix}
\]

\[
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-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
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-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta y} A_{yy} \sin \phi & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix} & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
-Q_{xx} & Q_{yy}
\end{pmatrix}
\]

\[
\times \begin{pmatrix}
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-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
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+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta y} A_{yy} \sin \phi & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix} & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
-Q_{xx} & Q_{yy}
\end{pmatrix}
\]

\[
\times \begin{pmatrix}
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-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
++ & ++ & ++ & ++ \\
+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta y} A_{yy} \sin \phi & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix} & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
-Q_{xx} & Q_{yy}
\end{pmatrix}
\]

\[
\times \begin{pmatrix}
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+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
++ & ++ & ++ & ++ \\
+- & ++ & ++ & ++ \\
-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta y} A_{yy} \sin \phi & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix} & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
-Q_{xx} & Q_{yy}
\end{pmatrix}
\]

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\times \begin{pmatrix}
++ & ++ & ++ & ++ \\
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-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
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-+ & ++ & ++ & ++ \\
-+ & ++ & ++ & ++
\end{pmatrix} \begin{pmatrix}
P_{\Delta y} A_{yy} \sin \phi & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix} & \begin{pmatrix}
P_{\Delta y} C_{yy}
\end{pmatrix}
\end{pmatrix} \begin{pmatrix}
-Q_{xx} & Q_{yy}
\end{pmatrix}
\]


Vector and tensor polarization lifetimes for a stored deuteron beam

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The time dependence of the vector and tensor polarization of a 270 MeV stored deuteron beam was measured near a depolarizing resonance, which was induced by an oscillating, longitudinal magnetic field. The distance to the resonance was varied by changing the oscillation frequency. The measured ratio of the polarization lifetimes is $\tau_{\text{vector}}/\tau_{\text{tensor}} = 1.9 \pm 0.2$. Assuming that the effect of the resonance is to induce transitions between magnetic substates $m_j$, we find that the transition rate between neighboring states (+1 and 0 or −1 and 0) is four times higher than between the states with $m_j = +1$ and −1.

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I. INTRODUCTION

Since polarized beams were first cooled in a storage ring, the question of how long the stored beam remains polarized has been of considerable interest [1]. It was found that usually the polarization lifetime is large compared to the beam lifetime; however, as a depolarizing resonance is approached, the polarization lifetime decreases sharply. Previously, the polarization lifetime has been studied for stored proton beams as a function of the distance from intrinsic as well as induced depolarizing resonances [2,3].

With the recent interest in deuteron beams for nuclear physics experiments (e.g., at COSY [4]), the behavior of the polarization of stored spin-1 particles has become important. In particular, the question arises whether, in the vicinity of a depolarizing resonance, the lifetimes of vector and tensor polarization differ.

In a recent experiment at the Indiana Cooler, we measured the vector and tensor polarization of a 270 MeV stored deuteron beam before and after adiabatically crossing an induced depolarizing resonance [5]. For a slow crossing rate, it was found that the sign of the vector polarization is reversed (flipped) while the sign of the tensor polarization is unchanged. At a slower crossing rate, corresponding to an incomplete rotation of the spin closed orbit, the observed vector polarization was zero and the tensor polarization reversed sign but was reduced by a factor of 2 ([5], Fig. 3, and [6]).

Since vector and tensor polarization behave so differently when crossing a depolarizing resonance, it can be expected that the polarization lifetimes also differ. The present article describes the measurement of the vector and tensor polarization lifetime near an induced depolarizing resonance at 270 MeV.

II. PREPARATION OF THE BEAM

Polarized deuterons from a pulsed atomic beam source (CIPIOS) were accelerated to 100 MeV in the Indiana Cooler injector synchrotron (CIS) and then transferred to the Cooler where they were further accelerated to 270 MeV. The betatron tunes of the Cooler were adjusted to avoid the $G\gamma + 5 = \nu_t$ intrinsic resonance during acceleration. The fractional betatron tunes measured after acceleration were $Q_x = 0.188$ and $Q_y = 0.214$.

In order to minimize systematic errors, the experiment was carried out with beams of different combinations of vector and tensor polarization (called “beam states” in the following), as well as with an unpolarized beam. The four beam states were injected into the Cooler in sequence, using a different state for each new cycle. Resetting the Cooler magnets prior to injection ensured that no beam from the previous cycle remained in the ring. In the following we describe briefly how the polarized beam was prepared. More information about the polarized source (CIPIOS) can be found in [7].

The process starts with the dissociation of deuterium atoms. These atoms emerge through a cooled nozzle and then encounter two stages consisting of a set of permanent sextupole magnets followed by a rf transition unit. These devices affect the populations of the six hyperfine states of the deuterium atom. In the following, we use the conventional numbering of these states in order of decreasing energy in a nonzero magnetic field [8]. The sextupole magnets separate the atoms according to their electron polarization, i.e., states 1,2,3 are focused and states 4,5,6 are defocused. The rf transitions cause the inversion of the populations of pairs of states. The first stage is equipped with a medium field transition (MFT) and the second stage with a weak field transition (WFT), immediately followed by a strong field transition (SFT). Table I illustrates how the four different beam states (column 1) are prepared. Columns 2 through 6, respectively, list the hyperfine states that remain after the first sextupole, the states that are exchanged in the MFT, the states that remain after the second sextupole, and the transitions made in the WFT and the SFT. Finally, the hyperfine states that are populated when the beam emerges from the source are listed in column 7. The atoms are ionized in a strong field (where the spins of the electrons and the nuclei are decoupled). This leads to the nominal vector ($P_z$) and tensor ($P_{zz}$) polarization shown in columns 8 and 9. For a number of

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reasons, the actual polarizations are less than their nominal values: after acceleration in the Cooler the measured vector polarization (beam states 1 and 2) typically was \( \pm 0.6 \), the tensor polarization in beam states 1–3 was \( \pm 0.8 \), and the tensor polarization in beam state 4 was \( \pm 1.6 \).

III. SPIN MOTION

In a ring with only vertical fields the magnetic moments of the beam particles precess about the vertical direction. The eigenvector of the precession of the magnetic moment in a single revolution around the ring is called the spin closed orbit. The precession frequency of the magnetic moment in the particle rest frame, the spin tune, is given by

\[ n_s = G \gamma \]

where \( G \) is the anomalous magnetic moment \( (G_{\text{deuteron}} = -0.1430) \) and \( \gamma \) is the usual relativistic Lorentz factor. A depolarizing resonance occurs when nonvertical magnetic fields are encountered in resonance with the spin tune. A depolarizing resonance can be induced by a longitudinal oscillating magnetic field. On resonance, the frequency of the oscillating field is given by

\[ f_r = f_c (k \pm n_s) \]

where \( f_c \) is the circulation frequency of the stored beam and \( k \) is an integer. Far away from the resonance the spin closed orbit is vertical. As the resonance is approached the spin closed orbit tilts and precesses around the vertical direction. On resonance, the tilt is \( \pi/2 \) and the spin closed orbit lies in the ring plane. Depolarization occurs because near a resonance the spin motion becomes sensitive to the betatron amplitude of individual particles and is no longer coherent.

IV. EXPERIMENT

A. Setup

The experiment was performed at the Indiana Cooler. A layout of the setup is shown in Fig. 1. The target consists of an open ended tube, 1.2 cm in diameter and 27 cm long, made from 0.05 mm thick aluminum. Hydrogen gas is fed into the center of this target cell, resulting in a target thickness of \( \sim 5 \times 10^{13} \) atoms/cm\(^2\). The outgoing proton and deuteron from \( pd \) elastic scattering were detected in coincidence in a stack consisting of a 0.635 cm thick \( \Delta E \) scintillator \((F)\), two wire chambers \((WC1, WC2)\), and a 15.24 cm thick stopping scintillator \((K)\) [9].

B. Event selection

Elastically scattered events were distinguished from breakup events and from background originating from the cell wall by the following software conditions. Two opposite elements of the stopping detector were required to have a signal. A least-\( \chi^2 \) fit to the wire chamber hit pattern had to be consistent with two prongs having a common vertex. This vertex was restricted to the target position. The correlation between the polar angles of the two prongs had to be consistent with \( pd \) elastic scattering and the azimuthal angle difference was required to be within \( \pm 10^\circ \) of 180°. In addition, the energy deposited in the stopping scintillator for both prongs and the angles measured by the wire chambers had to be consistent with \( pd \) scattering. For the analysis that follows, events in the center-of-mass angle range \( 90^\circ \leq \theta_{\text{c.m.}} \leq 130^\circ \) were used.

C. Polarization measurement

A description of the formalism to describe spin-1 polarization and of the method to measure vector and tensor analyzing powers with polarized deuterons can be found in the literature [10,11]. The spin-dependent cross section is given by

\[
\sigma = \sigma_0 \left[ 1 + 2 i T_{11} \text{Re}(i t_{11}) + T_{20} T_{20} + 2 T_{21} \text{Re}(t_{21}) + 2 T_{22} \text{Re}(t_{22}) \right].
\]

(4.1)

Here, \( \sigma_0 \) is the unpolarized cross section, \( i T_{11} \) and \( T_{2k} \) are

![Diagram of Indiana University Cyclotron Facility Cooler ring and experimental setup. The components are described in the text.](image-url)
the vector and tensor analyzing powers, and \( t_{11} \) and \( t_{2k} \) the corresponding beam moments, which are related in a known way to the polarizations \( P_z \) and \( P_{zz} \) of the beam. The beam moments depend on the orientation of the symmetry axis of the polarized ensemble (called the spin alignment axis) with respect to the scattering plane. In our case, with a vertical spin alignment axis, the \( t_{21} \) moment vanishes, \( t_{11} \) is proportional to \( \cos f \), where \( f \) is the azimuth of the scattering plane, \( t_{22} \) is proportional to \( \cos(2f) \), and \( t_{20} \) does not depend on \( f \). In the present experiment we make use of the \( f \) dependence of the observed yields to pick out the terms \( t_{11} iT_{11} \) and \( t_{22} T_{22} \) in Eq. \( \sim 4.1 \), which for beam states 1 and 2 are present simultaneously. Typical \( f \) distributions for the four beam states are shown in Fig. 2. Note that the offset of the \( f \) distribution is smaller for state 4, because the magnitude of \( t_{20} \) is about twice as large as for the other beam states. Each term in Eq. \( \sim 4.1 \) is the product of a beam moment and an analyzing power. The analyzing powers of \( pd \) scattering at \( 270 \) MeV are known \[12\]. Averaging over the \( \theta \) acceptance yields the effective analyzing powers \( iT_{11} = -0.31, T_{20} = -0.22, \) and \( T_{22} = -0.28 \). Thus, the moments \( i_{11} \) and \( t_{22} \) are deduced and the vector and tensor polarizations \( P_z \) and \( P_{zz} \) are obtained.

D. Induced resonance

The rf solenoid used to generate an oscillating magnetic field along the beam axis has been described elsewhere \[13\]. In this experiment, the rms value of the field integral was \( fB dl = 0.7 \) T mm when the solenoid was operated at full power. On resonance (see Sec. III) the solenoid frequency is given by \( f_{\text{sol}} = f_r(1 + \nu) \). The circulation frequency of the deuterons in the Cooler was \( f_r = 1 677 551 \) Hz. From \( f_r \) and the known circumference of the Cooler of \( 86.77 \pm 0.01 \) m \[14\], one obtains \( \gamma = 1.143 \) 88 and \( \nu_r = G \gamma = -0.163 56 \). The expected resonance frequency of the solenoid is then \( f_{\text{sol}} = 1 403 171 \pm 10 \) Hz. Experience shows that the calculated frequency is close to the actual resonance frequency, but an experimental determination is still necessary. In the first step, the resonance is located approximately by a method described in \[15\]. The precise determination of the resonance location requires a measurement of the beam polarization as a function of solenoid frequency. This was done as follows. After accelerating and cooling the stored beam, the rf solenoid was turned on by linearly ramping the amplitude from zero to full power within 50 ms. The solenoid was then operated for 1 s, after which the amplitude was ramped down, mirroring the turn-on procedure. This was followed by the polarization measurement. Figure 3 shows the vector polarization from beam states 1 and 2, the positive tensor polarization, averaged over beam states 1–3, and the negative tensor polarization from beam state 4 as a function of solenoid frequency. The curves in Fig. 3 are first- and second-order Lorentzian fits with widths of 75 \pm 4 and 101 \pm 4 Hz for the vector and tensor polarizations, respectively. The central resonance frequency is \( f_{\text{sol}} = 1 403 002 \pm 14 \) Hz.

V. Measurement

The polarization lifetime is measured using the following procedure. After the beam is injected, accelerated, and cooled, the solenoid is turned on and left on while data are taken. This is done with all four beam states and with the
unpolarized beam. While resetting the ring magnets and injecting beam for the next cycle, the solenoid is turned off. In order to improve statistics, data collected during ten Cooler cycles are added for each beam state.

It is obvious that by this method only polarization lifetimes that are comparable to the duration of a cycle (on the order of 100 s) are measurable. To achieve this, the distance $\Delta f_{\text{sol}}$ to the resonance is chosen accordingly. The experiment was carried out at the three values for $\Delta f_{\text{sol}}$: 0.38, 0.20, and 0.10 kHz. The duration of the cycle was adjusted for best coverage of the decay of the polarization. The data are then binned in time such that the data-taking period is divided into about 15 bins. For the longest lifetime this results in bins 20 s wide while for the shorter lifetimes the bins are 4 s wide. The beam polarization for each spin state is determined from the $f_d$ distributions associated with each time bin as described earlier. As with the resonance search described above, the vector polarization results from beam states 1 and 2, the positive tensor polarization is an average over beam states 1–3, and the negative tensor polarization is obtained from beam state 4. Figure 4 shows $P_z$ and $P_{zz}$ as a function of time for $\Delta f_{\text{sol}}=0.38$ kHz away from the resonance.

In order to determine the lifetime $\tau_p$ of the polarization an exponential function $P(t) = P_i e^{t/\tau_p}$ was fitted to the vector and the two tensor polarizations as shown by the lines in Fig. 4. Figure 5 shows the vector and tensor polarization lifetimes as a function of distance from the resonance. The lowest frequency ($\Delta f_{\text{sol}}=0.1$ kHz) is at approximately half the maximum of the resonance (see Fig. 3). As expected, the lifetimes for the two tensor polarizations are consistent with each other. Since systematic errors decrease with increasing magnitude of the polarization, the agreement between the two measured tensor polarization lifetimes that differ in magnitude by a factor of 2 suggests that systematic errors are negligible. In the following we average the two values.

It is clear from Fig. 5 that vector polarization lasts longer than tensor polarization. Figure 6 shows the polarization lifetime versus the vector polarization lifetime. The solid line represents the result of a least-$\chi^2$ fit of the points to a straight line going through zero. The ratio of the lifetimes resulting from this fit is $\tau_{\text{vector}} / \tau_{\text{tensor}} = 1.9\pm0.2$. The long-dashed line represents equal lifetimes while the short-dashed line represents a lifetime ratio of 3, which is the value one expects if transitions occur only between neighboring substates (Sec. VI and [6]).

VI. THEORETICAL INTERPRETATION AND CONCLUSIONS

The beam polarization can be described in terms of the fractional populations ($n_-, n_0$, and $n_+$) of the three mag-
netic substates of the deuteron along a quantization axis that is along the spin alignment axis which coincides with the spin closed orbit. With \( n_0 + n_+ + n_- = 1 \), we have \( P_z = n_+ - n_- \) and \( P_{zz} = 1 - 3n_0 \).

Let \( q_1 \) be the transition rate between neighboring substates (\( n_+ \) and \( n_0 \) or \( n_- \) and \( n_0 \)) and \( q_2 \) be the transition rate between \( n_+ \) and \( n_- \). The evolution of the populations of the three substates is then described by

\[
\begin{align*}
\dot{n}_+ &= -n_+ (q_1 + q_2) + n_0 q_1 + n_- q_2, \\
\dot{n}_0 &= -n_0 (2q_1) + n_+ q_1 + n_- q_1, \\
\dot{n}_- &= -n_- (q_1 + q_2) + n_0 q_1 + n_+ q_2.
\end{align*}
\]

It is straightforward to calculate the derivatives of the vector and tensor polarizations:

\[
\begin{align*}
\frac{dP_z}{dt} &= -(q_1 + 2q_2)P_z, \\
\frac{dP_{zz}}{dt} &= -3q_1 P_{zz}.
\end{align*}
\]

From this we get the ratio of the lifetimes

\[
\frac{\tau_{\text{vector}}}{\tau_{\text{tensor}}} = \frac{3}{1 + 2q_2/q_1}.
\]

This equation relates the ratio \( \tau_{\text{vector}}/\tau_{\text{tensor}} \) to the relative transition rates between substates. One can easily verify that if \( q_1 \) and \( q_2 \) were equal the vector and tensor lifetimes also would be the same. It is also clear that \( \tau_{\text{vector}}/\tau_{\text{tensor}} = 3 \) is the theoretical maximum, attained when \( q_2 = 0 \). The ratio \( \tau_{\text{vector}}/\tau_{\text{tensor}} = 2 \) that results from this experiment implies that \( q_1 \) is four times larger than \( q_2 \). This result may help to understand the detailed depolarization mechanism, which at this time is unknown.

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Spin exchange in polarized deuterium

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We have measured the vector and tensor polarization of an atomic deuterium target as a function of the target density. The polarized deuterium was produced in an atomic beam source and injected into a storage cell. For this experiment, the atomic beam source was operated without rf transitions, in order to avoid complications from the unknown efficiency of these transitions. In this mode, the atomic beam is vector and tensor polarized and both polarizations can be measured simultaneously. We used a 1.2-cm-diam and 27-cm-long storage cell, which yielded an average target density between 3 and 9 × 1011 at/cm3. We find that the tensor polarization decreases with increasing target density while the vector polarization remains constant. The data are in quantitative agreement with the calculated effect of spin exchange between deuterium atoms at low field.

I. INTRODUCTION

When two atoms with antiparallel electron spins collide, both spins flip with a large probability while conserving the longitudinal component of the total spin angular momentum. Due to this effect, the populations of the hyperfine states tend towards equilibrium, the so-called spin-temperature distribution. The rate at which the equilibrium is approached depends on the collision rate.

It is known that spin-exchange collisions may affect the polarization of polarized gas targets. For instance, laser-driven deuterium targets rely on spin-exchange collisions of optically pumped, polarized potassium atoms with deuterium atoms and subsequent spin-exchange collisions between deuterium atoms [1]. The spin-exchange rate is proportional to the number density of the atoms.

If an atomic beam source is used to inject a storage cell target with polarized atoms, spin-exchange effects are usually thought to be unimportant since such a target is much less dense than an optically pumped target. However, even in this case, significant depolarization occurs for tensor-polarized deuterons, as we will demonstrate in this paper. Our study has been prompted by a departure of the tensor polarization as a function of the magnetic field at the target are found to be in qualitative agreement with the theoretical expectation [4] (for a comparison with the present experiment, see Sec. VI).

There exists one previous observation of spin-exchange effects in a deuterium target [3], where measured changes of the tensor polarization as a function of the magnetic field at the target are found to be in qualitative agreement with the theoretical expectation [4] (for a comparison with the present experiment, see Sec. VI).

II. EXPERIMENTAL SETUP

The experiment was performed at the Indiana Cooler with a stored, unpolarized, 135-MeV proton beam. A layout of the experiment is shown in Fig. 1. Polarized deuterons are produced in an atomic beam source (ABS) [5]. The atoms emerge from the dissociator (a) through an aluminum nozzle, which is cooled to liquid nitrogen temperature. The atomic beam passes along the axis of a set of sextupole magnets (b), which defocus one of the two electron-spin substates. The remaining beam contains deuterium atoms in hyperfine states 1, 2, and 3 (it is customary to number the states in decreasing

FIG. 1. The PINTEX facility at the Indiana Cooler. a, dissociator; b, sextupole system; c, remotely controlled transition units; d, feedtube and target cell; e, silicon barrel; f, beam position monitors; g, Helmholtz coils; h, compensating coils; i, z-field coil; j, ΔE scintillator; k, l, wire chambers; m, stopping scintillator; n, veto scintillator. The beam direction is from right to left.
order of their energy in a magnetic field; for more detail, see, e.g., Ref. [6]. When the states 1, 2, and 3 are equally populated, and the ambient field is “weak” (i.e., does not decouple the electron from the nucleus), the nuclear vector polarization \( P_z \) equals \( \frac{1}{4} \) and the tensor polarization \( P_{zz} \) equals \( \frac{1}{2} \). In reality, the polarization is lower because the spin state separation by the sextupoles is not perfect: a molecular component may be present and wall collisions lead to some depolarization. Larger polarization can be achieved by inducing transitions between substates. In this experiment, however, such rf transitions were not used in order to avoid an assumption about their efficiency.

The internal target consists of a storage cell, located in a weak holding field of 0.3 mT, generated by a set of Helmholtz coils \((i,g)\). A storage ring, a weak field is preferred because it avoids significant orbit distortions. The atomic beam enters the storage cell through a 13.0-cm-long, 1.1-cm-diam feed tube \((d)\). The target cell is a 27-cm-long tube of 1.2-cm diam made from 0.05-mm-thick aluminum, coated with Teflon in order to minimize depolarization by wall collisions \([7]\). The length of the cell between the feed tube and the downstream end is 12.5 cm; the upstream part is 14.5 cm long. The sum of the conductances of the three legs for atomic deuterium at 300 K is 15.0 l/s [calculated by using Eq. (1) of Ref. \([8]\)]. The atomic beam current \( J \) (at/s) divided by the conductance equals the target density in the center of the cell. The average target density is half this value.

The atomic beam current depends on the flow rate of \( D_2 \) gas into the dissociator. This is demonstrated by the open symbols in Fig. 2, which show a measurement \([5]\) of the current \( J \) as a function of the gas input when the source is operated with hydrogen. Normally, the gas flow is chosen such that \( J \) is optimized. However, in order to vary the density of the target, we operated the ABS at “normal” gas flow, as well as at half and at a quarter of the normal gas flow. The solid symbols in Fig. 2 show the relative target thickness during this experiment, obtained by dividing the event rate by the stored proton current. After arbitrary normalization, these data obviously confirm the dependence of \( J \) on the gas flow into the dissociator.

While thus the relative atomic beam current \( J \) for the three flow rates is well known, its absolute value is not accurately known. Experience shows that routine operation corresponds to about \( \frac{1}{3} \) of the peak performance shown in Fig. 2. In addition, replacing \( H_2 \) by \( D_2 \) in the dissociator is known to reduce the flux by roughly a factor of 0.7. Altogether, this leads to a scaling factor of \( f_J=0.46 \) for the current \( J \) in Fig. 2. Since this figure is based on a rough estimate, we explore in Sec. V the dependence of \( J \) on scaling.

The outgoing proton and deuteron from \( pd \) elastic scattering are detected in coincidence. The forward going particle (either \( p \) or \( d \)) is detected in a stack consisting of a \( \Delta E \) scintillator \((j)\), two wire chambers \((k,l)\), and a stopping scintillator \((m)\) \([9]\). Laboratory polar angles from 10° to 45° are covered. The recoil particle is detected in the so-called silicon barrel \((e)\), an array of 18 silicon strip detectors surrounding the target cell. The strips are oriented in such a way that they measure the azimuth of the recoil, enabling us to impose a coplanarity condition on the two outgoing particles, in order to reject break-up events. Elastic scattering events are further selected by particle identification via \( \Delta E-E \) in the forward detector, and by the correlation between energy and angle of the forward particle and between forward scattering angle and recoil pulse height in the silicon detector.

During a given measurement, the holding field, and thus the direction of the spin alignment axis, is cycled between horizontal, vertical, and longitudinal and from positive to negative in 2 s intervals. When the field is reversed, the vector polarization reverses sign, whereas the tensor polarization does not.

III. MEASUREMENT AND ANALYSIS

For the three target densities mentioned in the previous section, the vector and tensor polarization of the target are measured simultaneously as follows.

To evaluate the elastic scattering yields, laboratory polar angles \( \theta \) from 19° to 33° are accepted. The cross section with a polarized target depends on the azimuth \( \varphi \) of the scattering plane. In the case of a vertical spin alignment axis, for instance, vector moment \( iT_{11} \) of the polarization induces a term that is proportional to \( \cos \varphi \), while the tensor moment \( T_{22} \) contributes a \( \cos 2\varphi \) term. The angle \( \varphi \) is measured by the wire chambers. The accepted \( \varphi \) range is divided into four sections centered about 0°, 180°, 90°, and 270°, corresponding to left, right, up, and down, respectively. The limits for the four sections are set to \( \pm 44° \) about the center values when evaluating the vector polarization, and \( \pm 22° \) in the case of tensor polarization. The vector polarization is determined from the left/right and up/down asymmetries for the vertical and horizontal holding field, respectively. The tensor polarization is determined from the sum of the up and down yields and the sum of the left and right yields. The effective analyzing powers \( iT_{11} \) and \( T_{22} \) are deduced from the known...
vector and tensor analyzing powers in $pd$ elastic scattering at
135 MeV [10], averaged over the respective $\theta$ and $\varphi$ accept-
tances. The average is weighted with the $\theta$ and $\varphi$ dependence
of the total yield, which represents the detector efficiency.

Varying the $\varphi$ integration limits changes the effective ana-
lyzing powers, but not the resulting polarizations. Variations
in the acceptance criteria for an elastic event have no signifi-
cant effect on the measured polarizations. This demonstrates
that a possible background contamination can be discounted.

IV. COMPARISON WITH THEORY

Walker and Anderson [4] predict the loss of polarization
of a deuterium target due to spin-exchange collisions by
calculating—starting from a given initial state—the evolu-
tion of the population of the substates in terms of the para-
meter $t/T_D$, where $t$ is the average dwell time of an atom in
the target and $1/T_D$ is the DD spin exchange rate. The average
dwell time is estimated from $t=N_c(d/v)$, where $N_c$ is the
average number of collisions with the cell walls, $d$ is the
average distance traveled between collisions, and $v$ is the
average velocity. A Monte Carlo tracking calculation leads to
$N_c\approx 200$ for our cell geometry [8]. Assuming that the prob-
ability of emission after a wall collision follows Lambert’s
 cosine law of ideal diffuse reflection, one can calculate geo-
metrically the average distance $d$ to the next collision. For an
infinitely long cylinder, one finds that $d$ equals the diameter
of the cylinder. The finite-length correction for our target cell
is less than 1%. Thus, we set $d=1.2$ cm.

The spin-exchange rate $1/T_D$ is the product of the spin-
exchange cross section, $\sigma_{SE}(DD)=2\times 10^{-15}$ cm$^2$ [4],
the number density of atoms $n_D$, and the velocity $v$ of the
atoms. The spin-exchange cross section is not expected to sig-
nificantly depend on either the velocity or the magnetic field,
nor has there ever been any experimental evidence of such a
dependence. We therefore take it as constant, $\sigma_{SE}(DD)=2\times 10^{-15}$ cm$^2$ [4]. The parameter $t/T_D$ that is rel-

vant for a spin-exchange calculation is then proportional to
the number density, independent of the velocity, and is given by

$$t/T_D=\sigma_{SE}(DD)n_DN_c d\xi_B.$$   \hfill (1)

The factor $\xi_B$ takes into account the slowing down of the
relaxation rate at magnetic fields large enough to decouple
the electron and nuclear spins. For vanishing external mag-
netc field, $\xi_B=1$. From Eq. (1) in Ref. [4] one obtains $\xi_B$
\hfill (1)

$$=\left\{1+\left[g_s\mu_B(B/\delta_e)^2\right]^{-1}\right\},$$

where $g_s=2.002$ is the electron $g$ factor, $\mu_B=5.79\times 10^{-11}$ MeV/T is the Bohr magneton,
and $\delta_e=1.66\times 10^{-13}$ MeV is the hyperfine splitting for the
deuteron. With these values, we find that our guide field of
0.3 mT hardly affects the spin-exchange rate ($\xi_B=0.96$).

Assuming that the density $n_D$ in Eq. (1) can be replaced
by the average over the length of the target cell, we evaluate
$t/T_D$ for the three target densities mentioned earlier (see Sec.
II). Figure 3 shows the measured vector polarization $P_x$
and negative tensor polarization $-P_{zz}$ as a function of $t/T_D$.

Walker and Anderson calculated the evolution of the six
substates in a vanishing field for the case where initially the
states 1, 2, and 3 are equally populated while the other three
are empty (see Fig. 6 of Ref. [4]). Using the known vector
and tensor polarization of individual substates in a weak field
[6], the evolution of $P_x$ and $P_{zz}$ can then be easily calculated.
The result is shown as dotted lines in Fig. 3. As can be seen,
the vector polarization is hardly affected by spin exchange (it
actually increases slightly with density).

The solid lines are the predictions each normalized by an
arbitrary factor such that the $\chi^2$ between the data and the
calculation is minimized. The normalization factors (0.56 for
the tensor polarization and 0.64 for the vector polarization)
are discussed in the next section. It can be seen from Fig. 3
that the expected dependence on $t/T_D$ (or target density) of
both vector and tensor polarization is consistent with the
data.

V. DEPENDENCE OF THE RESULT ON TARGET

DENSITY

In Fig. 3, the predictions for the vector and tensor analyze-
powers have been individually normalized to fit the data.
The normalization accounts for the loss of polarization due
to wall collisions, recombination, and incomplete rejection of
unwanted states in the sextupoles. Earlier we saw that the
number density $n_D$ of atoms in the cell depends on the factor
$f_J$ used to normalize the ordinate of Fig. 2. According to Eq.
(1), the ordinate of Fig. 3, $t/T_D$, scales with the same param-
eter. Since the vector analyzing power is practically indepen-
dent of $t/T_D$, the normalization factor for the vector polar-
ization, $r_z=0.643\pm 0.007$, is unaffected by a change in $f_J$.
However, this is not the case for the best-fit normalization $r_{zz}$
of the tensor polarization. Figure 4(a) shows $r_{zz}$ for five as-
sumed values of $f_J$. Figure 4(b) shows the $\chi^2$ per degree of
freedom of the data relative to the normalized calculations
for the same five values of $f_J$, indicating a preference for
$f_J=0.38$, corresponding to $r_{zz}=0.56$. We thus find that
the best-fit normalizations $r_z$ and $r_{zz}$ are significantly differ-
ent from each other, or that the loss of polarization with respect
to the ideal value is different for vector and tensor polariza-
tion. That this finding is indeed expected can be understood
with a simple model for depolarization.
Assume that the depolarizing mechanism is the exchange of the atomic electron with one of random orientation (e.g., during a wall collision). This means that either nothing happens or that the electron spin is flipped. In the latter case, the atom makes a transition, for instance from state 1 to state 6 or 2→5 or 3→4, or vice versa. Thus the initial-state occupations evolve. The final polarization depends on the initial state, the number of electron spin flips, and the magnetic field. It is easy to see that the evolution is not the same for vector and tensor polarization. For equal initial population of states 1, 2, and 3, for a weak field, and for a number of electron spin flips that would lower the vector polarization by a factor of 0.643 (our observed value for \( r_z \)), one finds that, at the same time, the tensor polarization would have decreased by 0.561. This value is shown as a dashed line in Fig. 4(a). To be sure, the depolarization mechanism postulated here is speculative, and we assume that all polarization loss is by this mechanism, however it is remarkable that the same density normalization \( f_J \) that yields a minimum in \( \chi^2 \) also supports the relative vector-tensor depolarization predicted by our simple model.

VI. CONCLUSION

We have measured the vector and tensor polarization of a polarized deuterium target in a weak magnetic field as a function of target density. The measured polarizations agree well with a model calculation taking into account spin exchange between deuterium atoms. This demonstrates that even for densities below \( 10^{12} \text{ cm}^{-3} \), depolarization by spin exchange may be sizeable.

In comparison to an earlier study of the change in tensor polarization as a function of the magnetic field at the target [3], the present evidence is based on a direct measurement of the target polarization via the known \( pd \) scattering analyzing powers, without assumptions about the efficiency of the transition units. The simultaneous measurement of vector and tensor polarization also confirms the predicted relative effect of spin exchange where only the tensor polarization is affected significantly.

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Polarization Experiments with Storage Rings

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Abstract

The technical issues are summarized that arise in hadron storage ring experiments with a polarized beam and a polarized internal target. This is followed by a synopsis of polarization experiments in storage rings.

1. Polarized beam in a ring

1.1. Spin dynamics

1.1.1. Spin closed orbit

The magnetic moments of particles stored in a ring precess around the fields they encounter along their orbit. If the particles are polarized their polarization vector precesses accordingly. This spin motion must repeat on each revolution. For any point along the beam, this condition defines a stable polarization direction, called the spin closed orbit. If there are fields other than the bending field, and one knows them, one can calculate the spin closed orbit. This is an eigenvalue problem that is best solved by representing rotations as unitary, complex $2 \times 2$ matrices (one of the few classical applications of the SU(2) formalism). A clear account of this topic has been given by Montaigle [1].

The ring environment determines most beam properties. Their values before injection into the ring are lost. This is true for energy, time structure and emittance, but also for the polarization direction: any initial polarization component that is not along the spin closed orbit is quickly washed out. If the field integral along the ring is vertical, the spin closed orbit is vertical everywhere, but spin rotators (so-called snakes) can be used to produce non-vertical spin orientation. A longitudinal solenoid field is an example of a spin rotator.

It is interesting that polarization, in principle, can be transferred from a polarized, internal target to the stored beam [2,3]. This is a small effect that can be observed only because it accumulates as the beam passes through the target many times.

1.1.2. Resonances

In a machine where the average field is vertical, the magnetic moments of the beam particles in the particle frame precess with a frequency $f_{orb}G\gamma$, where $f_{orb}$ is the orbit frequency, $\gamma$ is the Lorentz factor, and $G = 1/2(1-g-2)$ follows from the g-factor (for protons, $G_p = 1.79285$). Depolarizing spin resonances arise when this precession is consonant with the particle motion. So-called imperfection resonances occur at energies for which $G\gamma$ has an integer value. Intrinsic resonances are caused by the focusing fields and depend on the vertical and horizontal machine tunes $v_v$ and $v_h$. They occur when $G\gamma = n + m\nu_h + kv_v$ ($n$, $m$, $k$ are integers). Finally, induced resonances are produced by longitudinal or transverse oscillating fields when the frequency $f_{osc}$ satisfies $f_{osc} = f_{orb}(m + nG\gamma)$.

1.2. Stored polarized protons

1.2.1. Polarization reversal

Crossing a spin resonance at slow speed (adiabatic passage) causes the spin closed orbit to flip by 180°. It has been demonstrated [4–6] that the crossing of an induced resonance (by varying the rf frequency) flips the sign of the polarization (transverse as well as longitudinal) of protons with better than a few percent loss in polarization. This can be an important tool in eliminating systematic errors in polarization experiments.

1.2.2. Polarization lifetime

Near a resonance, the spin closed orbit deviates from its off-resonance value and becomes sensitive to the tune and the betatron amplitude, which can be slightly different for individual particles. Thus, the ensemble de-coheres and polarization is lost. This process is not understood in detail but it is still possible to predict the polarization lifetime as a function of the “distance” to the resonance (for an intrinsic resonance, that would be the difference between the actual and the resonant tune). The polarization lifetime for protons has been measured near an intrinsic resonance [7] and an induced resonance [8].

One finds that the effect of a resonance is very localized. Thus, the polarization of a stored beam is remarkably stable, and if one is not extremely close to a resonance the polarization lifetime is much longer than the beam intensity lifetime.

1.2.3. Acceleration of polarized beam

In the absence of resonances, changing the energy of the beam hardly affects its polarization. This was demonstrated by measuring the polarization of a 200 MeV beam, accelerating it to 450 MeV, decelerating back to 200 MeV, and repeating the polarization measurement [9]. This persistence of polarization makes it possible to export a polarization standard (a known analyzing power at a given energy) to any other energy that can be reached by accelerating or decelerating the stored beam.

When accelerating the beam to higher energies, eventually depolarizing resonances have to be crossed. In order to preserve the polarization, imperfection resonances may be enhanced to affect a complete spin flip [10], while a sudden tune change may be used to jump over intrinsic resonances (as is done at COSY). Another method to avoid depolarizing resonances involves the use of spin rotators.
(so-called Siberian snakes). The effectiveness of such a device has been demonstrated for the first time at the Indiana Cooler [11,12] (see Fig. 1). At this time, RHIC is operating a multitude of snakes to preserve beam polarization up to 500 GeV.

1.3. Stored polarized deuterons

1.3.1. Polarization of spin-1 particles

While the three components of the polarization vector are sufficient to describe an ensemble of spin-1/2 protons, eight real numbers are in general required for spin-1 deuterons. However, any ensemble has a symmetry axis $S$ (i.e., is invariant under rotation around $S$), called the spin alignment axis. When $S$ is taken as the quantization axis, the relative populations $n_+ = n_0$, and $n_-$ of the magnetic substates with $m = +1$, 0, or $-1$, completely define the polarization of the ensemble. For an unpolarized ensemble the substates are equally populated ($n_+ = n_0 = n_- = 1/3$). If the +1 sub-state is enriched at the expense of the $-1$ substate, the system is said to have vector polarization, $P_v = n_0 - n_-$. If the population $n_0$ differs from 1/3, the assembly has tensor polarization, $P_t = 1 - 3n_0$. Obviously, the system can be vector- and tensor-polarized at the same time.

With an arbitrary quantization axis, three vector components and five components of a second-rank tensor are required to describe spin-1 polarization, but these so-called beam moments can be deduced if one knows the vector and tensor polarizations, $P_v$ and $P_t$, and the orientation of the spin alignment axis.

1.3.2. Orbiting polarized deuterons

The magnetic moment of deuterons is smaller than that of protons ($\mu_d/\mu_p = 0.3070$) and the mass of the deuteron is larger ($m_d/m_p = 1.999$), so the precession of deuterons is slower, but the spin closed orbit is defined just as it is for protons. Since, for symmetry reasons, the spin alignment axis is collinear with the magnetic moment, the spin closed orbit represents the orientation of $S$, and one can evaluate the beam moments at any point in the ring. The formal treatment of spin-1 particles in a ring has been discussed by Bell [13], and Huang [14].

Since the relevant parameter, $G_d = -0.14299$, is more than an order of magnitude smaller for deuterons than for protons, there are much fewer deuteron depolarizing resonances (the lowest imperfection resonance occurs at 11.3 GeV). There are also fewer intrinsic resonances but they can occur at low energy since the tune can be a small number. A polarized deuteron beam has been accelerated and decelerated through an intrinsic resonance at KEK [15]. Very recently, polarized deuterons have been stored at COSY and at IUCF.

1.3.3. Reversal of deuteron polarization

In Section 1.2.1, I mentioned the reversal of spin-1/2 polarization by crossing an induced depolarizing resonance. The effect of resonance crossing is related to the crossing speed [4]. Thus, one can compensate for the smaller deuteron magnetic moment by increasing the rf field strength or by decreasing the crossing speed. The effect of an induced resonance on a polarized deuteron beam has recently (March 2002) been studied for the first time at the Indiana Cooler [16]. The experiment used a 270 MeV deuteron beam with both, vector and tensor polarization. It was verified that a complete reversal of the spin closed orbit changes the sign of the vector polarization, but does not affect the tensor polarization, as expected. It was also shown that when a resonance is only partly crossed the spin closed orbit may end up in the ring plane when the resonance releases it. Then, the vector polarization vanishes and the tensor polarization is decreased by a factor of two, but has changed sign, also as expected [17].

1.3.4. Vector and tensor polarization lifetime

Interpreting the depolarization of a stored beam as random walk of the spin closed orbit leads to the prediction of different lifetimes for vector and tensor polarization [17]. A recent experimental test with 270 MeV deuterons near an induced rf solenoid resonance indicates that the vector polarization lifetime is about twice that of tensor polarization [16].

2. Polarized, internal targets

2.1. General remarks

The attainable production rate by standard methods for atoms with nuclear polarization satisfies the requirements for an internal target. The atomic beam is either crossed with the beam in the ring or injected into a storage cell (see 2.2.1). A guide field over the target region defines the spin alignment axis. If the atoms are in a pure spin state (maximum total angular momentum), a weak field of 0.2 0.5 M, essentially to overcome the earth’s field, is sufficient. Corresponding compensating fields can be set up to practically eliminate the effect of transverse guide fields on the beam orbit. A review of polarized internal targets has been given by Rathmann [18].

Polarized internal targets are pure, not susceptible to radiation damage, and offer complete freedom in choosing the direction of the polarization, including rapid reversal of its sign. The ability to use such targets is perhaps the most

Fig. 1 The Indiana Cooler, 1988–2002.
important benefit of the storage ring environment for nuclear physics.

Until recently, it was assumed that atoms that recombine into molecules lose their polarization. However, it has been demonstrated by a study at the Indiana Cooler [19] that $\text{H}_2$ molecules may well retain some nuclear polarization.

At high atomic density, when mixed hyperfine states are present, their populations are modified by spin-exchange collisions, reducing the resulting nuclear polarization [20].

2.2. Technical realization

2.2.1. Storage cells

For a given flux of polarized atoms, the target thickness can be enhanced by several orders of magnitude by directing the atomic beam from the side into a narrow, open-ended channel through which the stored beam passes. The purpose of such a so-called storage cell is to provide gas flow impedance. This scheme was first used at the VEPP-3 electron storage ring in Novosibirsk [21]. To avoid depolarization of the atoms by wall collisions, the cell walls are coated with Teflon, dri-film, frozen water, or some other material [22]. The conflicting criteria that enter the design of a storage cell target are discussed in Ref. [23].

2.2.2. Sources of polarized atoms

The preferred method to produce polarized H and D atoms is to dissociate the molecules, form a beam and then select a single hyperfine state or a desired mixture of states. This selection is affected by a combination of sextupole fields (to discard one electron sub-state) with rf-induced transitions between magnetic sub-states. It is possible to generate pure deuteron vector or pure tensor polarization. The Wisconsin atomic-beam source [24] that is in use at the Indiana Cooler, generates a beam of about 1 cm diameter, suitable for injection into a storage cell, with a flux of about $3 \times 10^{14}$ polarized H atoms/s with nuclear polarization $P = 0.75$. For a typical storage cell of 1 cm diameter and 25 cm length, the target thickness then amounts to between $10^{13}$ and $10^{14}$ atoms/cm$^2$.

An alternative method to produce polarized atoms makes use of spin exchange between H (or D) atoms and a small admixture of optically pumped potassium atoms [25]. This method produces a higher flux, but (so far) quite low nuclear polarization. This and the unavoidable potassium contamination of the target are serious disadvantages of this method.

Polarized $^3\text{He}$ is obtained by producing meta-stable atoms by optically pumping the $^3\text{S}_1 \rightarrow ^1\text{P}_0$ transition, and then transferring the nuclear polarization by “metastability exchange” collisions to the ground state. A flow into the storage cell of $10^4$ atoms/s with a polarization of $P = 0.4$ has been reported [26].

3. Nucleon–Nucleon reactions

3.1. Proton–proton elastic scattering

3.1.1. What can be measured? (spin-$1/2$ on spin-$1/2$ with a two-particle final state)

The beam polarization as well as the target polarization may have non-zero components along the axes of a fixed Cartesian frame. Either beam or target or both may also be unpolarized. Thus, there are 16 combinations of beam and target polarization states, each of which corresponds to an observable. These observables include the unpolarized cross section $\sigma_{00}$, the analyzing powers $A_{00}$ and $A_{0k}$ and the spin correlation coefficients $C_{ik}$ ($i$ and $k$ stand for $x$, $y$, or $z$, where the $z$-axis is along the beam, the $y$-axis is up, and the $x$-axis completes a right-handed coordinate frame). Four of these observables are related to others by a simple rotation of the frame around the beam axis (e.g., $C_{xz}$, rotated by $90^\circ$ is the same as $C_{yz}$). Another four observables vanish if parity is conserved. The remaining seven observables are the beam and the target analyzing power, $A_{00}$ and $A_{0k}$, and the correlation coefficients $C_{xx}$, $C_{xy}$, $C_{xz}$, $C_{yz}$, and $C_{zz}$. Each of these is a function of the polar angle $\theta$ and the azimuth $\varphi$. The dependence on $\theta$ may be complicated (depending on the number of participating partial waves), but the $\varphi$ dependence are simple trigonometric functions. Each observable is associated with a known, characteristic $\varphi$ dependence. This dependence is crucial in distinguishing observables from each other when analyzing the data. For this reason, it helps to have a detector with full azimuthal coverage. Ohlsen [27] has given a complete treatment of the formal aspects of spin correlation measurements.

If the particles in the initial state are both protons, two of the seven observables listed above become redundant ($A_{00}$ and $C_{zz}$ are equivalent to $A_{00}$ and $C_{zz}$) due to the identity of the collision partners.

3.1.2. $p + p$ elastic scattering at IUCF and COSY

A fair number of experiments with the Indiana Cooler were aimed at the analyzing power $A_{00}$ and three of the four spin correlation coefficients, $C_{xx}$, $C_{xy}$, $C_{xz}$ (in pp scattering, often called $A_{xx}$, $A_{xy}$, $A_{xz}$, or $A_{SS}$, $A_{NN}$, $A_{SL}$). Initial measurements near 200 MeV [28–31] were accompanied by the development of the new technology and novel analysis tools [32]. Finally, a general survey of these observables from 200 to 450 MeV [33], using up- and down-ramping to export the polarization calibration, resulted in an impressive body of data with statistical uncertainties of about 0.01 and an overall normalization error of 2.4%. A measurement of the fourth correlation coefficient, $C_{yy}$, at 200 MeV [34] made use of a solenoid snake to generate beam with longitudinal polarization at the target.

The EDDA collaboration at COSY subsequently continued these measurements to higher energies, resulting in analyzing power and spin correlation coefficients between 450 MeV and 2.5 GeV. A noteworthy achievement was that continuous excitation functions were obtained by taking data while the beam was accelerated [35].

3.1.3. Physics interest: NN phenomenology

The high precision of the data, and the fact that the clean definition of the scattering events made the small angles of the Coulomb-nuclear interference region accessible, made it possible to demonstrate, for the first time, a significant effect due to the interaction of the magnetic moments. However, a more important consequence of the Indiana data was that they required an update of the phenomenological description of $NN$ scattering in terms of empirical phase shifts. This is even truer at GeV energies where the
new COSY data had a significant impact on the pp phase shift analysis in this energy range [36]. The NN phase shifts constitute the basis for numerous models in nuclear physics.

3.2. Meson production in $p + p$ collisions

3.2.1. What can be measured? (spin-1/2 on spin-1/2 with a three-particle final state)

Compared to a two-particle final state, we need to specify a third momentum vector to describe the final-state kinematics. This brings the number of kinematic variables to a total of five. These can be $\theta_1$, $\varphi_1$ (direction of particle 1, e.g., the pion), $\theta_2$, $\varphi_2$ (direction of the relative momentum between particle 2 and 3, e.g., the two nucleons), and $\varepsilon$ (energy sharing between the three particles). Again the azimuthal dependences are predictable and useful for data analysis. Thus, the seven observables defined for the two-body case (Section 3.1.1) are now five-dimensional ($\theta_1$, $\theta_2$, $\varphi_1$, $\varphi_2$, $\varepsilon$). One can reduce this information systematically into a (fairly large) set of one-dimensional quantities by integrating either over $\theta_2$ and $\varepsilon$, or over $\theta_1$ and $\varepsilon$ [37].

But this is not all. Since in a three-body final state parity conservation no longer constrains any observables, we must now also consider the longitudinal analyzing powers $A_{L0}$ and $A_{L0}$, and the correlation coefficient $C_{xy} = C_{yx}$. In addition, one must address the dependence on the energy-sharing parameter $\varepsilon$ (for instance, by studying the dependence on $\varepsilon$ of the unpolarized cross section and the spin-dependent total cross sections $\Delta \sigma_T$ and $\Delta \sigma_L$). The formal aspects of polarization observables in reactions with a three-particle final state are treated in detail in Ref. [37].

3.2.2. Physics issues

In meson production near threshold relatively few partial waves contribute. For the reaction $pp \rightarrow pp\pi^0$, up to about 400 MeV bombarding energy, the final-state angular momenta of the two nucleons ($L_{NN}$) and of the pion ($L_{\pi}$) are both either 0 or 1, and within the first 20 MeV above threshold just a single partial wave ($P_{\pi} \rightarrow S(\ell_{\pi} = 0)$) is important. Early pion-production experiments in this energy region yielded the first evidence for the importance of heavy meson exchange [38] and the associated enhancement of the axial current in a nuclear system. But the theoretical situation is far from settled, since other contributions (such as off-shell pion rescattering) may also be relevant. A meaningful test of models, at this stage, needs more that just the strength of the $L_{NN} = \ell_{\pi} = 0$ amplitude. Such information is provided by polarization observables.

3.2.3. Pion production experiments at the Indiana Cooler

The first pion production polarization experiments at the Cooler were measurements near threshold of cross section and analyzing power for the reactions $pp \rightarrow dx^+ [39]$ and $pp \rightarrow p\pi^+ [40,41]$. After the polarized internal target became available, a program to study spin correlation in $pp \rightarrow pp\pi^0$ was initiated [42,43], and it was demonstrated that model-independent information on the contribution of a single partial-wave amplitude can be deduced from the spin-dependent cross sections $\Delta \sigma_T$ and $\Delta \sigma_L$ alone [44]. This effort eventually led to a complete measurement of all observables of $pp \rightarrow pp\pi^0$ with polarization in the initial state, everywhere in the three-body phase space, up to 400 MeV [37]. This data set provides sufficient information to experimentally determine all 12 amplitudes for which $L_{NN}$ and $\ell_{\pi}$ are either 0 or 1.

Perhaps the single most interesting result was the observation that the longitudinal analyzing power $A_L$ (normally forbidden by parity conservation) can well be large in a reaction with a three-body final state [45]. Concurrently with the $pp \rightarrow pp\pi^0$ experiment, data on the spin correlation coefficients in $pp \rightarrow dx^+$ [46] and in $pp \rightarrow p\pi^+$ [47,48] were also obtained.

3.2.4. Meson production at COSY

At COSY, the first polarization data in meson production begin to emerge: very recently the COSY-11 collaboration has reported a measurement of the analyzing power (with respect to the meson) of $pp \rightarrow pp\eta$, 40 MeV above threshold [49].

4. P + D reactions below the pion threshold

4.1. Proton–deuteron elastic scattering

4.1.1. What can be measured? (spin-1/2 on spin-1 with a two-particle final state)

When the spin-1 particles are vector-polarized, the situation is analogous to the spin-1/2 on spin-1/2 case: there are a total of seven observables, namely two analyzing powers and five ‘vector’ spin correlation coefficients.

For tensor-polarized spin-1 particles (see Section 1.3.1) there is some added complexity. Good introductions to tensor observables, terminology and conventions are given, e.g., by Haeburni [50], and Darden [51].

In Cartesian notation, a tensor-polarized beam (or target) has five independent moments, $p_{xx}$, $p_{xy}$, $p_{xz}$, $p_{yy}$, and $p_{yz}$ ($p_{zz}$ is not independent because $p_{xx} + p_{yy} + p_{zz} = 0$). There are 20 ways to combine these with the four possibilities for the spin-1/2 particle (indices 0, x, y, z). Of these combinations, seven are related to others by rotation around the beam axis, and three are forbidden by parity conservation.

The remaining observables are the three tensor analyzing powers $A_{xyz}$, $A_{yzx}$, and $A_{3z}$, and the seven ‘tensor’ correlation coefficients $C_{xx,y}$, $C_{xy,x}$, $C_{yy,x}$, $C_{xz,y}$, $C_{xz,z}$, $C_{zy,y}$, and $C_{zy,z}$. Thus, apart from the unpolarized cross section, there are a total of 17 polarization observables that can be measured in $p + d$ elastic scattering with polarized collision partners (for a formal discussion of spin correlation measurements, see Ohlsen [27]).

4.1.2. Polarization data for $p + d$ scattering

A number of measurements of the proton analyzing power, and the deuteron vector and tensor analyzing powers exist between proton bombarding energies between 100 and 200 MeV (see references given in Ref. [52]). The first spin correlation measurement ($C_{x,y}$), together with the deuteron vector analyzing power, has been carried out at 200 MeV [53] at the Indiana Cooler, using a laser-driven polarized target. More recently, the PINTEX group has embarked on a program to measure all but two of the 17 possible polarization observables. This experiment has been carried out at two beam energies (135 and 200 MeV stored,
polarized protons including longitudinal polarization) and
an atomic-beam deuterium target with either pure vector or
pure tensor polarization. The analysis of these data is in
progress; preliminary results from a subset of the data are
in hand.

4.1.3. Physics interest: the three-nucleon force

It is believed that today’s state-of-the-art Faddeev cal-
dulations, solving the three-nucleon problem with a phenom-
enological NN input, are telling us how nature would behave if there were only two-nucleon forces. It has been
shown that these calculations depend only weakly on the
choice of the underlying NN potential. They neglect the
Coulomb force but there is evidence that this does not
matter above 100 MeV and for $\theta_{\text{cm}} > 30^\circ$. Comparing such
calculations with a complete set of polarization data thus
promises to reveal information about the three-nucleon
force, in particular its spin dependence.

The general features of our preliminary $p + d$ elastic
scattering results are reproduced rather well by the most
recent 2N Faddeev calculations [54]. The remaining
discrepancies are most pronounced for center-of-mass
angles greater than $70^\circ$ and forward of $30^\circ$ (the latter
presumably because of Coulomb effects). The observables
with the largest discrepancies are the tensor analyzing
powers. Inclusion of either the Tucson–Melbourne 3NF or
the Argonne IX three-nucleon force often (but not always)
reduces the discrepancy. Clearly, much more theoretical
work is needed to quantitatively explain the disparity
between experiment and 2NF calculations. It may also be
worthwhile to search for a phenomenological three-nucleon
potential that would explain the data, much along the lines
of the phenomenological potentials that we employ to
describe the NN interaction.

4.2. Deuteron break-up reaction

4.2.1. Physics issue: axial observables

When searching for 3NF effects, the break-up into three
nucleons has an advantage since the kinematic freedom in
the final-state offers the possibility to select configurations,
which differ in their sensitivity to 3N forces. For instance,
3NF effects predicted for spin correlation observables by
the Tucson–Melbourne force are negligible near the quasi-
free peak, but are quite large in the “FSI” configuration
where two of the outgoing nucleons are at relative rest [55].

On the other hand the complexity of the spin-1/2–spin-1
spin space combined with a three-body phase space is
mind-boggling, and a measurement clearly needs some
guidance. Such guidance is provided, for instance, by the
following argument.

It was pointed out by Knutson [56] that three-body
potentials involve spin operators of a type that is not
allowed for ordinary two-body interactions. These opera-
tors affect the so-called “axial” observables. An example of
an axial observable is the longitudinal analyzing power $A_z$.
In reactions with two outgoing particles this observable
must vanish by parity conservation, but it is unconstrained
when there are outgoing particles with lab momenta that are
not co-planar.

An attempt to observe a non-zero $A_z$ in $pd \rightarrow ppn$ with a
longitudinally polarized 9 MeV proton beam [57] failed (an
upper limit of 0.003 was established, in agreement with
Faddeev predictions). Somewhat surprisingly, the same
calculations at 135 MeV predict a much larger value for $A_z$.
This opens up the exciting possibility of an experiment that
addresses the 3NF where the choice of the quantity to be
measured is motivated by a theoretical argument.

4.2.2. Measurement of $A_z$ at 135 MeV

One of the last experiments with the Indiana Cooler (ce64)
aims at a measurement of $A_z$ in $pd \rightarrow ppn$ at 135 MeV
(proton energy). The experiment is carried out with a
270 MeV deuteron beam on a proton target. Even though
the focus is the longitudinal analyzing power, vector- and
tensor-polarized beam is used and the target polarization is
not just pointed along the $z$-axis but also along the $x$- and
the $y$-axis. Thus, given the interest and the manpower,
much more than $A_z$ could be extracted from the data. The
data taking is completed. From a first look at the data, it is
evident that the observed $A_z$ is indeed sizeable.

5. More few-nucleon reactions

5.1. Pion production in $p + d$ collisions

5.1.1. What can be measured? (spin-1/2 on spin-1 with a	hree-particle final state)

Combining the complexity of spin-1/2 on spin-1 with a
five-parameter three-body phase space results in an over-
whelming number of polarization observables, making a
complete study a monumental task. Let me therefore
discuss here just the spin-dependent total cross section,
which has relatively few terms. To measure the total cross
section, we integrate over all degrees of freedom of the final
state by having a detector that covers all (or, most) of
phase space. Then, there are only four polarization
observables left, namely the tensor analyzing power $A_z$
(no spin-1/2 polarization), integrated over phase space,
two-spin-dependent total cross sections $\Delta \sigma_T$ and $\Delta \sigma_L$
measured with vector polarized spin-1 particles, and the
integrated tensor correlation coefficient $C_{yz} = C_{xz,z}$.

5.1.2. Physics issues

Pion production in the three-nucleon system is interesting
because it represents the simplest case where one may study
the effect of the ‘medium’ (actually, one more nucleon) on
the production of mesons. The nuclear wave functions of
the three-nucleon system are presumably under control.
Nevertheless, microscopic calculations have not been very
successful explaining the cross section and even less the
available polarization data. For this reason, a number of
theoretical studies attempted to describe $p + d$ pion
production in terms of the amplitudes of the underlying
elementary process $NN \rightarrow d\pi$ (see, e.g., [58] and references
therein). Spin-correlation data (e.g., a measurement of $\Delta \sigma_T$
and $\Delta \sigma_L$), would provide model-independent information
on the importance of different reaction mechanisms.

Based on qualitative agreement with the $pd \rightarrow t\pi^+$ cross
section at 800 MeV it has been argued that mechanisms
involving all three nucleons are important [59]. This offers
some hope that eventually information about the three-
nucleon force might also be obtained from pion production
in the three-nucleon system.
5.1.3. Polarization measurements

Below 500 MeV, the proton analyzing power and vector- and tensor analyzing powers have been measured for pd → 3He\(^0\) before the advent of storage rings. At the Indiana Cooler the proton analyzing power for pd → 3He\(^0\) has been measured at energies from 199.4 to 210 MeV [60]. More recently, also at the Cooler, the first spin correlation data have been obtained for the reaction pd → τ\(^{+}\)τ\(^{-}\). In this experiment the spin-dependent total cross sections \(\Delta \sigma_{T}\) and \(\Delta \sigma_{L}\) were measured at 250 and 275 MeV. The analysis of this experiment is in progress.

5.2. \(p + d\) reactions at GeV energies

5.2.1. \(p + d\) break-up

As the bombarding energy increases beyond the pion threshold, Faddeev calculations can not yet be used to describe the three-nucleon continuum. On the other hand, one might hope that few-nucleon reactions get simpler as the energy is increased. However, this does not seem to be the case, given the very limited success of the impulse approximation to describe, e.g., p + d break-up. We are thus relegated to models, evaluating contributions from various postulated reaction mechanisms. Selective tests of such models make use of a specific choice of the final-state kinematics, and of the observation of polarization observables. Recently, the ANKE experiment at COSY has measured the analyzing power of pd break-up with a proton pair with low relative energy (presumably in a 1\(^S_0\) state) in the final state [49]. Eventually, these experiments will also cover spin-correlation observables.

5.2.2. \(p + d\) total cross section: time reversal invariance

It is commonly thought that many of the systematic errors that make the classical tests for charge symmetry, parity violation and time reversal invariance hard, are easier to handle in a storage ring environment. Yet such experiments are slow in coming. Here, I would like to mention one experiment that probably can only be done in a storage ring.

In Section 5.1.1 I have discussed the total cross section of the p + d reaction. One of the contributions to the spin-dependence of the total cross section, the tensor correlation coefficient \(C_{T,22} - C_{T,11}\), can be shown to vanish under time reversal if one includes all exit channels (but only then) [61]. There exists a proposal to exploit this observable for a time reversal invariance test at COSY [62], using a vertically polarized proton beam and a tensor-polarized, internal deuteron target. A similar experiment at very high energy has also been discussed for RHIC.

5.3. Spin-dependence of bound-state wave functions

5.3.1. Physics issues

Many experimental queries in nuclear physics, from \(NN \rightarrow X\pi\) reactions, to studies of parton polarization with high-energy electrons at JLAB and HERA, to polarization studies at very high energy at RHIC, require polarized neutrons as collision partners. This can be achieved by using the neutrons in polarized D or \(^3\)He nuclei. Neutrons in vector-polarized deuterons are polarized in the same direction, except for the d-state where the neutron polarization is opposite. Similarly, the unpaired neutron and a singlet np pair, except for admixtures of other configurations, give the \(^3\)He He polarization. Thus, such measurements with polarized neutrons require knowledge of the spin-dependent nuclear wave functions, and a model is needed to determine the effective nucleon polarization under the kinematic constraints of the experiment in question. Spin correlation measurements can test such models.

5.3.2. Spin correlation in quasi-free \(p + ^3\)He scattering

The difference between the single-nucleon momentum-density distributions with the nucleon spin parallel or anti-parallel to the \(^3\)He spin has been studied at the Indiana Cooler. To this effect, the spin correlation coefficients for quasi-elastic np or pp scattering were measured with a polarized proton beam on an internal \(^3\)He target, polarized by optical pumping [63]. The data were analyzed assuming the validity of the plane-wave impulse approximation [64] and good agreement with theoretical models of \(^3\)He was found up to about 300 MeV/c Fermi momentum.

6. Summary and outlook

The first measurement with a polarized beam in the Indiana Cooler (see Fig. 1) took place in 1988, and the first polarized internal target \(^3\)He was in use by 1994. Most of the storage ring experiments with polarized hadrons to date, and much of the development of the associated technology, were carried out at Indiana. This article has been an attempt to summarize this work.

As of this year, the “Cooler” is no more. The future in polarization studies at a storage ring belongs to COSY. The only other storage ring in the world that features polarized, hadronic collision partners is RHIC, where very different physics questions will be investigated using many of the techniques and methods developed and first tested at IUCF.

References

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Author Please resupply Fig. 1 if quality not good enough
Spin correlations in $pp \to pn \pi^+$ pion production near threshold

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A first measurement of longitudinal as well as transverse spin correlation coefficients for the reaction $\bar{p}p \to pn\pi^+$ is made using a polarized proton target and a polarized proton beam. We report kinematically complete measurements for this reaction at 325-, 350-, 375-, and 400-MeV beam energies. The spin correlation coefficients $A_{xx} + A_{yy}, A_{xx} - A_{yy}, A_{xz},$ and $A_{xz}$ and the analyzing power $A_x,$ as well as angular distributions for $\sigma(\theta_p)$ and the polarization observables $A_{ij}(\theta_p),$ are extracted. Partial wave cross sections for dominant transition channels are obtained from a partial-wave analysis that included transitions with final-state angular momenta of $l \leq 1.$ The measurements of the $pp \to pn\pi^+$ polarization observables are compared with the predictions from the Jülich meson exchange model. The agreement is very good at 325 MeV, but it deteriorates increasingly for the higher energies. At all energies agreement with the model is better than for the reaction $pp \to pp\pi^0.$

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I. INTRODUCTION

Pion-nucleon interaction has provided increasingly sensitive tests of nuclear theory. One of the challenges yet to be met is to understand the polarization observables for pion production in $\bar{p}p$ collisions. This is especially interesting near threshold, where few partial waves contribute and where calculations should be more manageable and more conclusive.

After the initial theoretical work in the 1950s by Gell-Mann and Watson [1] and Rosenfeld [2], more than a decade elapsed before explicit $pp \to pp\pi^0$ and $pp \to pn\pi^+$ cross sections for $S$ states ($I_{\pi N} = 0, I_{\pi N} = 0$) transitions were predicted by Kolton and Reitan [3] in 1966 and by Schillaci, Silbar, and Young [4] in 1969. When the small cross sections very close to threshold could finally be measured 20 years later [5,6], it turned out that these calculations had missed the true cross sections by factors up to 5. This realization spurred much new theoretical research.

To date the Jülich meson exchange model [7–11] has yielded the most successful calculations. This model represents a much advanced development of the approach of Ref. [3] and builds on the insights of the 1990s (e.g., those of Lee and Riska [12] and many others). It permits detailed calculations beyond $I_N = 0$ transitions, and provides analyzing powers and spin correlation coefficients for the near-threshold region. The Jülich model incorporates all the basic diagrams: realistic final-state interactions, off-shell effects, contributions from the delta resonance, and the exchange of heavier mesons. With the exception of the heavy meson exchange term there are no adjustable parameters. At this time it is the only model with predictions that can be compared to our measurements. However, the Jülich model does not account for quark degrees of freedom, the potential study of which had motivated our experiment initially.

Ideally, one would interpret the basic pion production reactions in a framework compatible with QCD, e.g., calculations using chiral perturbation theory ($\chi PT$). However, with one exception [13], the $\chi PT$ calculations published to date are still restricted to $I_N = 0.$ Moreover, for all three $pp \to X\pi$ reactions, the $\chi PT$ cross sections remain a factor of 2 or more below experiment [14]. This shortcoming may be attributable to the difficulties of $\chi PT$ for momentum transfers larger than $m_\pi.$ The $\chi PT$ calculations published to date are best viewed as works in progress [15].

Calculations and experiments very close to threshold require great care. For $S$ states ($I_N = 0, I_{\pi N} = 0$) in $pp \to pn\pi^+$ only one amplitude is calculated, and the angular dependence is trivial. However, the near-threshold cross sec-
tion and its energy dependence are significantly modified by “secondary” effects, such as final-state interactions that are particularly important for $l_{pn}=0$.

Measurements very close to threshold can present difficulties because the cross sections are small, of the order of 1 $\mu$b, and change rapidly with energy. The energy of the interacting nucleons for reactions very close to threshold must be precisely known and maintained. At the Indiana University Cyclotron Facility (IUCF) this was accomplished by the use of a very thin internal target and the precise beam energy control of the Cooler (storage) Ring. The IUCF Cooler also generates a low background.

The earliest studies of $pp \rightarrow pn \pi^+$ very close to threshold [6,16–18] had available a stored IUCF beam of $\leq 50$ $\mu$A. They used an unpolarized gas jet target and measured cross sections and analyzing powers from 293 MeV (i.e., 0.7 MeV above the $\pi^+$ production threshold) to 330 MeV. These experiments deduced cross sections for $Ss$ pion production very close to threshold. As long as $Ss$ production of pions strongly dominated, analyzing powers also provided information for $Sp$ ($l_{s}=1$) admixtures [18]. At 325 MeV and above, higher partial waves enter significantly, but the larger cross sections make it practical to explore analyzing powers and spin correlation coefficients, which allow a much more detailed comparison of theory and experiment. At the upgraded IUCF Cooler Ring, an intense polarized proton beam with a large longitudinal component and an efficient windowless polarized hydrogen target now permit measurements of all spin correlations coefficients for $\vec{p}p \rightarrow pn \pi^+$. Some initial results for transverse spin correlations were reported in Ref. [19].

The goal of the present study is to quantify the growing importance of higher partial waves ($Sp$, $Ps$, $Pp$, and, potentially, $Sd$ transitions) by measuring analyzing powers and spin correlation coefficients as a function of energy. These polarization observables are sensitive indicators of the reaction mechanism and the contributing partial waves [18,20,21], and are a powerful tool in determining transition amplitudes empirically. In this experiment we measured $A_{xi}+A_{yi}$, $A_{xi}-A_{yi}$, $A_{zi}$, $A_{zi}$, as well as the polarization observables $A_{x}$ and $A_{z}$ for the energy region 325–400 MeV.

II. EXPERIMENT

A. Experimental considerations

The Cooler Ring of the IUCF produces protons of energies up to 500 MeV, with polarization of $P=0.65$, low emittance, and low background. This permits in-beam experiments of reactions with microbarn cross sections. The improvement of beam intensity at the IUCF over time now allows the use of very thin polarized targets. During the $\vec{p}p \rightarrow pn \pi^+$ experiment typical intensities of the stored polarized beam ranged from 100 to 300 $\mu$A.

The apparatus for polarized internal target experiments (PINTEX) makes use of a windowless target cell continuously filled by a polarized atomic hydrogen beam. The measurements cycle through a full set of relative beam and target spin alignments. The technical aspects of beam preparation, electronics, and target and detector properties were reported previously in Ref. [22]. As is customary, the beam is defined to travel in the positive $z$ direction, $y$ is vertical, and $x$ completes a right-handed coordinate system. Below is a brief review of parameters pertinent to the data analysis.

In the experiment we used the Madison atomic beam target with a storage cell of very low mass [23,24]. The storage cell had a length of 25 cm and a diameter of 1.2 cm. This open-ended cylindrical cell produces a triangular shape of the target density distribution with its maximum at the center ($z=0$). It was made of a thin (25 $\mu$m) aluminum foil to keep background events caused by the beam halo to a minimum. A Teflon coating was used to inhibit depolarization of the target atoms. Sets of orthogonal holding coils surround the storage cell. The coils are used to align the polarized hydrogen atoms in the $\pm x$, $\pm y$, and $\pm z$ directions. Typical target polarizations were $Q=0.75$, and the approximate target density was $1.4 \times 10^{13}$ atoms/cm$^2$.

The target spin alignment can be changed in less than 10 ms. During runs the target polarization direction was changed every 2 sec, and followed the sequence $\pm x$, $\pm y$, and $\pm z$. Each data-taking cycle had a constant beam polarization, and was set to last 5–8 min, after which the remaining beam was discarded. The beam polarization was reversed with each new cycle to minimize the effect of apparatus asymmetries. In the first phase of the experiment (run a) the beam spin directions were alternated between $+y$ and $-y$. In the more recent runs (b) solenoid spin rotators were used to give the beam spin a large longitudinal component. This spin rotation was energy dependent and produced roughly equal longitudinal ($z$) and vertical ($y$) spin components and a very small component in the ($x$) direction, as shown in Table I.

Elastic $\vec{p}p$ scattering was used to measure and monitor the three beam polarization components as well as the luminosity. Elastic protons were detected with four plastic scintillators mounted at $\theta=45^\circ$, with $\phi=\pm 45^\circ$ and $\pm 135^\circ$. Coincident protons striking these monitor detectors (labeled $S$ in Fig. 1) pass through wire chamber 1, so the needed tracking information is available. The product $PQ$ of beam polarization ($P$) and target polarization ($Q$) was deduced from the large known spin correlation $A_{x}-A_{y}$ in elastic scattering [25]. A three-dimensional sketch of the detector system is shown in Fig. 1.

The reaction pions in this study had lab energies from 0.1 to 120.5 MeV, and were emitted at polar lab angles from 0$^\circ$ to 180$^\circ$. By contrast, the reaction nucleons remain constrained by kinematics to forward angles below 31.2$^\circ$ and to lab energies from 20.8 to 227.9 MeV. This range of angles and energies affects the choice of detectors that can be employed. If both outgoing nucleons are protons as in $pp \rightarrow pp \pi^0$, one can ignore the pion and use a moderate size forward detector to intercept almost all ejectiles of interest [22]. This procedure was used for the simultaneously measured $pp \rightarrow pp \pi^0$ reaction [26]. The corresponding procedure for $pp \rightarrow pn \pi^+ \pi^-$ is to mount a large area neutron hodoscope behind the proton detectors, and determine the energies of the detected neutrons by time of flight. The con-
TABLE I. Beam energies, integrated luminosities for the $p + \pi^+$ measurements, and the products of beam and target polarization for runs $a$ and $b$. (No $p + n$ data were taken in run $a$. The $p + n$ measurements began in the middle of run $b$ and have correspondingly lower integrated luminosities.)

<table>
<thead>
<tr>
<th>Run $a$</th>
<th>Run $b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (MeV)</td>
<td>$\int L dt$ (nb$^{-1}$)</td>
</tr>
<tr>
<td>325.6</td>
<td>2.163</td>
</tr>
<tr>
<td>350.5</td>
<td>0.901</td>
</tr>
<tr>
<td>375.0</td>
<td>3.024</td>
</tr>
<tr>
<td>400.0</td>
<td>0.831</td>
</tr>
</tbody>
</table>

structure and operation of the neutron hodoscope were described previously in Ref. [27].

All detectors are segmented because the energies of the coincident reaction particles need to be measured independently. Monte Carlo calculations suggest that eight $\Delta \phi$ segments are sufficient, because of the tendency of the ejectiles to have significantly different azimuthal angles. The $K$ detector was needed to obtain the necessary stopping power for the more energetic pions and protons. Identification of the charged particles was usually accomplished by their time of flight vs energy correlation, where the start signal was supplied by the $F$ detector and the stop signal was provided by the $E$ detector. The pion and proton distributions were generally well separated. Figure 2 shows a typical particle ID spectrum for accepted $p + \pi^+$ coincidence events.

The more energetic ejectiles stop in the $K$ detector. Superior particle identification is obtained by comparing the energies deposited in the $K$ vs $E$ detectors, as seen in Fig. 3.

We measured the polarization observables $A_{ij}$ in two different ways: (1) by measuring the pions directly, in coincidence with protons (the $p + \pi$ method), and (2) by reconstructing pion momenta from the measured proton and neutron momenta (the $p + n$ method). The first method had the advantage of simplicity and a high count rate, but we cannot measure pions at large angles due to the limited detector size. Therefore, the spin-dependent cross-section ratios could be compared with theoretical spin correlation coefficients only at forward angles. The second method is free from this limitation, but at the cost of the low neutron detection efficiency and therefore much lower statistics.

B. Measurement of $p + \pi^+$ coincidences

We accept events with two charged reaction particles ($p$ and $\pi^+$) in coincidence. They must show separate tracks in the wire chambers WC1 and WC2, trigger separate sections of the $E$ detectors, and at least one section of the $F$ detector, but not the scintillator (V) veto. The trajectories of the protons and pions are deduced from the wire chamber position readings. Their angular resolution was limited primarily by multiple scattering in the 1.5-mm-thick $F$ detector and in the 0.18-mm-thick stainless-steel exit foil. Approximate angular resolutions (in the lab system) are $\sigma = 0.5^\circ$ for protons and $1^\circ$ for pions. This resolution was fully sufficient for the angular variations expected.

FIG. 1. The PINTEX detector for the experiment: $F$ is a thin timing detector. $S$ labels one of the four detectors for the elastic $pp$ scattering monitor. WC1 and WC2 are wire chambers. $E$ and $K$ are segmented plastic scintillator stacks that determine the energy of the charged reaction products. $V$ is the charged particle veto detector, and $H$ is the neutron hodoscope.

FIG. 2. Raw ejectile time of flight (channels) vs energy deposited in the $E$ detector by pions, protons, and deuterons. Triggers from two charged particles tracks and (no $K$) were a prerequisite. This spectrum was used for identification of protons and pions.
The good intrinsic angular resolution of the wire chambers was used to check the consistency of the pion and neutron position readouts by tracing elastic protons to the hodoscope bars, and comparing predicted and observed position readings. It was found that from run to run the beam axis and detector symmetry axis could differ slightly in direction and also in their relative $y$ and $z$ coordinates at $z=0$ (the target center). We could also cross check the nominal $z$ separation of the wire chambers, since the separation and location of the hodoscope bars was fixed and well known. Small corrections of 1–3 mm had to be applied in software to the detector positions. After such corrections the remaining systematic angular error of the measured polar angles is about 0.04°.

Charged particles that do not stop in or before the $K$ detector trigger the $V$ detector and are tagged as likely elastic events and generally vetoed. At 400 MeV we reach the design limit of the charged-particle detectors, and the veto detector begins to see (and reject) the most energetic pions at small angles. In deducing the energy spectra account was taken of the differing nonlinearity of light production for protons and pions by the plastic scintillators, as well as of energy losses in the exit foil, the $F$ detector, air, and other materials between the scintillators. After calibrations of all detector segments the detector stack provided an energy resolution for typical reaction protons and pions of about $\Delta E/E = 0.09$ [full width at half maximum (FWHM)]. The missing mass spectra contain a background continuum (see Fig. 4), which at higher beam energies stretches slightly beyond the missing mass peak.

The trajectory traceback indicates that this background is primarily caused by beam halo hitting the Al and teflon components of the target cell. Without a target gas (and the beam heating normally produced by it) almost no background is seen. In order to obtain a realistic background shape near and below the missing mass peak, the target cell was filled with N$_2$. This gas will produce some background of its own, but just as importantly it heats the circulating beam (as the hydrogen gas would) and reproduces the ordinary beam halo. We found that the “N$_2$ spectra” “seen” after the common software cuts looked identical to the background “tail” in the hydrogen missing mass spectra. Therefore, N$_2$ spectra were measured with good statistics, and their shape was later used to correct for background under the missing mass peak. Our statistically most accurate measurements were obtained in the $p + \pi$ mode, i.e., by observing pions and protons in coincidence.

C. Measurement of $p+n$ coincidences

Reaction neutrons in coincidence with protons were detected in a large hodoscope consisting of 16 long plastic scintillator bars. The bars were placed symmetrically about the beam direction in a plane defined by $z=1.48$ m. They were 15 cm deep, and mounted so that their dimension in the $y$ and $x$ directions were 120 and 5 cm, respectively (see Fig. 1). The position in the $y$ direction was determined from the differing arrival times of the scintillator light pulses read out by the top and bottom photomultipliers. The $y$-position resolution was $\sigma \approx 1.7$ cm. At 325 MeV the geometric acceptance for $p+n$ detection is comparable to that for the $pp \rightarrow pp\pi^0$ branch; however, the achievable event detection rate is much smaller because of the low neutron detection efficiency. The neutron pulse height threshold was set as low as practical, and corresponds to 5-MeV electrons for all bars. At this threshold a 15-cm-thick plastic scintillator averages a neutron detection efficiency of about 0.17 for the neutron energies of this experiment [27].

A thicker neutron detector would be more efficient, but along with technical problems it would produce a correspondingly poorer time of flight resolution, since the length of the available flight path was limited to 1.5 m. In this experiment an additional reduction of the neutron detection efficiency arose because the $E$ and $K$ proton detectors are located in front of the neutron hodoscope, and represent a
26-cm-thick (polystyrene) absorber for the reaction neutrons. Resulting neutron losses in this “absorber” range from 30% for the highest energy neutrons to about 90% for those at the very lowest energies. As a consequence the energy-averaged effective neutron detection efficiency was reduced to a value of about 0.07. Since neutron energies are measured and neutron reaction cross sections are known, the effective neutron detection efficiency was reduced to a value of about 0.07. Since neutron energies are measured and neutron reaction cross sections are known [28], corrections for energy-dependent efficiency losses can and have been made, but the loss in the counting rate seriously limited the statistics obtained.

The neutron energy was measured by neutron time of flight. In applying this method we use the correlated proton trigger from $pp \rightarrow pn \pi^+$ in the $F$ detector. Since the proton arrival at the $F$ detector is delayed, one has to use a two-step process: First, the trigger time difference ($F$ detector time minus hodoscope mean time) is measured. Next the timing must be corrected for the proton flight time to the $F$ detector, since the $F$ detector is triggered by the proton after it has traveled about 30 cm before reaching the $F$ detector. This correction is based on the measured proton energy and reconstructed track length. Neutron times of flight (TOF’s) range from 5 to 12 ns.

The dominant contribution to the TOF resolution comes from the 15-cm bar thickness, which constitutes 10% of the flight path and cannot be overcome with the available detectors. Smaller contributions come from the intrinsic timing resolution of the hodoscope (0.4-ns FWHM) and the $F$ detector (0.5-ns FWHM, after amplitude walk correction). We note that the raw time resolution of the $F$ detector is worse than the figure quoted above because of the trigger walk in the electronics and because of the light loss and travel delay of light from parts of the large four-section $F$ detector more distant from the photomultipliers. A substantial improvement was achieved by employing a pulse height compensation function. Overall, we see a neutron time of flight resolution with $\Delta T/T = 0.1$. Therefore, the missing mass (MM) peak for $\pi^+$ from $p+n$ detection is not as sharp as for the corresponding neutron missing mass derived from $p+\pi^+$ events.

III. ANALYSIS

A. Monte Carlo simulations

Our Monte Carlo (MC) simulations of the experiment used the event generator GENBOD of the CERN library. The simulation was used to determine various limiting effects of the apparatus, and to derive corresponding corrections. The code contained the detailed geometry of the detector systems and the density distribution of the gas target. In the MC simulation we took into account the loss of energy of the charged particles before entering the detectors, detector resolutions, charged particle multiple scattering, pion decay in flight, energy-dependent neutron detection efficiency and the probability of nuclear reactions of the reaction neutrons in the $E$ and $K$ detectors. In the MC simulation we have used a $pn$ final-state interaction (FSI) based on the Watson-Migdal theory, and the equations were derived following Morton [29]. We found that at the lower energies the FSI has a large effect on the overall coincidence acceptance. The simulation also provided a guide to the expected energy and angular distribution of the reaction products.

Pion counting losses caused by the limited detector depth are not large enough to be detectable in the shape of spectra; however, the Monte Carlo simulation shows that they must be considered. At 400 MeV the loss for pions is 14% because this fraction of the forward pions is too energetic to stop in the K detector. Only about 0.2% of the reaction protons penetrate past the K scintillators and are vetoed. The loss of high-energy pions at small lab angles may create a small distortion of the 400 MeV $p+\pi$ data. Corrections to the 400-MeV spectra were not made since they would have to be very model dependent. We note parenthetically that the 400-MeV data from $p+n$ coincidences do not have this systematic error, but within statistics they agree with overlapping $p+\pi^+$ results. No “veto” losses are seen at 375 MeV or below.

The finite size of the individual detector segments produces some counting losses, since two sections have to trigger for acceptable events. However, systematic effects for the polarization observables are unlikely since the protons have no strong $\phi$ correlations with the pions. The segmentation used leads to a loss of about 7% in counting statistics for the $p+\pi^+$ branch. There is no such loss for $p+n$ detection. The charged particle detectors cover polar angles between $5^\circ$ and $40^\circ$ in the laboratory frame. Hence a large number of pions miss the detector. The total $p+\pi^+$ coincidence acceptance ranges from 21% at 325 MeV to 15% at 400 MeV.

For $p+n$ detection the MC simulation shows that the acceptance is symmetric about $90^\circ$ although not quite isotropic. [See Fig. 5(a)]. Acceptance losses for $p+n$ coincidences attributable to the detector geometry alone are of the order of 25%. The major cause is the central hole in the proton detectors. After all geometric acceptance losses and detector inefficiencies for neutron detection are taken into account, the computed overall detection efficiency for $pn$ coincidence events is 3.5%. It is seen in Fig. 5(a) that the angular variations of the coincidence efficiency for the reconstructed pion are small. This is so despite the fact that we cannot detect protons at angles $\approx 5^\circ$ and neutrons at angles $\approx 2.5^\circ$, and have reduced coverage by the hodoscope of some azimuthal angles for large neutron polar angles. The Monte Carlo ac-

FIG. 5. Monte Carlo simulation for (a) $p+n$ and (b) the $p+\pi^+$ acceptances, in the center of mass system. The partial acceptance for pions seen in the $p\pi$ diagram at $\theta_{\pi} \approx 70^\circ$ results from the dominating forward boost for low energy pions. The cutoff at $\cos \theta_{\pi} \approx -0.5$ is caused by detector thresholds for the lowest ejectile energies.
ceptance curves for $p+n$ detection suggest that within the statistical accuracy of the experiment the spin-correlation parameters integrated over $\theta_p$ and $\phi_p$ would need no significant correction. Figure 5(b) shows the Monte Carlo simulation for $p+\pi^+$ acceptance as a function of $\cos \theta_p$. For $p+\pi^+$ coincidences the apparatus acceptance is only useful for $\theta_p = 70^\circ$. Therefore, the integrated spin correlation coefficients will be deduced from the combined sets of the $p+\pi^+$ and $p+n$ coincidences.

B. Analysis of $p+\pi^+$ coincidences

The energies of the charged particles are measured by the plastic scintillator systems $E$ and $K$. The calculated momenta of the unobserved particles strongly depend on the energies of the detected ejectiles, so considerable attention was given to a careful energy calibration of all detectors. The complex geometry of the segmented plastic detectors required corrections for light collection that primarily were derived from the observation of elastically scattered protons. An $xy$-position correction factor was applied to account for this dependence. A second pulse height correction factor was applied to compensate for a variation of phototube gains with the orientation of the magnetic guide field for the target polarization. For details see Ref. [22].

The corrected pulse heights $L$ were converted into the deposited energy $E$ using

$$ E = L + k_1 \sqrt{L} + k_0. $$

The nonlinear term corrects for light quenching in plastic scintillators. $k_0$ and $k_1$ are calibration constants. $L$ is the sum of the light pulse from the $E$ and $K$ detectors in MeV, and is given by $L = c_1(E_{\text{light}} + c_2 K_{\text{light}} + c_3)$. The constants $c_1$, $c_2$, and $c_3$ are gain matching constants, and $E_{\text{light}}$ and $K_{\text{light}}$ correspond to the observed light pulses in the $E$ and $K$ detectors, respectively. The constant $c_3$ corrects for small energy losses in the material between the $E$ and $K$ detectors. It is small and set equal to zero when there is no $K$ trigger.

The total kinetic energy of the charged particle was calculated by also taking account of the energy lost by the charged particle on its way to the $E$ detector. The calibration constants were fine tuned by utilizing kinematical relations. We required that the missing mass centroid was at its predicted value and that the angular distribution of the pions from the simultaneous measurement of the reaction $pp \rightarrow pp \pi^0$ was symmetric in the center-of-mass (c.m.) system about $\theta_\pi = 90^\circ$. This symmetry was sensitive to the relative size of the calibration coefficients. However, the variation of the deduced spin correlation coefficients under different reasonable combinations of the calibration constants was small and less than the statistical errors.

Figure 6 shows the directly observed $\pi^+$ differential cross sections plotted against $\cos \theta_\pi$ in the c.m. coordinate system. We note that there are almost no counts for pion back angles $-1 < \cos \theta_\pi < 0$, as expected from the apparatus acceptance [compare Fig. 5(b)].

Figure 7 shows missing mass spectra seen at 325 MeV for four combinations of vertical beam and target polarization at equal integrated luminosity. The polarization observables are obtained from the ratio of “yields” for different spin orientations. The yields to be used are the integrated counts inside the missing mass gates minus background. In order to estimate the error from uncertainties in the background we varied the background subtraction by \pm 25%. The effect on the final results was smaller than the statistical error. At 325 MeV the off-line resolution of the neutron missing mass peak was $\sigma = 1.4$ MeV/c². Even before software cuts and background correction it is apparent from Fig. 7 that different spin combinations produce very different yields.

It turns out that the decay in flight of pions plays a negligible role for these data. It will appreciably affect only the (undetected) backward scattered pions as these have much

![FIG. 6. Detected pions at 325 MeV as a function of $\theta_\pi$. Only events with $\cos \theta_\pi \geq 0.4$ were used for the analysis.](image)

![FIG. 7. Distributions of the calculated missing mass $m_x$ for $p + \pi^+$ detection at 325-MeV bombarding energy, for the four combinations of vertical beam and target polarization. A sharp peak ($\approx 3.5$ MeV/c² FWHM) is seen at 939.6 MeV/c², the neutron rest mass. The shaded region indicates the background distribution.](image)
lower lab energies than the forward pions. For the final analysis we selected the $pp \rightarrow pn \pi^+$ events of interest by using a gate of 30 MeV or wider over the relevant missing mass peak. Gates as narrow as 10 MeV did not produce systematic changes, and neither did they measurably reduce background induced errors. However, the narrower gates lead to some loss of statistics.

C. Analysis of $p+n$ coincidences

This detection channel has the advantage that the acceptance for the detection of $p+n$ coincidences has little angular variation. So the $\theta_{\pi}$- and $\phi$-dependent acceptance corrections generally can be ignored. Therefore, the $p+n$ coincidences importantly complement the $p+\pi^+$ channel. Reliance on $p+n$ angular distributions at large angles leads to larger statistical error bars relative to the $p+\pi^+$ (forward) region. However, the combination of the two detection modes provides data for the full angular range, and so keeps the integrated spin correlation coefficients model independent.

In the $p+n$ analysis we first analyzed only those events where all three reaction particles ($p$, $n$, and $\pi^+$) were detected (the triple coincidence). Next we evaluated the case where the pions missed the $E$ detector, but a proton and a neutron were detected (double coincidence). The energy of reaction protons was determined using the calibration constants described above. The energy of the neutrons was determined by measuring their TOF to the hodoscope. The MC simulation showed that, although the $F$ detector was always triggered by protons for a $p+n$ double coincidence, in the case of a $pn \pi^+$ triple coincidence it was triggered by the faster pions. Therefore, depending on the event class, we corrected the neutron TOF by adding the time it takes either for the coincident proton or the pion to reach the $F$ detector. A calculated offset was added to the timing signal of each hodoscope bar in order to make the timing information independent of the bar electronics. This correction was obtained by calibrating the timing circuits with elastic proton scattering.

For $\theta_{\pi,lab} \leq 40^\circ$ we observe $pn \pi^+$ triple coincidences, which are practically free of background. The absence of accepted events from the $N_2$ gas target showed that the triple $pn \pi^+$ coincidence signal contains no background from any competing reaction. The missing mass peak as in Fig. 8, but there is no “background tail” at all. The triple coincidence spectrum confirms the background subtraction shown in Figs. 4 and 7.

If the same triple coincidence events are used to calculate the (pion) missing mass by using the proton and neutron momenta (i.e., ignoring the simultaneously known pion momenta) we obtain the spectrum shown in Fig. 8. This spectrum can be used as a standard for the missing mass that $p+n$ (double coincidence) events would have in the absence of background.

The MM distribution peaks at the true pion mass of 139.6 MeV, but there also is a “tail” over a wide range of the missing mass spectrum which is not background related. We conclude that the counts in the MM tail of Fig. 8 represent genuine $pn \pi^+$ events from the hydrogen target, albeit events with poorly determined neutron momenta. We estimate that up to 20% of the $p+n$ coincidences contain neutron observables that are distorted by interactions of neutrons with the $K$ or $E$ detectors. That is, neutrons can undergo small angle elastic and inelastic scatterings, but still reach the hodoscope. This would lead to incorrect readings for polar and azimuthal neutron angles and hence to an incorrect missing mass calculation.

Such events with poorly determined missing masses were excluded from further analysis. For all $p+n$ events we reduce genuine background and avoid analyzing measurably distorted $p+n$ events by using a missing mass gate from 100 to 160 MeV.

Using the triple coincidence MM spectrum as a standard, the background under the missing mass peak for two-particle $p+n$ coincidences was deduced by adding a fraction of the measured unstructured $N_2$ background continuum to the “standard” MM spectrum until the observed $p+n$ MM spectrum shape was reproduced. The tail in the latter is flatter and more pronounced because of actual background contributions. To estimate the error in this procedure we varied the match until it became unrealistic ($\pm 15\%$). A typical missing mass spectrum for $p+n$ (double coincidence) detection is shown in Fig. 9. In the final result the uncertainty from this background subtraction was about half as large as the statistical error.

At 375 MeV the resolution of the pion MM peak was $\sigma = 9$ MeV/c$^2$. Pion angular and energy distributions from $p+n$ detection were computed using only events inside this missing mass gate. Some resulting distributions are compared with Monte Carlo projections for the laboratory coordinate system in Fig. 10. The end points of these distribu-
tions agree well with the kinematics of the experiment as they must. The solid curves represent pure \( l_x=0 \) MC calculations. Although \( l_x=0 \) makes the major contribution, this MC assumption produces oversimplified energy and angular distributions. Nevertheless, the simulated distributions agree reasonably well with the data.

Figure 11 shows the deduced pion angular distribution in the center-of-mass system. The reconstructed \( \pi^+ \) distribution is plotted against \( \cos \theta_x \) in the center of mass [corrected for background and for the slightly nonuniform acceptance shown in Fig. 5(a)]. As expected, it is nonisotropic and symmetric about \( \theta_x=90^\circ \) within statistical errors.

IV. POLARIZATION OBSERVABLES

A. Formalism for spin correlation coefficients

The meaning of the symbols \( A_{ij} \) used for polarization observables is defined by Eq. (2). In terms of the “Cartesian polarization observables” the spin-dependent cross section is written as

\[
\sigma(\xi, \bar{P}, \bar{Q}) = \sigma_0(\xi) \left[ 1 + \sum_i P_i A_{i0}(\xi) + \sum_j Q_j A_{j0}(\xi) \right. \\
+ \sum_{i,j} P_i Q_j A_{ij}(\xi) \left. \right].
\]

where \( \xi \) stands for the pion coordinates \( \theta_x \) and \( \varphi_x \), the energy defining pion momentum \( p_x \), and the proton coordinates \( \theta_p \) and \( \varphi_p \). The unpolarized cross section is \( \sigma_0(\xi) \), and the polarization of the beam and the target is denoted by the vectors \( \bar{P}=(P_x, P_y, P_z) \) and \( \bar{Q}=(Q_x, Q_y, Q_z) \). The subscripts \( i,j \) stand for \( x, y, \) or \( z \), and the sums extend over all possibilities. The resulting 15 polarization observables include the beam analyzing powers \( A_{i0} \), the target analyzing powers \( A_{0j} \), and the spin correlation coefficients \( A_{ij} \).

The partial wave analysis for \( \bar{p}p \rightarrow pn \pi^+ \) is similar to that for \( \bar{p}p \rightarrow pp \pi^0 \) in terms of transition amplitudes. However, the \( pp \rightarrow pn \pi^+ \) transitions have isoscalar as well as isovector components. The different isospins in \( \bar{p}p \rightarrow pn \pi^+ \) modify the selection rules for the reaction, and lead to polarization observables that are different. The general relations between reaction amplitudes and angular distributions, however, remain almost identical. The applicable partial wave formalism was discussed in detail in Ref. [26]. We use the same notation as in Ref. [26], and reiterate some relevant definitions and theoretical relations below. Several names are in use for polarization observables. Their meaning is as defined below:

\[
A_2(\xi) = A_{xx}(\xi) + A_{yy}(\xi),
\]

\[
A_4(\xi) = A_{xx}(\xi) - A_{yy}(\xi),
\]

\[
A_6(\xi) = A_{xy}(\xi) - A_{yx}(\xi).
\]

For identical particles in the entrance channel there are seven independent polarization observables:
\[ A_{\gamma 0}(\xi), \ A_{\Sigma}(\xi), \ A_{zz}(\xi), \ A_{x}(\xi), \ A_{\Delta}(\xi), \]
\[ A_{\Xi}(\xi), \ A_{\zeta 0}(\xi). \]
(4)

This paper addresses the first five observables of this set. The remaining two, \( A_{\Xi}(\xi) \) and \( A_{\zeta 0}(\xi) \), can be nonzero only for noncoplanar final states. In the following we will integrate over the angles of the nucleon, and thus these two observables vanish if parity is conserved.

\[ p_{\pi, \text{max}} = \frac{1}{2\sqrt{s}} \sqrt{[s - (m_\pi + m_\pi^\pm)^2][s - (m_n + m_p - m_\pi^\pm)^2]}, \]
(6)

where \( \sqrt{s} \) is the total center-of-mass energy, and \( m_p, m_n \), and \( m_\pi^\pm \) are the masses of the proton, neutron, and pion, respectively. (We explicitly labeled the pion as \( \pi^\pm \) to emphasize that the \( \pi^\pm \) and \( \pi^0 \) mass difference matters here.)

Below we quote some useful relations between integrated spin correlation coefficients and some directly observable spin dependent cross sections. For two colliding spin-1/2 particles, one can define three total cross sections, two of which depend on the spin. The total cross sections are related to the observables above by

\[ \sigma_{\text{tot}} = \int \sigma_0(\xi) d\Omega_\rho d\Omega_\sigma dp_\pi, \]
(7a)

\[ \Delta \sigma_T = - \int \sigma_0(\xi) A_{\Sigma}(\xi) d\Omega_\rho d\Omega_\sigma dp_\pi, \]
(7b)

\[ \Delta \sigma_L = - 2 \int \sigma_0(\xi) A_{zz}(\xi) d\Omega_\rho d\Omega_\sigma dp_\pi. \]
(7c)

Here \( d\Omega = d\cos \theta d\varphi \), and the integration extends over \( 0 \leq \theta \leq \pi \), and all pion momenta. \( \Delta \sigma_T/\sigma_{\text{tot}} \) and \( \Delta \sigma_L/\sigma_{\text{tot}} \) can have values between -2 and +2.

The integrated spin correlation coefficients are defined as

\[ A_{\Sigma} = \int [\sigma_0(\xi) A_{\Sigma}(\xi) d\Omega_\rho d\Omega_\sigma dp_\pi]/\sigma_{\text{tot}}, \]
(8a)

\[ A_{zz} = \int [\sigma_0(\xi) A_{zz}(\xi) d\Omega_\rho d\Omega_\sigma dp_\pi]/\sigma_{\text{tot}}, \]
(8b)

\[ A_{\Delta} = \int [\sigma_0(\xi) A_{\Delta}(\xi) \sin \theta_\pi d\theta_\pi]/\sigma_{\text{tot}}, \]
(8c)

\[ A_{xz} = \int [\sigma_0(\xi) A_{xz}(\xi) \sin \theta_\pi d\theta_\pi]/\sigma_{\text{tot}}, \]
(8d)

\[ A_{\gamma 0} = \int [\sigma_0(\xi) A_{\gamma 0}(\xi) \sin \theta_\pi d\theta_\pi]/\sigma_{\text{tot}}. \]
(8e)

We note that \( A_{\Sigma} \) and \( A_{\Delta} \) differ by a scale factor from \( \Delta \sigma_T \) and \( \Delta \sigma_L \). These quantities can in principle be measured directly, although in this study they are derived from integration over \( A_{\Sigma}(\cos \theta_\pi) \) and \( A_{zz}(\cos \theta_\pi) \). The remaining three integrals must be defined differently. Here the spin correlation \( A_{ij} \) are taken at \( \phi_\pi = 0 \). (They cannot be integrated over the variable \( \phi_\pi \), since they would vanish, as will be seen below).

Based on the dominance of \( S_s \), \( S_p \), \( P_s \), and \( P_p \) transitions, general symmetries and spin coupling rules [26], the cross sections and spin correlation coefficients must have the general forms:

\[ \sigma_0(\xi) = a_{00} + b_{00}(3 \cos^2 \theta_\pi - 1) + c_0(3 \cos^2 \theta_\pi - 1) + d_0(3 \cos^2 \theta_\pi - 1)(3 \cos^2 \theta_p - 1) + e_0 \sin 2 \theta_p \sin 2 \theta_\pi \cos \Delta \varphi \]
\[ + f_0 \sin^2 \theta_\pi \sin^2 \theta_p \cos 2 \Delta \varphi, \]
(9a)

\[ \sigma_0(\xi) A_{\Sigma}(\xi) = [a_{\Sigma} + b_{\Sigma}(3 \cos^2 \theta_\pi - 1)] \sin \theta_\pi + c_{\Sigma}(3 \cos^2 \theta_\pi - 1) + d_{\Sigma}(3 \cos^2 \theta_\pi - 1)(3 \cos^2 \theta_p - 1) + e_{\Sigma} \sin 2 \theta_p \sin 2 \theta_\pi \cos \Delta \varphi \]
\[ + f_{\Sigma} \sin^2 \theta_\pi \sin^2 \theta_p \cos 2 \Delta \varphi, \]
(9b)

\[ \sigma_0(\xi) A_{zz}(\xi) = a_{zz} + b_{zz}(3 \cos^2 \theta_\pi - 1) + c_{zz}(3 \cos^2 \theta_\pi - 1) + d_{zz}(3 \cos^2 \theta_\pi - 1)(3 \cos^2 \theta_p - 1) + e_{zz} \sin 2 \theta_p \sin 2 \theta_\pi \cos \Delta \varphi \]
\[ + f_{zz} \sin^2 \theta_\pi \sin^2 \theta_p \cos 2 \Delta \varphi, \]
(9c)

\[ \sigma_0(\xi) A_{\gamma 0}(\xi) = a_{\gamma 0} + b_{\gamma 0}(3 \cos^2 \theta_\pi - 1) + c_{\gamma 0}(3 \cos^2 \theta_\pi - 1) + d_{\gamma 0}(3 \cos^2 \theta_\pi - 1)(3 \cos^2 \theta_p - 1) + e_{\gamma 0} \sin 2 \theta_p \sin 2 \theta_\pi \cos \Delta \varphi \]
\[ + f_{\gamma 0} \sin^2 \theta_\pi \sin^2 \theta_p \cos 2 \Delta \varphi, \]
(9d)

It is common to display the bombarding energy dependence of the observables in terms of the dimensionless parameter \( \eta \), which is defined as

\[ \eta = p_{\pi, \text{max}} / m_\pi^+. \]
(5)

The term “near threshold” is meant to include the energy region with \( \eta < 1 \), i.e., below 400 MeV. Setting \( c = h = 1 \), the maximum value of the \( \pi^+ \) momentum is found from
\[ \sigma_0(\xi)A_\Delta(\xi) = [a_\Delta + b_\Delta(3\cos^2\theta_p - 1)]\sin^2\theta_p\cos 2\varphi_p + e_\Delta\sin 2\theta_p\sin 2\varphi_p(\cos \varphi_p + \varphi_p), \]  
\[ \sigma_0(\xi)A_{\Delta z}(\xi) = [(a_{\Delta z} + b_{\Delta z}(3\cos^2\theta_p - 1))\sin \theta_p + e_{\Delta z}\cos \theta_p + f_{\Delta z}\sin \theta_p(\cos \varphi_p + \varphi_p)] \times \sin^2\theta_p\cos 2\varphi_p + f_{\Delta z}\sin 2\theta_p\sin^2\varphi_p(\cos 2\varphi_p - \varphi_p). \]

Here we have used the abbreviation \( \Delta \varphi = \varphi_p - \varphi_p \). Equations (9) explicitly depend on the four angles \( \theta_p, \varphi_p, \theta_\varphi, \) and \( \varphi_\varphi \). The energy-dependent parameter \( p_\varphi \) is contained in the coefficients. Statistics in this experiment are not sufficient to present double or higher differential cross sections. Therefore, we integrate over the angles of the proton and use energy and momentum conservation to eliminate all angles except \( \theta_\varphi \) and \( \varphi_\varphi \). This leads to a set of much simpler equations:

\[ \sigma_0(\xi) = a_{00} + b_{00}(3\cos^2\theta_p - 1), \]

\[ \sigma_0(\xi)A_{\varphi_\varphi}(\xi) = [a_{\varphi_\varphi} + c_{\varphi_\varphi}\sin 2\theta_p]\cos \varphi_\varphi, \]

\[ \sigma_0(\xi)A_\varphi(\xi) = [a_\varphi + b_\varphi(3\cos^2\theta_p - 1), \]

\[ \sigma_0(\xi)A_{\varphi_\varphi}(\xi) = [a_{\varphi_\varphi} + b_{\varphi_\varphi}(3\cos^2\theta_p - 1), \]

\[ \sigma_0(\xi)A_\Delta(\xi) = [a_\Delta \sin \theta_p + b_\Delta \sin 2\theta_p]\cos 2\varphi_\varphi, \]

\[ \sigma_0(\xi)A_{\Delta z}(\xi) = [a_{\Delta z} \sin \theta_p + b_{\Delta z} \sin 2\theta_p]\cos \varphi_\varphi. \]

The symbol \( \xi \) now represents the reduced set of variables \( \{p_\varphi, \theta_\varphi, \varphi_\varphi\} \). These equations display a simple and characteristic \( \varphi_\varphi \) dependence of the different polarization observables, and show the expected \( \theta_\varphi \) dependence. The coefficients \( a_{e^1}, b_{e^1}, \ldots \) for set (10) correspond to those in Eqs. (9). They are obtained by one- or two-parameter fits to the observed angular distributions, separately for each observable.

### B. Extraction of polarization observables

The data analysis, as described in the previous sections, identifies the reaction particles, assesses the background for each spectrum, and calculates the kinematic variables and spin-dependent cross sections of the reaction products. It produces event files which contain kinematically complete information for all detected reaction particles. For each beam energy there are 12 such event files, one for each combination of beam and target spin. These yields are first corrected for the beam luminosity, which can vary for beam “spin-up” and “spin-down” subcycles, and for the background measurement. The background correction was made for each selected \( \theta_\varphi \) angle bin individually. The ratios \( R_i \) of yields for different spin combinations, integrated over a chosen \( \theta_\varphi \) range, are then analyzed as a function of \( \varphi_\varphi \), because the allowed \( \varphi_\varphi \) dependence can be predicted from spin coupling rules [21]. For this energy range, only final states with \( p_\varphi \) or pion angular momenta of 0 and 1 are expected to be significant. In a previous measurement of \( pp \to d^+ \pi^- \) at 400 MeV, it was found that any \( l_\varphi = 2 \) contribution is very small [30]. This allows us to consider only transitions to \( S \), \( S_p \), \( S_\varphi \), and \( Pp \) final states in the analysis. \( Sd \) and \( Ds \) transitions would affect the energy dependence of the coefficients only, and so are very difficult to separate from \( Pp \) transitions [26]. They will be ignored in this analysis. We then have explicit predictions for the expected \( \theta \) and \( \phi \) dependences from Eq. (10).

The combination of \( p + \pi^+ \) and \( p + n \) measurements provides model-independent values for the polarization observables for all polar and azimuthal angles of the pion. The low neutron detection efficiency and the resulting low statistical accuracy of the \( p + n \) data make it advisable to display the combined data using some theoretical guidance. As shown below, \( A_\varphi(\theta_\varphi), A_\Delta(\theta_\varphi), \) and \( A_{\Delta z}(\theta_\varphi) \) must be symmetric about \( \theta_\varphi = 90^\circ \) for the transitions considered. So a good analysis in terms of the pion coordinates does not require the (redundant) data at large polar angles. This simplification, and the fact that all published theoretical predictions have been presented in terms of the pion coordinates, make these coordinates our preferred system for the analysis.

The microscopic relations between the coefficients and the transition amplitudes can be derived from the partial-wave expansion described in the Appendix of Ref. [26], but they are complicated. Moreover, the number of individual \( pp \to pn \pi^+ \) amplitudes contributing above 350 MeV has become too large (19 rather than 12 for \( pp \to pp \pi^+ \)), since isospin 1 and 0 are present in the final state. They could not be deduced individually from the \( pp \to pn \pi^+ \) data available.

When calculating the value of a polarization observable from Eqs. (9) or (10), one evaluates the ratio \( A_{ij}(\xi) = \sigma_0(\xi)A_{ij}(\xi)/\sigma_0(\xi) \), so the overall normalization of all terms in these equations cancels. As seen from Eqs. (10) the yield ratios \( R_i(\varphi_\varphi) \) could either be constant or have a \( \varphi_\varphi \) or \( 2 \varphi_\varphi \) dependence. This is borne out by the data (compare Fig. 12).

The polarization observables were deduced by evaluating the observed \( \varphi_\varphi \) dependences of the ratios \( R_i \) for selected beam and target spin combinations. This evaluation is complex when longitudinal as well as transverse beam polarizations are present at the same time. Therefore, the devolution process uses the computerized fitting routine BMW [31], which was written for this purpose.

Figure 12 shows the \( \varphi_\varphi \) dependence of six spin-dependent yield ratios. The data for the beam (first arrow) and target spin combinations indicated have been integrated over all coordinates other than the coordinate \( \varphi_\varphi \). The curves are fits using one to three components of Eqs. (10). The first three rows present different ways to extract the analyzing power \( A_{ij}(\varphi_\varphi) \). The lower three rows contain information on \( A_\varphi = A_{\varphi\varphi} + A_{\varphi\gamma}, \) \( A_\Delta = A_{\Delta\varphi} - A_{\Delta\gamma}, \) and contribu-
was obtained to sufficient accuracy from the p spin, and the detector acceptances cancel out for a given angle.

The correlation we use repeatedly is
detectors only cover forward polar angles for the direct detection of pions. The correlation we use repeatedly is

tions from $A_{x\Sigma}$ and $A_{x\Delta}$. For some ratios the statistics are marginal, and one cannot exclude the potential presence of l components higher than included in the analysis, but the $\phi_\pi$-dependent fits show that the inclusion of $S_\pi$, $S_p$, $P_\pi$, and $P_p$ transitions is sufficient to reproduce the data within experimental errors.

For the simultaneous detection of neutrons and protons, our data sample the full range for $\theta_\pi$, although with low statistics. We combine the $p+\pi$ and $p+n$ data sets to obtain optimal spin correlation coefficients for the full angular region. We avoid difficulties generated by the nonuniform detector acceptances in $\theta_\pi$ by evaluating the $p+n$ and $p+\pi$ relations $A_{ij}(\cos \theta_\pi)$, which are ratios of cross sections at a given angle. [Our detection efficiency does not depend on spin, and the detector acceptances cancel out for $A_{ij}(\cos \theta_\pi)$.] The combined $p+n$ and $p+\pi^+$ sets yield complete angular distributions with their best statistics at forward angles. The unpolarized angular distribution $\sigma_0(\theta_\pi)$ was obtained to sufficient accuracy from the $p+n$ branch. The angular distributions can now be integrated. To best account for experimental errors, we have chosen to integrate Eqs. (10) directly after the fitting coefficients are deduced.

Some of the polarization ratios measured are not independent, as the first three rows in Fig. 12 show. The reaction has additional redundancies. If parity is conserved and if we have identical particles in the entrance channel, this redundancy can give us back-angle information for $A_{1\pi}$ even though our detectors only cover forward polar angles for the direct detection of pions. The correlation we use repeatedly is

$$A_{ij}(\theta_\pi, \phi_p, \theta_\pi, \phi_\pi) = A_{ij}(\pi - \theta_\pi, \phi_p + \pi, \pi - \theta_\pi, \phi_\pi + \pi).$$

This relation holds for $i \neq j$ and also for $i = j$. That is, since both $A_{x\Sigma}$ and $A_{x\Delta}$ are measured at forward angles, we will obtain the back angle information for $A_{x\pi}$ from the $A_{x\pi}$ measurement at forward angles. The polarization observables $A_{x\Sigma}$ and $A_{x\Delta}$ are not symmetric about $\theta_\pi = 90^\circ$, so this redundancy becomes very useful.

V. RESULTS

A. Polarization observables

It follows from Eq. (11) that the observables $A_{x\Sigma}$, $A_{x\Delta}$, and $A_{x\pi}$ are symmetric about $\theta_\pi = 90^\circ$ ($\cos \theta_\pi = 0$). Within statistical errors the experimental data agree with this expectation. In Fig. 13 we have reduced the scatter from the low statistics of the $p+n$ coincidences by combining the corresponding data for forward and backward polar angles. The data for $\cos \theta_\pi = 0.5$ are dominantly determined by events from $p+\pi$ coincidences. In agreement with theoretical expectations, there is only a slow dependence on the polar angle, so the lack of good statistics near $\theta_\pi = 90^\circ$ does not impede comparison with theory or the extraction of good values for the integrated polarization observables.

Figure 14 shows results for $A_{x}(\theta_\pi)$ and $A_{x\pi}(\theta_\pi)$ for the full angular range, so potential asymmetries can be seen. The statistically most accurate data were obtained for 375 MeV. Here and at 400 MeV the Jülich model is at odds with the data. The fit with Eq. (10) (solid lines) does much better. Still, a close inspection of the fits shows some small, but statistically significant differences between the partial wave curve and the data at very small and very large angles. We also see from Table II that the $\chi^2$ value for the $A_2$ fit has become large. The $A_2$ data suggest that higher partial waves enter at 375 MeV, but the experimental uncertainties discourage the extraction of relatively small contributions.

The fits obtained with Eq. (10) are good (i.e., $\chi^2 \approx 1$ for all curves except for $A_{x\pi}$ at 375 and 400 MeV). Therefore Eqs. (10) together with the coefficients of Table II can be used to represent the new data. The coefficients in these equations are bilinear sums of the reaction amplitudes. Their experimental values are given in Table II. This set is also used to obtain the integrated spin correlation coefficients. Integration of the angular distributions shown above produces the spin correlation coefficients in Cartesian coordinates. These coefficients were the original objective of this experiment. They are now known with good statistical accuracy, and are given in Table III. A comparison of these integrated polarization observables as a function of beam energy with predictions of the Jülich model is shown in Fig. 15.

For completeness we note that our attempt to extract noncoplanar angular distributions for $A_{1\Sigma}$ produced only small negative values with large statistical errors (not shown). At 375 MeV our results are consistent with zero. They still agree with the $pp \rightarrow pp \pi^0$ results, provided we assume a negligible contribution for the isoscalar component.

It is clear from Figs. 13, 14, and 15 that the distributions
based on the Jülich model are in good agreement with the data at 325 MeV. However, above $\eta=0.7$ they produce ever larger $\chi^2$ values when compared to the data. These disagreements become striking for $A_S$ and $A_y$. The failures are most visible for $A_y$, an observable sensitive to admixtures of higher partial waves. (More serious disagreements with this model have been seen for the isovector production in $pp \rightarrow pp\pi^0$ [26]. However, as discussed below, in this energy region isovector terms contribute less than 10% to the $pp \rightarrow pn\pi^+$ cross section. The observed differences in $pp \rightarrow pn\pi^+$ grow well beyond this level.)

Our partial wave analysis, which includes $S_s$, $S_p$, $P_s$, and $P_p$ transitions, generally provides fits to the measured angular distributions with $\chi^2$ (per degree of freedom) values near 1. The exceptions are $A_y$ at 375 and at 400 MeV, where the cross sections are largest and the statistics are good. Some $\chi^2$ values as large as 3.9 are found if only statistical errors are considered.

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TABLE II. Coefficients for the fits with Eqs. (10) that reproduce the measured angular distributions of the polarization observables. The associated Legendre polynomials used for the fits are determined by selection rules for $Ss$, $Sp$, $Ps$, and $Pp$ transitions. The unpolarized angular distribution $\sigma_d(a_{00},b_{00})$ is given in arbitrary units by setting $a_{00}=1$. The errors listed refer to the individual fitting coefficients. The $\chi^2$ numbers give the overall quality of the fit to the data per degree of freedom. The fits are shown in Figs. 11, 13, and 14.

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<th>Name</th>
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<td>0.1</td>
<td>-0.177</td>
<td>0.047</td>
<td>0.8</td>
<td>-0.310</td>
<td>0.018</td>
<td>1.0</td>
<td>-0.431</td>
<td>0.037</td>
<td>3.1</td>
</tr>
<tr>
<td>$a_{zz}$</td>
<td>-0.188</td>
<td>0.054</td>
<td>0.1</td>
<td>-0.257</td>
<td>0.057</td>
<td>-0.233</td>
<td>0.021</td>
<td>-0.199</td>
<td>0.043</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{zz}$</td>
<td>-0.247</td>
<td>0.015</td>
<td>0.5</td>
<td>-0.255</td>
<td>0.013</td>
<td>0.9</td>
<td>-0.276</td>
<td>0.005</td>
<td>2.4</td>
<td>-0.285</td>
<td>0.008</td>
<td>3.9</td>
</tr>
<tr>
<td>$c_{yz}$</td>
<td>0.007</td>
<td>0.013</td>
<td>0.5</td>
<td>0.050</td>
<td>0.010</td>
<td>0.44</td>
<td>0.005</td>
<td>-0.032</td>
<td>0.007</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{xz}$</td>
<td>-0.051</td>
<td>0.042</td>
<td>0.7</td>
<td>0.021</td>
<td>0.042</td>
<td>1.2</td>
<td>0.053</td>
<td>0.021</td>
<td>1.1</td>
<td>-0.041</td>
<td>0.041</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The values for the product $P^*Q$ are known to good precision (see Table I), but errors for the beam ($P$) or target ($Q$) polarization individually are not negligible at the lower energies. Changes in $P$ and $Q$ affect only the analyzing powers $A_s(\theta)$. They could reduce or increase the asymmetry of the angular distributions. Typically, the uncertainties in $P$ are smaller than the statistical errors.

B. Discussion and comparison with other work

The statistical and fitting errors listed in Tables II and III include all known and estimated random errors. As explained above, all angles were measured simultaneously, and systematic normalization errors for $A_{ij}$ are unlikely. Based on the detector design and redundant measurements, we expect that all systematic errors have remained small. In the center region ($\cos \theta=0$) the angular distributions show large statistical errors. However, these data points do not materially affect the partial wave fits or the integrals. We note that our initial results reported in Ref. [19] were subject to some model dependence that is absent here. Nevertheless, they are consistent with the final results presented here. Noticeable asymmetries around 90° have been seen for $A_\gamma$ above 350 MeV. In the framework of our partial-wave analysis this asymmetry must be produced by $Pp$ transitions. (At higher energies such asymmetries can also be produced by $Ds$ and $Sd$ transitions.) With the possible exception of the analyzing powers $A_s(\theta)$ at 375 and 400 MeV, the pion production data are well represented by the partial wave predictions based on the assumptions of $Ss$, $Ps$, $Sp$, and $Pp$ transitions. The Jülich model predictions and the data agree for $A_\Delta$. However, we see serious disagreements for $A_S$ and $A_p$ as the beam energy increases. Reference [11] included more amplitudes than our analysis, but the calculations predicted little asymmetry for $A_s(\theta)$. The differences for $A_\gamma$ and the increasing divergence with energy are also seen in Fig. 15. At this time there are no predictions available for $A_{xz}$ and $A_{z0}$.

C. Deduction of important partial waves

The number of contributing partial waves grows rapidly with energy. If we restrict ourselves to $Ss$, $Sp$, $Ps$, and $Pp$ contributions as above, the 19 individual amplitudes listed in Table IV are needed for a detailed interpretation of the data. There are 12 isoscalar amplitudes and seven isovector amplitudes. The experimental information available includes the

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TABLE III. Beam energy, the $\eta$ parameter, and the deduced integrated spin correlation coefficients. The table gives the weighted average of all runs as shown in Fig. 15.

<table>
<thead>
<tr>
<th>$T$ (MeV)</th>
<th>$\eta$</th>
<th>$\overline{A}_S$</th>
<th>$\overline{A}_\Delta$</th>
<th>$\overline{A}_{zz}$</th>
<th>$\overline{A}_{z0}$</th>
<th>$\overline{A}_{xz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>325.6</td>
<td>0.464</td>
<td>-0.533±0.046</td>
<td>-0.027±0.064</td>
<td>0.148±0.041</td>
<td>-0.209±0.011</td>
<td>-0.043±0.044</td>
</tr>
<tr>
<td>350.5</td>
<td>0.623</td>
<td>-0.761±0.046</td>
<td>0.001±0.070</td>
<td>-0.143±0.040</td>
<td>-0.218±0.011</td>
<td>0.018±0.036</td>
</tr>
<tr>
<td>375.0</td>
<td>0.753</td>
<td>-0.945±0.068</td>
<td>0.062±0.036</td>
<td>-0.283±0.016</td>
<td>-0.237±0.004</td>
<td>0.045±0.019</td>
</tr>
<tr>
<td>400.0</td>
<td>0.871</td>
<td>-1.026±0.023</td>
<td>0.056±0.034</td>
<td>-0.414±0.032</td>
<td>-0.244±0.011</td>
<td>-0.035±0.036</td>
</tr>
</tbody>
</table>
three cross sections $\sigma_{tot}$, $\Delta \sigma_T$, and $\Delta \sigma_L$ for $pp \rightarrow pn\pi^+$ that are related to these amplitudes. In addition a recent $pp \rightarrow pp\pi^0$ study provides the three relevant isovector cross sections $\sigma'_{tot}$, $\Delta \sigma'_T$, and $\Delta \sigma'_L$ (Ref. [26], Table V). As long as isospin is a good quantum number these cross sections also give the isovector part of the $pp \rightarrow pn\pi^+$ reaction if taken at the same $\eta$. So one has six new measurements for 19 variables. This necessitates some restriction of the further analysis. In a previous $pp \rightarrow pn \pi^+$ study [18], closer to threshold, the partial-wave space was restricted to the lowest isoscalar amplitudes $a_0, a_1, a_2$, and to the lowest known isovector amplitudes. With this simplification and with reliance on the measured analyzing powers three amplitudes were deduced for $\eta \leq 0.5$. Some of these earlier results will be shown below. It will become apparent in comparison with our data that the angular momentum space considered in Ref. [18] is too small for $\eta > 0.3$. For $0.3 < \eta < 0.9$ it becomes necessary to consider all $Ss$, $Sp$, $Ps$, and $PP$ contributions. In order to reduce the number of variables we use similarities in the spin algebra coefficients for the 19 amplitudes of interest. A suitable combination of the 19 partial cross sections into six groups allows us to find the $Sp$ and $Ps$ strengths separately to deduce the lowest $Pp$ isoscalar partial cross section for the amplitude $a_3$ directly, and to put a close upper limit on the $Ss$ contributions. We will identify the isoscalar partial wave cross sections by $\sigma(a_0)$, $\sigma(a_1)$, $\sigma(a_2)$, etc. and the isovector partial wave cross sections by $\sigma(b_0)$, $\sigma(b_1)$, $\sigma(b_2)$, etc. as in Table V. Generally, $\sigma(a_i) = C_i |a_i|^2$, where the $C_i$ factor is a combination of factors like $\pi$ and Clebsch-Gordan coefficients, which can differ from amplitude to amplitude. (Therefore, the partial cross sections listed in Table V do not provide the magnitude of the corresponding amplitudes without further work.) The notation $\sigma(a_{i_1, a_{i_2, \ldots}})$ implies that we could not separate the cross sections for $a_1$, the $Ss$ component, from the $PP$ components $a_4$ to $a_6$. Hence $\sigma(a_{i_1, a_{i_2, \ldots}}) = \sigma(a_{i_1}) + \sigma(a_{i_2}) + \sigma(a_{i_3}) + \sigma(a_{i_4})$. The partial cross section groups that could be isolated are given in Eqs. (12):

\[
\begin{align*}
Ss & \text{ isoscalar terms: } \\
\sigma(a_0, a_2) & = \frac{1}{8} (\Delta \sigma_L + 2 \Delta \sigma_T + 2 \sigma_{tot} - 2 \Delta \sigma'_T - 2 \Delta \sigma'_L), \\
\sigma(a_0, a_1) & = \frac{1}{8} (\Delta \sigma'_L + 2 \Delta \sigma'_T + 2 \sigma_{tot}'), \\
PS & \text{ isovector terms: } \\
\sigma(b_1, b_2) & = \frac{1}{8} (\Delta \sigma'_L + 2 \Delta \sigma'_T + 2 \sigma_{tot}).
\end{align*}
\]

<table>
<thead>
<tr>
<th>Type</th>
<th>Label</th>
<th>$2s_{iJ_j}^{+}2s_{iJ_j}^{+}I_jI_j, I_{iJ_j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_s$ isoscalar</td>
<td>$a_0$</td>
<td>$^3P_0 \rightarrow ^3S_{0}, s$</td>
</tr>
<tr>
<td>$S_s$ isoscalar</td>
<td>$a_0$</td>
<td>$^1S_0 \rightarrow ^3S_{1}, p$</td>
</tr>
<tr>
<td>$S_s$ isoscalar</td>
<td>$a_0$</td>
<td>$^1D_2 \rightarrow ^3S_{1}, p$</td>
</tr>
<tr>
<td>$P_s$ isovector</td>
<td>$b_1$</td>
<td>$^1S_0 \rightarrow ^3P_{0}, s$</td>
</tr>
<tr>
<td>$P_s$ isovector</td>
<td>$b_1$</td>
<td>$^1S_0 \rightarrow ^3P_{0}, s$</td>
</tr>
<tr>
<td>$P_s$ isovector</td>
<td>$b_1$</td>
<td>$^1S_0 \rightarrow ^3P_{0}, s$</td>
</tr>
<tr>
<td>$P_s$ isovector</td>
<td>$b_1$</td>
<td>$^1S_0 \rightarrow ^3P_{0}, s$</td>
</tr>
<tr>
<td>$P_s$ isovector</td>
<td>$b_1$</td>
<td>$^1S_0 \rightarrow ^3P_{0}, s$</td>
</tr>
<tr>
<td>$P_s$ isovector</td>
<td>$b_1$</td>
<td>$^1S_0 \rightarrow ^3P_{0}, s$</td>
</tr>
</tbody>
</table>

FIG. 15. Energy dependence of the integrated spin correlation coefficients $A_{c1}$, $A_{c2}$, $A_{\alpha \beta}$, $A_{\lambda \mu}$, and $A_{\nu \nu}$, and the peak analyzing power $A_{y,\max}$ for $pp \rightarrow pn\pi^+$. The diamond shape symbols represent measurements at lower energies and are taken from Ref. [18]. The solid lines are predictions of the Jülich meson exchange model. (There is no prediction for $A_{c1}$.)
also hold for each group are indicated on the left side of Eqs. vantage over relying on analyzing powers, which are sensi-

higher lying weak amplitudes is minimized. This is an ad-

absolute cross sections. So our experimental

pp

much easier to measure accurate cross section ratios than

pp

presented before. It is identical to Eq. ~13! .

Of these six equations, which hold for

Ss + Pp  isoscalar terms:

\[
\sigma(a_1, a_4 - 6) = \frac{1}{4} \left( -\Delta \sigma_L + 2 \Delta \sigma_{tot} + \Delta \sigma'_L - 2 \sigma'_{tot} \right),
\]  \hspace{1cm} (12c)

Ss + Pp  isovector terms:

\[
\sigma(b_0, b_3) = \frac{1}{8} \left( \Delta \sigma'_L - 2 \Delta \sigma'_T + 2 \sigma'_{tot} \right),
\]  \hspace{1cm} (12d)

Pp  isoscalar terms:

\[
\sigma(a_3) = \frac{1}{8} \left( \Delta \sigma_L + 2 \Delta \sigma_T - 2 \sigma_{tot} + 2 \sigma'_L - 2 \sigma'_{tot} \right),
\]  \hspace{1cm} (12e)

Pp  isovector terms:

\[
\sigma(b_{4-1}) = \frac{1}{4} \left( -\Delta \sigma'_{tot} + 2 \sigma'_{tot} \right).
\]  \hspace{1cm} (12f)

Of these six equations, which hold for

pp → pn π^+ , three also hold for

pp → pp π^-. We note that Eq. ~12b! has been

presented before. It is identical to Eq. ~13! in Ref. ~26!. The six equations now permit a calculation of partial wave cross sections to the specified groups of final states from the measured spin-dependent cross sections. The sum of these partial cross sections equals the total π^+ production cross section. Since the partial cross sections add incoherently the effect of higher lying weak amplitudes is minimized. This is an advantage over relying on analyzing powers, which are sensitive to even small admixtures. The amplitudes included in each group are indicated on the left side of Eqs. ~12!.

In many experiments, including the present one, it is much easier to measure accurate cross section ratios than absolute cross sections. So our experimental

pp → pn π^+ quantities are given as a fraction of the total π^+ production cross section σ_{tot}. Equations ~12! are easily rewritten in terms of partial wave strengths by dividing both sides by σ_{tot}. For the p + π^+ branch the values Δσ_T/σ_{tot} and

Δσ_L/σ_{tot} were calculated from Eqs. ~7! and \( A_\Sigma(\hat{\theta}) \), \( A_\Delta(\hat{\theta}) \), and \( \sigma_0(\hat{\theta}) \). In figures and tables we will generally use the ratios of partial-wave cross sections to total cross sections. We refer to them as partial-wave strengths.

For use in this study the total

pp → pp π^0 and

pp → pn π^+ cross sections were taken from the literature and interpolated for the present \( \eta \) values. We obtained the

pp → pp π^0 information needed from Ref. ~26! and the

pp → pn π^+ total cross sections from Ref. ~18! and from Fig. 2 in Ref. ~6!. The accuracy of the total cross section ratios so obtained is not very high, but it will suffice here because the isoscalar terms of interest are an order of magnitude larger than the isovector terms. The partial cross section strengths derived with Eqs. ~12! are displayed in Fig. 16 and listed in Table V.

The primed cross sections are the (pure isovector) cross sections measured for

pp → pp π^0 , which are also more accurately given as fractional strengths. To work in terms of

pp → pn π^+ partial wave strengths the

pp → pp π^0 strengths of Ref. ~26! have to be multiplied by the ratio of the

pp → pp π^0 and

pp → pn π^+ unpolarized cross sections, taken at the same relevant \( \eta \) values.

Figure 16 shows the change of partial wave strength with energy for

Sp , \( Ps \), and other groups. It is immediately ap-

parent that for the energy region studied the leading isoscalar partial cross sections are an order of magnitude larger than the isovector ones. It helps our discussion that lowest-lying

Pp  isoscalar partial wave strength \( Pp(a_3) \) could be re-

solved. It is much smaller than the \( Ss \) and \( Sp \) strengths. So is the sum of all isovector cross sections for \( b_4 \) to \( b_{11} \). The \( Pp \) strengths attributable to \( b_5 \) can be assessed by comparing \( Pp(b_{4-11}) \) from this work with the heavy dash-dotted curve derived from Ref. ~26! for the full \( Pp \) isovector strength \( Pp(b_{3-11}) \).

Therefore, it seems reasonable to assume that the \( Pp \) contributions from \( a_4, a_5, a_6, \) and \( b_3 \), which could not be dis-
entangled from the $S_s$ amplitudes, are also much smaller than the $S_s$ terms. On this basis we estimate that they make up no more than 5–10% of the "$P_P$ entangled" $S_{max}$ cross-section curve. For $\eta = 0.9$ the $S_p(a_0, a_2)$ cross section has become dominant. As seen in Fig. 16, it is very much larger than the $P_s$ isovector contribution. It would be of interest to resolve the isoscalar component $a_0$, because it can be used to constrain the strength of three-body forces [13]. However, in this analysis $a_0$ and the much larger amplitude $a_2$ always appear together. The $S_s$ fraction, including the unresolved $P_P$ contributions, has fallen to less than 0.3. This is consistent with the work at 420 MeV [32].

In Fig. 17 the data points give the summed $S_p + P_s$ strengths, the upper limit for the summed $S_s$ strengths, and a lower limit for the $P_P$ strength. The heavy dashed curve shows the likely energy dependence of the actual $S_s$ strength. The divergence of the old and new $pp \rightarrow pn\pi^+$ interpretation near $\eta = 0.45$ serves as a reminder that a partial wave analysis is only model independent if it fully encompasses all contributing amplitudes. This apparently was no longer true for the 320 MeV data ($\eta = 0.42$) of Ref. [18].

In this respect our present difficulty to perfectly reproduce $A_y$ at 375 and 400 MeV in the $S_s$, $S_p$, $P_s$, and $P_P$ frameworks (see Table II) should be taken as a warning. At these energies some higher partial waves may contribute enough so that they must be considered, at least for the analyzing powers.

VI. CONCLUSIONS

We have measured the spin correlation coefficients $A_x = A_{xx} + A_{yy}$, $A_\Delta = A_{xx} - A_{yy}$, $A_z$, and $A_y$, as well as angular distributions for $\sigma(\theta_\pi)$ and the polarization observables $A_j(\theta_\pi)$ at energies from 325 to 400 MeV. At the lowest energies the results are in agreement with prediction of the Jülich meson exchange model. The agreement deteriorates considerably at energies where $S_s$ transitions no longer dominate. At 375 and 400 MeV some physics aspects in $pp \rightarrow pn\pi^+$ apparently are missed by the model. This suspicion is supported by the even poorer agreement of the model with the $pp \rightarrow pp\pi^0$ data [26].

The $pp \rightarrow pp\pi^0$ and $pp \rightarrow pn\pi^+$ reactions are found to differ greatly in the relative importance of $S_p$, $P_s$, and $P_P$ transitions. $S_p$ strongly feeds the delta resonance in $pp \rightarrow pn\pi^+$, but this transition is forbidden for $pp \rightarrow pp\pi^0$. By contrast, $P_s$ contributions in $pp \rightarrow pn\pi^+$ are no larger than $P_P$ contributions, as seen in Fig. 16. In $pp \rightarrow pn\pi^+$ the $S_s$ and $S_p$ isoscalar terms are most important while the $P_P$ transitions just begin to contribute. For $pp \rightarrow pp\pi^0$ $P_P$ becomes dominant at $\eta = 0.7$. 

FIG. 16. Partial-wave strengths for six groups of amplitudes as function of $\eta$. The isoscalar cross sections are connected by solid lines, the isovector ones by dashed lines. The contributing amplitudes, including the (small) $P_P$ contributions not resolved from the dominant $S_s$ cross sections are indicated in the legend. The dashed line represents the full $P_P$ isovector strength contribution in $pp \rightarrow pn\pi^+$, as derived from the results of Ref. [26].

FIG. 17. Sums of isoscalar and isovector partial wave strengths as function of $\eta$. The $S_p + P_s$ sum is measured directly. The points labeled "$S_s$ max" represent a close upper limit to the sum of the $S_s$ partial cross sections. Any correction for the unresolved $P_P$ amplitudes ($a_1, a_5$, and $b_3$) would lower the $S_s$ curve (as indicated by the estimated errors). The admixtures can be expected to be smaller than $P_p(a_3)$. The lower points show the documented $P_P$ strengths only. The data at $\eta = 0.22$ and 0.42 are from Ref. [18].
The partial wave analysis was able to reproduce almost all polarization observables within experimental errors. This supports the postulated adequacy of considering only $Ss$, $Sp$, $Ps$, and $Pp$ transitions in the near-threshold region. However, this angular momentum space may not be adequate to explain details of analyzing powers, because they can be affected by small admixtures of higher-lying transitions. Even in this limited space the number of individual partial waves for $pp \rightarrow pn\pi^+$ at 400 MeV is too large to deduce all individual amplitudes. Some interesting sum rules for groupings of amplitudes were found [Eqs. (12)], and the corresponding partial cross sections could be extracted. They show, e.g., that for $\eta<1$ $Pp$ and $Ps$ terms play a considerably smaller role in $pp \rightarrow pn\pi^+$ than in $pp \rightarrow pp\pi^0$.

Further progress may come from improved theoretical models that can accurately predict the new data at hand. It is interesting to note again that the Jülich model does well at 325 MeV where $Ss$ dominates, but it increasingly fails for

$pp \rightarrow pn\pi^+$ (as well as for $pp \rightarrow pp\pi^0$) as higher angular momenta become important.

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Nuclear Polarization of Hydrogen Molecules from Recombination of Polarized Atoms

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The nuclear polarization of H\textsubscript{2} molecules formed by recombination of polarized H atoms on a Cu surface was measured as a function of external magnetic field and of temperature of the surface. The proton polarization of the molecules was determined by scattering of a longitudinally polarized 203-MeV proton beam in the Indiana University Cyclotron Facility storage ring. The nuclear polarization of the molecules, relative to the polarization of the atoms before recombination, increased from near zero in a weak magnetic field to 0.42 ± 0.02 in a 0.66 T field. A simple model of the relaxation accounts quantitatively for the observations.

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Conventional spin-physics experiments in high energy and nuclear physics use as targets hydrogenous substances such as NH\textsubscript{3} at low temperatures, whose protons are polarized by microwave irradiation. However, during the past few years, increased use has been made of polarized hydrogen and deuterium gas targets, which are placed in the circulating beam of storage rings. In order to increase the target thickness over that obtained by a jet of polarized H atoms, the beam from atomic beam sources is directed into an open cell (“storage cell”) in which the atoms make several hundred collisions before escaping from the target [1]. One example is the series of experiments by the HERMES Collaboration, which has studied, e.g., deep inelastic scattering of 27.6 GeV electrons from a polarized H target at DESY [2]. Other applications are measurements of spin correlation parameters in proton-proton scattering, as described in more detail below.

A simplified scale drawing of the equipment is shown in Fig. 1. A beam of polarized H atoms in a pure spin state (state 1, \(m_I = 1/2, m_J = 1/2\), see Ref. [8]) passes through a 13 cm long, tapered entrance tube and enters the recombination cell. In the recombiner, which is made of copper and is lined with copper mesh, the atoms encounter enough wall collisions to lead to almost complete recombination. The H\textsubscript{2} molecules (and some remaining atoms) diffuse out of the recombiner into a thin-walled Al target cell (length 25 cm, diameter 1.2 cm) through which the proton beam passes. The recombination cell is separated from the target cell by a remotely operated valve. To measure \(P_{at}\) the valve is closed, in which case the atoms from the polarized beam source diffuse into the target cell with negligible loss in polarization, because they encounter only the Teflon-coated surfaces of the recombiner valve and the target and entrance tubes, which are known [9,10] to strongly inhibit depolarization and recombination. The
present measurements determine the polarization \( P_{\text{mol}} \) of the \( \text{H}_2 \) molecules, relative to the polarization of the atoms before recombination:

\[
R = \frac{P_{\text{mol}}}{P_{\text{at}}}.
\]

The magnetic field is provided by a superconducting coil (Fig. 1) which produces fields up to 0.66 T at the recombiner. Thus, the available field is large compared to the critical field of the hyperfine interaction in H of 50.7 mT [8]. To avoid the large distortion of the proton closed orbit that a transverse \( B \) field would have entailed, \( B \) was chosen longitudinal (\( z \) direction). Forward wire chambers and scintillators detected protons from \( pp \) elastic scattering at lab angles in the range 30°–60°. Event identification depended on detection of two protons in coincidence, with the requirement that they be coplanar.

Parity conservation requires the longitudinal analyzing power to be zero. Consequently, to measure the longitudinal target polarization, one needs to resort to a spin correlation measurement (polarized beam and polarized target).

The number of observed \( pp \) coincidences for longitudinal beam polarization \( p \) and target polarization \( P \) can be written as

\[
Y = k t(\alpha) [1 + p \rho A_{zz}(\theta)],
\]

where the dependence of \( Y \) on proton beam current was removed by dividing the observed number of counts by the integrated beam current obtained from a current transformer. In the above equation, target polarization \( P \) stands for either \( P_{\text{open}} \) or \( P_{\text{closed}} \), depending on whether the recombiner valve is open or closed, and the constant \( k \) contains factors such as the \( pp \) cross section and detector efficiencies. The spin correlation coefficient \( A_{zz} \) has recently been shown [11] to be near 1 in the vicinity of \( \theta_{\text{lab}} = 45^\circ \), but the precise values of \( p \) and \( A_{zz} \) are unimportant here because they cancel in the ratio \( P_{\text{open}}/P_{\text{closed}} \).

The target thickness \( t(\alpha) \) is a function of the degree of dissociation of the target gas because the conductance depends on the mass of the target particles.

Data acquisition cycles were 4 min long. During each cycle, the recombiner valve was switched between the open and the closed position twice. At the end of a cycle, the stored proton beam was dumped. For every third cycle, the polarization of the injected beam was turned off. This allowed taking the ratio of \( Y \) [Eq. (2)] for polarized and unpolarized beams to determine \( p P_{\text{open}} A_{zz}(\theta) \) or \( P_{\text{closed}} A_{zz}(\theta) \), depending on whether the valve-open or the valve-closed part of the cycle is evaluated. As a check, the beam polarization \( p \) was reversed in sign after every polarized cycle.

Ideally, the degree of dissociation, \( \alpha \), defined as the fraction of hydrogen which is in atomic form,

\[
\alpha = \frac{n_H}{(n_H + 2n_{H2})},
\]

should be \( \alpha_{\text{closed}} = 1 \) when the recombiner valve is closed, and \( \alpha_{\text{open}} = 0 \) when the valve is open. Under this condition, \( R = P_{\text{mol}}/P_{\text{at}} = P_{\text{open}}/P_{\text{closed}} \). Prior to installation in the IUCF storage ring, the recombiner was tested by analyzing the gas emerging from the target tube with a quadrupole mass analyzer. Recombination \( >85\% \) was achieved, but only after lining the inside of the recombiner with copper mesh to increase the number of wall collisions. To guard against changes in the surface properties of the recombiner, the degree of recombination was monitored throughout the measurements, and the deviation from the ideal case was taken into account in the data analysis, as described below.

To monitor \( \alpha \), we made use of the yields [Eq. (2)] measured with an unpolarized beam, \( Y = k t(\alpha) \). In the ideal case, \( Y \) would increase by a factor \( \sqrt{2} \) when the recombiner valve is opened, because the gas conductance is proportional to \( 1/\sqrt{m} \), where \( m \) is the mass of the particles in the target. In practice, the increase will be \( <\sqrt{2} \). The value of \( Y_{\text{open}}/Y_{\text{closed}} \) found here is 1.364 ± 0.004, independent of the strength of the magnetic field \( (\chi^2 = 0.9 \) for the entire data set) which is 3.7% below the ideal \( \sqrt{2} \). This can be due either to molecules present in the target even with the recombiner valve closed \( (\alpha_{\text{closed}} = 0.879 \pm 0.010, \alpha_{\text{open}} = 0) \), or to incomplete recombination inside the copper recombiner when the recombiner valve is open \( (\alpha_{\text{open}} = 0.089 \pm 0.007, \alpha_{\text{closed}} = 1) \), or to some combination of the two.

The data analysis was carried out for both extreme assumptions about \( \alpha \). The values of \( R \) are shown in Figs. 2 and 3, where the error bars are probable errors that include the uncertainty from statistics as well as the systematic uncertainty from the incomplete knowledge of \( \alpha_{\text{open}} \) and \( \alpha_{\text{closed}} \) discussed above.

The data analysis took into account corrections for contributions from background (2%) and loss of counts from
As the magnetic field is varied from 5 mT to 0.66 T, the direct dipole-dipole interaction (\(B' = 2.7\) mT) and the direct dipole-dipole interaction (\(B'' = 3.4\) mT) [12] cause the nuclei to rapidly precess about a direction that is skew to the external field by \(\theta = B' \cdot B / B''\). The orientation of \(B_c\) is randomized by each wall collision. Between successive wall collisions, the component of the polarization along the external field decreases by an amount \(B_c / B^2\); thus, after \(n\) wall bounces, the nuclear polarization retained by the molecules rises from near zero to \(R = 0.42 \pm 0.02\).

If no loss in proton polarization occurred, one would observe \(R = 1\). Assume for a moment that, in a sufficiently strong external magnetic field, the proton spins of the \(H_2\) molecules and of the \(H\) atoms in the gas phase are preserved. However, recombination \(H + H \rightarrow H_2\) requires at least one of the atoms to be adsorbed on the wall prior to recombination, since, at our very small gas densities, volume recombination is completely negligible. The wall atoms are likely to be depolarized by fluctuating fields on the surface, so that the resulting net nuclear polarization of the molecules coming off the wall is half of the original proton polarization of the \(H\) atoms, i.e., \(R = 1/2\).

Additional loss of polarization from other processes may arise, e.g., when two atoms on the surface recombine with each other. Indeed, Fig. 2 is not inconsistent with \(R\) rising asymptotically to \(R \sim 1/2\) for large \(B\).

We interpret the decrease in \(R\) at low \(B\) as arising from spin relaxation of the ortho-\(H_2\) molecules during the time periods between wall collisions. In free flight, internal molecular fields \(B_c\) from the spin-rotation interaction (\(B' = 2.7\) mT) and the direct dipole-dipole interaction (\(B'' = 3.4\) mT) [12] cause the nuclei to rapidly precess about a direction that is skew to the external field by \(\theta = B_c / B\). The orientation of \(B_c\) is randomized by each wall collision. Between successive wall collisions, the component of the polarization along the external field decreases by an amount \(B_c / B^2\); thus, after \(n\) wall bounces, the nuclear polarization retained by the molecules rises from near zero to \(R = 0.42 \pm 0.02\).

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fraction of H that remained in the cell changed, but their polarization was retained. Only when the wall temperature was lowered below 70 K did the polarization of the atoms drop very rapidly. This observation is supported by results in the HERMES experiment: when the surface of the target cell was accidentally damaged by the circulating electron beam, about 80% of the atoms experienced recombination, but the remaining 20% of the atoms were fully polarized [16].

From the information given above, we can predict how $R$ should depend on wall temperature $T$ of the recombiner. Figure 3 shows data for a fixed magnetic field $B = 0.44$ T. The only temperature dependence in Eq. (4) arises from the decrease in the $J = 3$ population, causing a change in $B_c$ from 6.1 mT at 300 K to 5.4 mT at 40 K. The corresponding change in $R$ at $B = 0.44$ T is 3.1%, which is below the accuracy of our data. Thus $R \sim \text{const}$, in agreement with observation down to $T = 100$ K (Fig. 3). The rapid drop of $R$ below 70 K is almost certainly associated with the depolarization of the H atoms mentioned in the preceding paragraph. In Fig. 3 the curve shows measurements of the polarization of H atoms in a Cu cell with 80 wall collisions as a function of wall temperature. The larger number of wall collisions in the present experiment explains why the drop in $R$ occurs at a somewhat higher temperature. Note that, in the present measurements, $T_{\text{cell}}$ was held above 130 K so that depolarization in the measurement of $P_{\text{at}}$ is negligible.

In summary, measurements of high statistical accuracy have been made of nuclear polarization of $^2$H$_2$ molecules which result from the recombination of polarized H atoms on Cu. It is found that, for the strongest magnetic fields available (0.66 T), the nuclear polarization of the molecules is nearly half of the polarization of the recombining atoms, which we interpret to result from the recombination of a depolarized atom on the wall and a polarized atom in the gas phase. The decrease of the polarization at lower $B$ is consistent with the assumption that the polarization of the molecules is lost by coupling of the proton spin to the magnetic field of the molecule’s rotational motion. The decrease of molecular polarization below 100 K is expected from available data on the polarization loss of hydrogen atoms in collisions with cold walls.

Further studies are needed of the depolarization mechanism in recombination of H atoms. It would be interesting not only to vary the number of wall collisions but also the hyperfine state composition of the recombining atoms.

Also, measurements on the recombination of deuterium atoms would be of interest. For deuterium, the orthomolecules at room temperature and below are primarily in the $J = 0$ rotational state and thus should show less depolarization at low $B$.

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[13] The factor 1.19 accounts for the reduced coupling of the nuclei to the external field due to the rotational magnetic moment.


[15] J.S. Price, Ph.D. thesis, University of Wisconsin, 1993. [available as dissertation No. 9318638, from Dissertation Express (http://wwwlib.umi.com/dxweb/)]. For the curve in Fig. 3, the results were normalized by a factor of 0.46.

Complete set of polarization observables in $\vec{p}\vec{p} \rightarrow pp\pi^0$ close to threshold


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In a kinematically complete experiment we have measured the two analyzing powers and the five spin correlation coefficients of the reaction $\vec{p}\vec{p} \rightarrow pp\pi^0$ as a function of all five parameters of the three-body final state for bombarding energies between 325 and 400 MeV. The data are in disagreement with the theoretical predictions available at this time. Below 400 MeV, fewer than a dozen complex partial-wave amplitudes are likely to be significant, and it is expected that the present experimental information constrains these amplitudes. We also describe the formalism for an expansion of the spin observables into a complete set of angular functions and use this to completely characterize the polarization information obtainable from reactions with polarized spin-1/2 collision partners and a three-body final state.

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1. INTRODUCTION

The behavior of a system consisting of two nucleons and a pion is basic to classical nuclear physics. It is thus an important task to try to relate the process of pion production in a nucleon-nucleon ($NN$) collision to our understanding of the $NN$ interaction or to constraints given by basic symmetries, or, ultimately, to a model that features the constituents of nucleons and mesons. The theoretical task was expected to be relatively simple at energies very close to threshold because only a single angular momentum channel contributes.

Triggered by the advent of new cross section data close to threshold, there has been a flurry of theoretical activity during the past five years devoted to an understanding of the lowest partial wave (see Sec. V A for more details on the current status of the theory). Even though this work is still going on, it is clearly important to also investigate the higher partial waves which become active as the bombarding energy is increased. In order to identify the role of individual partial waves, the use of polarized collision partners is essential.

Each of the three periods of activity in the study of pion production in the nucleon-nucleon system is related to specific technical advances. The first was the development of accelerators with sufficient energy, which led to the first observation of the $pp \rightarrow pp\pi^0$ reaction [1] just a few years after the pion was discovered [2] and 17 years after it was predicted by Yukawa [3]. The second was the construction of meson factories with intense, well-defined proton beams that made possible accurate and kinematically complete cross section measurements, and the third was the advent of storage rings with electron-cooled beams and internal targets [4], which started to operate in the late 1980s, and which opened up the near-threshold region for experimental study.

Measurements of pion production in $pp$ collisions benefit from storage ring technology mainly in two ways. The first concerns the use of windowless internal gas targets. Such targets put only hydrogen gas into the path of the beam and make it possible to measure small $pp \rightarrow pp\pi^0$ cross sections very close to threshold with little contamination from undesired reactions. In addition, the amount of material between the target volume and the detector can be made small, and the momenta of both outgoing hadrons can be measured accurately. Thus, the complete kinematics of each event can be determined. Internal targets must be thin in order for the cooling process to keep up with target heating, but this limitation is offset by the intensity of the accumulated, stored beam. The second unique advantage of the storage ring environment concerns polarized atomic gas targets. It turns out that the maximum target thickness that can be achieved is a good match for the target thickness requirements of a medium-energy storage ring.

Close to threshold the number of participating partial...
waves is small. In fact, at energies below 320 MeV, only one partial wave is significant (the Ss partial wave with the angular momenta of the final-state pp pair as well as the pion equal to zero). In one of the first nuclear physics experiments with a stored, cooled beam [5], the total cross section in this energy region was measured, revealing a serious disagreement with the theory at that time (see Sec. V A). For bombarding energies larger than 320 MeV, additional partial waves come into play but their number is still relatively small since below about 400 MeV final-state angular momenta larger than one should be unimportant. With this limitation, it is possible to provide an expression for the most general dependence of any observable on the angles of the three outgoing particles. For the present study, this point is crucial for two reasons. First, we use the angular dependence given by these expressions to formulate a strategy to order and present the information available from an experiment with polarized beam and target by defining an appropriate set of single-valued “observables” that characterize the complete five-dimensional phase space. Second, it allows us to carry out an analysis of the data in terms of the coefficients that appear in these expressions. The resulting coefficients completely parametrize the polarization observables of the reaction and constrain participating amplitudes individually. This constitutes a powerful and detailed test of any theory.

Prior to this experiment, the world’s polarization data for the reaction \( pp \rightarrow pp \pi^0 \) below 400 MeV consisted of just two analyzing power measurements [6,7]. In this paper we describe a complete measurement of this reaction covering most of the available phase space, carried out with a polarized beam on a polarized target at bombarding energies between 325 and 400 MeV. All polarization observables allowed by parity conservation have been measured. Since we are dealing with a three-body final state, these observables depend on five kinematic variables. Section II of this paper is concerned with the definition of polarization observables and their dependence on the kinematics of the final state. Section III contains a description of the apparatus, an account of the acquired data, and a description of the method used to extract the observables from the measured quantities. In Sec. IV a scheme is introduced to completely map out the spin dependence of the reaction everywhere in the five-dimensional phase space, and results are presented. Finally, Sec. V is devoted to a discussion of the present status of the theory, a comparison of some of the data to recent calculations, and a list of conclusions from the present experiment.

II. POLARIZATION OBSERVABLES

A. Basic definitions

In a reaction with two outgoing particles it is customary to relate the coordinate frame to the reaction plane. With a three-body final state there is no such distinguished plane, so we use a Cartesian coordinate frame that is fixed in space. The \( z \) axis is along the beam direction, the \( y \) axis is vertical, pointing up, and the \( x \) axis completes the right-handed coordinate system. The polar angle \( \theta \) and azimuthal angle \( \phi \) as defined in Fig. 1, are used to specify the direction of any vector.

In this experiment we detect the energy and direction of the two final-state protons of the reaction \( pp \rightarrow pp \pi^0 \). Let the center-of-mass momentum of the two protons be \( \vec{b}_1 \) and \( \vec{b}_2 \). To describe the final-state kinematics we define the momenta \( \vec{p} \) and \( \vec{q} \), where \( \vec{p} = (\vec{b}_1 + \vec{b}_2)/2 \) (the proton momentum in the pp rest system) and \( \vec{q} = -(\vec{b}_1 + \vec{b}_2) \) (the center-of-mass momentum of the pion; see Fig. 2). Five independent parameters are needed to describe the final state, namely, the directions \( \hat{p} \) and \( \hat{q} \) and an “energy-sharing” parameter \( \epsilon \), which we will later define as the kinetic energy of the two final-state protons in their rest system [see Eq. (21)]. All five parameters follow from the observation of the two protons. For brevity, we sometimes denote the set \( \{ \theta_p, \varphi_p, \theta_q, \varphi_q, \epsilon \} \) by \( \xi \).

The largest possible value of the pion momentum is given by (we set \( c = \hbar = 1 \))

\[
q_{\text{max}} = \frac{1}{2\sqrt{s}} \sqrt{[s-(2m_p+m_\pi)^2][s-(2m_p-m_\pi)^2]},
\]

(1)
where $\sqrt{s}$ is the total center-of-mass energy, and $m_p$ and $m_\pi$ are the masses of the proton and the pion, respectively. Instead of the bombarding energy, one often quotes the parameter

$$\eta = q_{\text{max}}/m_\pi,$$

(2)

which vanishes at threshold. The term ‘near threshold’ loosely corresponds to the energy region with $\eta < 1$, i.e., below 400 MeV.

The polarization of an ensemble of spin-1/2 particles may be described by the expectation value of the three-component Pauli spin operator (see, e.g., Ref. [8]). In the following, we denote the polarization of the beam and the target by the two vectors $\vec{P} = (P_x, P_y, P_z)$ and $\vec{Q} = (Q_x, Q_y, Q_z)$, respectively.

B. Definition of observables

We abbreviate the differential cross section for the reaction, initiated by a polarized beam on a polarized target, by

$$\sigma(\xi, \xi, \tilde{P}, \tilde{Q}) = \frac{d\sigma(\theta_p, \varphi_p, \theta_q, \varphi_q, \epsilon, \vec{P}, \vec{Q})}{d\Omega_p d\Omega_q d\epsilon},$$

(3)

and write $\sigma_0(\xi)$ for the cross section that would be measured without polarization. In terms of the so-called Cartesian polarization observables, the spin-dependent cross section becomes

$$\sigma(\xi, \tilde{P}, \tilde{Q}) = \sigma_0(\xi) \left[ 1 + \sum_i P_i A_{i0}(\xi) + \sum_{ij} Q_j A_{ij}(\xi) \right].$$

(4)

Here, $i$ and $j$ stand for $x$, $y$, or $z$ and the sums extend over all possibilities. The resulting 15 polarization observables include the beam analyzing powers $A_{i0}$, the target analyzing powers $A_{0j}$, and the spin correlation coefficients $A_{ij}$. It is convenient to define the following combinations of spin correlation coefficients:

$$A_x(\xi) = A_{xx}(\xi) + A_{yy}(\xi),$$

(5a)

$$A_\Delta(\xi) = A_{xx}(\xi) - A_{yy}(\xi),$$

(5b)

$$A_\Xi(\xi) = A_{xy}(\xi) - A_{yx}(\xi).$$

(5c)

The 15 polarization observables of Eq. (4) are not independent. For instance, $A_{y0}$ and $A_{0y}$ are equivalent because the radiation pattern observed with a beam polarized along $\hat{y}$ is the same as when the beam is polarized along $\hat{x}$, except for a rotation by 90° around the $z$ axis. This and other, similar, “rotational” equivalences are given by [9]

$$A_{y0}(\theta_p, \varphi_p, \theta_q, \varphi_q) = A_{x0}(\theta_p, \varphi_p + \pi/2, \theta_q, \varphi_q + \pi/2),$$

(6a)

$$A_{xy}(\theta_p, \varphi_p, \theta_q, \varphi_q) + A_{yx}(\theta_p, \varphi_p, \theta_q, \varphi_q) = A_\Delta(\theta_p, \varphi_p, -\pi/4, \theta_q, \varphi_q - \pi/4).$$

(6b)
\( \alpha = \{ l, s, \ell, l_p, s_f, j, l_q \} \), \( \alpha = \{ l, s, \ell, l_p, s_f, j, l_q \} \),

fully identifies the amplitudes \( U_\alpha \) for transitions from a given initial to a given final state. These amplitudes are functions of the energy-sharing parameter \( \alpha \) and the total energy. The quantum numbers in Eq. (10) are constrained by angular momentum and parity conservation as well as by the Pauli principle. Because close to threshold it is realistic to assume that \( l_p \) and \( l_q \) are either 0 or 1, the possible choices for the angular momentum in the final state are then \( (l_p, l_q) = (0, 0), (1, 0), \) and (1, 1), or \( \text{SS}, \text{PS}, \) and \( \text{PP}. \) In \( pp \rightarrow pp \pi^0 \), there are no \( \text{SP} \) final states permitted by the usual symmetry constraints of parity and angular momentum conservation and the Pauli principle. A list of all transitions with these constraints can be found in Table I. For completeness, we have included in Table I the transitions with \( l_p = 2, l_q = 0 \) (\( \text{Ds} \)) and \( l_p = 0, l_q = 2 \) (\( \text{Sd} \)). Since these amplitudes can interfere with the important \( \text{SS} \) amplitude, their contribution might be non-negligible [10]. The list in Table I follows the conventional notation \( \Sigma_i I_j \rightarrow \Sigma_i + I_j \) where the spectroscopic notation, \( (l, l_p) = S, P, D, F, \ldots \) and \( l_q = s, p, d, f, \ldots \) is used.

2. Angular distributions of the observables

Since close to threshold only relatively few amplitudes contribute to \( pp \rightarrow pp \pi^0 \), it is feasible to expand the observables in terms of angular momentum. In the formalism we use, the expansion functions are products of two spherical harmonics with arguments \( \ell \) and \( \ell' \), and the expansion coefficients are a sum of terms, where each term contains the product of two amplitudes \( U_\alpha U_\alpha^* \) times an angular-momentum coupling factor. The coupling factor is often zero, reflecting the constraints arising from conservation laws and antisymmetrization. For instance, one finds that the amplitudes can be arranged into the two groups (\( \text{SS}, \text{SD}, \text{DS} \)) and (\( \text{PS}, \text{PP} \)), and only amplitudes within one group can interfere with each other. The details of such an expansion into partial waves are given in the Appendix.

\[
\sigma_0(\xi) = E^2 + F_1^2 + H_{00}^2 + (H_{00}^2 + I)(3\cos^2 \theta - 1) + (H_{10}^2 + F_2 + K)(3\cos^2 \theta - 1) + H_{30}^2(3\cos^2 \theta - 1)(3\cos^2 \theta - 1) + H_{40}^2\sin 2 \theta_p\sin 2 \theta_q\cos \Delta \varphi + H_{50}^2\sin^2 \theta_p\sin^2 \theta_q\cos 2 \Delta \varphi, \quad (11a)
\]

\[
\sigma_0(\xi)A_{y0}(\xi) = \left\{ G_{10}^2 + G_{20}^2(3\cos^2 \theta - 1) \right\}\sin \theta_q + \left\{ H_{10}^2 + I^2 + H_{20}^2(3\cos^2 \theta - 1) \right\}\sin 2 \theta_q \cos \varphi_q + [H_{50}^2 + K^2 + G_{10}^2\cos \theta_q + H_{30}^2(3\cos^2 \theta - 1)]\sin 2 \theta_p \cos \varphi_p + [G_{10}^2\sin \theta_q + H_{30}^2\sin 2 \theta_q \sin^2 \theta_p\cos (2\varphi_q - \varphi_p)] \sin^2 \theta_p \sin^2 \theta_q \cos (2\varphi_q - \varphi_p), \quad (11b)
\]

\[
\sigma_0(\xi)A_{z0}(\xi) = 2(E - F_1) + H_{00}^2 + (H_{00}^2 + 2I)(3\cos^2 \theta - 1) + (H_{20}^2 - 2F_2 + K)(3\cos^2 \theta - 1) + H_{30}^2(3\cos^2 \theta - 1)(3\cos^2 \theta - 1) + H_{40}^2\sin 2 \theta_p\sin 2 \theta_q \cos \Delta \varphi + H_{50}^2\sin^2 \theta_p\sin^2 \theta_q \cos 2 \Delta \varphi, \quad (11c)
\]

\[
\sigma_0(\xi)A_{zz}(\xi) = -E - F_1 + H_{00}^2 + (H_{00}^2 + 2I)(3\cos^2 \theta - 1) + (H_{20}^2 - 2F_2 - K)(3\cos^2 \theta - 1) + H_{30}^2(3\cos^2 \theta - 1)(3\cos^2 \theta - 1) + H_{40}^2\sin 2 \theta_p\sin 2 \theta_q \cos \Delta \varphi + H_{50}^2\sin^2 \theta_p\sin^2 \theta_q \cos 2 \Delta \varphi, \quad (11d)
\]

TABLE I. Angular momentum quantum numbers for the partial waves of the reaction \( pp \rightarrow pp \pi^0 \). The \( \text{Sd} \) and \( \text{Ds} \) amplitudes have been included for completeness sake; the present experiment finds no evidence for their significance.

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<tr>
<th>Type</th>
<th>( 2s_j + I_j \rightarrow 2s_j + 1l_p, l_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{SS} )</td>
<td>( 3P_0 \rightarrow 3S_0, s )</td>
</tr>
<tr>
<td>( \text{Ps} )</td>
<td>( 3P_0 \rightarrow 3P_0, s )</td>
</tr>
<tr>
<td>( \text{Ps} )</td>
<td>( 1D_2 \rightarrow 3P_0, s )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3P_0 \rightarrow 3P_1, p )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3P_2 \rightarrow 3P_1, p )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3P_2 \rightarrow 3P_2, p )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3F_2 \rightarrow 3P_1, p )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3F_2 \rightarrow 3P_2, p )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3P_1 \rightarrow 3P_0, d )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3P_1 \rightarrow 3P_1, p )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3P_1 \rightarrow 3P_2, p )</td>
</tr>
<tr>
<td>( \text{Pp} )</td>
<td>( 3F_3 \rightarrow 3P_2, p )</td>
</tr>
<tr>
<td>( \text{Sd} )</td>
<td>( 3P_2 \rightarrow 1S_0, d )</td>
</tr>
<tr>
<td>( \text{Sd} )</td>
<td>( 3F_2 \rightarrow 1S_0, d )</td>
</tr>
<tr>
<td>( \text{Sd} )</td>
<td>( 3P_2 \rightarrow 1D_2, s )</td>
</tr>
<tr>
<td>( \text{Sd} )</td>
<td>( 3F_2 \rightarrow 1D_2, s )</td>
</tr>
</tbody>
</table>
\[
\sigma(\xi)A_N(\xi) = [H_1^2 + H_2^2(3 \cos^2 \theta_p - 1)] \sin^2 \theta_q \cos 2 \varphi_q + [H_3^2 + H_4^2(3 \cos^2 \theta_q - 1)] \sin^2 \theta_p \cos 2 \varphi_p \\
+ H_5^2 \sin 2 \theta_p \sin 2 \theta_q \cos (\varphi_p + \varphi_q),
\]
\[
\sigma(\xi)A_{\pm}(\xi) = \{G_{1z}^2 + G_{2z}^2(3 \cos^2 \theta_p - 1)\} \sin \theta_q + \{H_1^2 + I_2^2(3 \cos^2 \theta_p - 1)\} \sin 2 \theta_q \cos \varphi_q \\
+ [H_3^2 + K_{2z}^2 + G_{2z}^2 \cos \theta_q + H_{2z}^2(3 \cos^2 \theta_q - 1) \] \sin 2 \theta_p \cos \varphi_p \\
+ [G_{4z}^2 \sin \theta_q + H_{2z}^2 \sin 2 \theta_q] \sin^2 \theta_p \cos (2 \varphi_p - \varphi_q) + H_{2z}^2 \sin 2 \theta_p \sin^2 \theta_q \cos (2 \varphi_q - \varphi_p),
\]
\[
\sigma(\xi)A_{\pm}(\xi) = [H_1^0 \sin 2 \theta_q + G_{1z}^0 \sin \theta_q] \sin 2 \theta_p \sin \Delta \varphi + H_2^0 \sin^2 \theta_p \sin^2 \theta_q \sin 2 \Delta \varphi,
\]
\[
\sigma(\xi)A_{\mp}(\xi) = G_{1z}^2 \sin 2 \theta_p \sin \theta_q \sin \Delta \varphi.
\]

Equations (11) explicitly depend on the four angles \( \theta_p, \varphi_p, \theta_q, \) and \( \varphi_q, \) while the energy-sharing parameter \( \varepsilon \) is contained in the coefficients. A discussion of the energy dependence is given in Sec. IV E.

When calculating the value of a polarization observable from Eqs. (11), one has to evaluate the ratio \( A_{ij}(\xi) = \sigma_0(\xi) \Lambda_{ij}(\xi) / \sigma_0(\xi), \) and an overall normalization of all terms in these equations cancels. Here, we choose to multiply all coefficients by \( 8 \pi^2 / \sigma_{\text{tot}}. \) This makes the coefficients dimensionless. The spin-averaged total cross section is then an incoherent sum of the partial total cross sections \( \sigma(S_s) / \sigma_{\text{tot}} = E, \sigma(P_s) / \sigma_{\text{tot}} = F, \) and \( \sigma(P_P) / \sigma_{\text{tot}} = H_0^{00}, \) involving the three final states with \( (Ss)^2, (Ps)^2, \) and \( (Pp)^2, \)

\[
E + F_1 + H_0^{00} = 1.
\]

The spin-dependent total cross sections are then given by

\[
\Delta \sigma_T / \sigma_{\text{tot}} = -2E + 2F_1 - H_0^Z, \\
\Delta \sigma_L / \sigma_{\text{tot}} = 2E + 2F_1 - 2H_0^Z.
\]

It should be noted that not all coefficients are independent. For instance, we know from the partial-wave analysis (see the Appendix) that for \( m = 0, \ldots, 5, \)

\[
H_m^{00} = H_m^Z + H_m^{Z-1}
\]

holds. Combining Eqs. (12b)–(12d) one easily derives the important relation

\[
\sigma(P_s) / \sigma_{\text{tot}} = \frac{1}{4} \left( 1 + \frac{\Delta \sigma_T}{\sigma_{\text{tot}}} + \frac{\Delta \sigma_L}{\sigma_{\text{tot}}} \right).
\]

This relation, which holds for \( pp \rightarrow pp \pi^0, \) allows one to determine, in a model-independent way, the total strength of the reaction going to a \( Ps \) final state directly from the measured total cross sections. This measurement of a partial wave has been presented in an earlier publication [11], where the relation given in Eq. (13) appears without proof.

---

**TABLE II.** Partial waves according to the final-state angular momenta. The column labeled \( L \) lists the symbol used in Eqs. (11) for a parameter of this type. The last column shows the power of \( \eta \) for the expected dependence on bombarding energy for the cases where neither \( l_p \) nor \( l'_p \) is zero.

<table>
<thead>
<tr>
<th>Final-state angular momenta</th>
<th>( l_p )</th>
<th>( l'_q )</th>
<th>( l_p' )</th>
<th>( l'_q' )</th>
<th>( L )</th>
<th>( w_L(\varepsilon) )</th>
<th>( \eta^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ss)^2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>E</td>
<td>( q \cdot p \cdot f(\varepsilon) ) ( d\varepsilon )</td>
<td>-</td>
</tr>
<tr>
<td>(Ps)^2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>F</td>
<td>( q \cdot p^3 ) ( d\varepsilon )</td>
<td>( \eta^6 )</td>
</tr>
<tr>
<td>PsPp</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>G</td>
<td>( q^2 \cdot p^3 ) ( d\varepsilon )</td>
<td>( \eta^4 )</td>
</tr>
<tr>
<td>(Pp)^2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>H</td>
<td>( q \cdot p^3 ) ( d\varepsilon )</td>
<td>( \eta^6 )</td>
</tr>
<tr>
<td>SsDd</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>I</td>
<td>( q^3 \cdot p \cdot f(\varepsilon) ) ( d\varepsilon )</td>
<td>-</td>
</tr>
<tr>
<td>SsDs</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>K</td>
<td>( q \cdot p^3 \cdot \sqrt{f(\varepsilon)} ) ( d\varepsilon )</td>
<td>-</td>
</tr>
</tbody>
</table>

---

064002-5
TABLE III. Bombarding energies used in this experiment, the $\eta$ parameter [Eq. (2)], and the upper bound $\epsilon_{\text{max}}$ on the energy-sharing parameter [Eq. (21)]. Also listed are the accumulated luminosities and the products of beam and target polarization for the two phases of the experiment (see Sec. III B).

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$\eta$ (MeV)</th>
<th>$\epsilon_{\text{max}}$</th>
<th>$fL dt$ (nb$^{-1}$)</th>
<th>$P_x Q$</th>
<th>$fL dt$ (nb$^{-1}$)</th>
<th>$P_x Q$</th>
<th>$P_y Q$</th>
<th>$P_z Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>325.6</td>
<td>0.560</td>
<td>21</td>
<td>2.163</td>
<td>0.456 (3)</td>
<td>3.0</td>
<td>0.059 (2)</td>
<td>0.333 (2)</td>
<td>0.296 (3)</td>
</tr>
<tr>
<td>350.5</td>
<td>0.707</td>
<td>33</td>
<td>0.901</td>
<td>0.342 (4)</td>
<td>1.3</td>
<td>0.053 (3)</td>
<td>0.316 (3)</td>
<td>0.267 (5)</td>
</tr>
<tr>
<td>375.0</td>
<td>0.832</td>
<td>44</td>
<td>3.024</td>
<td>0.514 (4)</td>
<td>4.1</td>
<td>0.041 (2)</td>
<td>0.333 (2)</td>
<td>0.266 (4)</td>
</tr>
<tr>
<td>400.0</td>
<td>0.948</td>
<td>55</td>
<td>0.831</td>
<td>0.526 (6)</td>
<td>1.1</td>
<td>0.039 (4)</td>
<td>0.289 (4)</td>
<td>0.203 (8)</td>
</tr>
</tbody>
</table>

III. MEASUREMENTS

A. Apparatus

The experiment was carried out with the Indiana Cooler storage ring. A detailed description of the apparatus has been presented previously in a technical paper [12]. In the following, we give an abbreviated description of the experimental setup, pointing out features that are especially important in appreciating the benefits and limitations of the technique employed.

1. Beam

A polarized 197 MeV proton beam from the IUCF cyclotron was accumulated in the Cooler ring, resulting in orbiting currents of 100–200 $\mu$A. The energy of the stored beam was then ramped to the desired value (for a list of energies, see Table III). The beam energy was known to better than 100 keV, and the polarization of the beam varied between 0.65 and 0.70.

The experiment was conducted in two phases. During the first phase, the beam polarization was vertical (along $\hat{y}$), while in the second phase nonvertical polarization was used. The latter is achieved with two spin-rotating solenoids. Their field is held fixed during acceleration. The field integral of these solenoids is limited, partly by the current limit of the solenoid, partly by difficulties in adjusting the ring optics to compensate for the additional focusing. The consequence of this limitation is that purely longitudinal beam polarization cannot be achieved for beam energies larger than 200 MeV. Instead, for the second phase of the experiment, the actual polarization direction is about $\vec{P}/P = (0.12, 0.75, 0.65)$, somewhat depending on beam energy (for actual values, see Table III).

The filling and ramping process takes 1–2 min, followed by 5–8 min of data taking. This beam cycle is then repeated. The sign of the beam polarization is changed every cycle.

2. Target

The stored beam passes through a target cell that consists of an open-ended 12 mm diameter cylindrical tube constructed from 25 $\mu$m aluminum foil. The tube is 25 cm long; the center of the cell defines the origin of the $z$ axis. Joined to the side of this tube, at $z = 0$, is a similar “feed” tube that is oriented towards the incident beam of polarized atoms. The target cell is supported by the end of the feed tube. It is possible to remotely adjust the cell position relative to the stored beam, in order to minimize the overlap between the beam halo and the cell wall. An atomic beam source [13] delivers the polarized hydrogen atoms. This source produces a beam of about 1 cm diameter with a flux of about $3 \times 10^{10}$ atoms per second in a pure spin state with a nuclear polarization of about $Q = 0.75$. The role of the target cell is to improve the utilization of the source output. The cell is coated with Teflon, which practically eliminates depolarization of the atoms during wall collisions. The total thickness of the target is a few times $10^{13}$ atoms/cm$^2$. The density of the target is determined by the gas flow through the cell, decreasing linearly from a maximum in the cell center to near zero at the open ends. The polarization direction is selected by a magnetic guide field of a few gauss in the region of the target. This field is generated by coils exterior to the scattering chamber, and can be oriented in the $\pm x$, $\pm y$, and $\pm z$ directions. It has been shown [14] that the magnitude of the target polarization does not vary significantly when the polarization direction is changed, and in the following we assume $Q = Q_x = Q_y = Q_z$ for the target polarization. During data acquisition the direction of the target polarization is changed every 2 s.

Internal polarized targets of this kind are pure and not susceptible to radiation damage, and they offer the possibility of rapidly changing the polarization direction.

3. Detector

The purpose of the detector is to measure the directions and energies of the two outgoing protons. This is accomplished with a stack of scintillators and wire chambers that are arranged as shown in Fig. 3. The directions of the two outgoing protons are determined by a set of four planes of wire chambers, and the “E” and the “K” scintillator arrays measure the energies of the protons.

The combined thickness of the E and the K detector planes is sufficient to stop the protons from the $pp \rightarrow pp \pi^0$ reaction for up to 400 MeV bombarding energy. The light from both planes is added and then converted to the energy of the stopped particle using a phenomenological expression for the light response, and a correction for the position-dependent light collection efficiency. The angular coverage of the detector depends on where along the target cell axis the event occurs. Seen from the center of the cell, the detec-
alternated in 2 s intervals between four directions. Elastic scattering is observed near 1.5-mm-thick scintillator target consists of a 0.18-mm-thick, stainless steel window. A sequence of incomplete acceptance are discussed in Sec. IV F. Departure of the detector acceptance from 100%. The consequences of incomplete acceptance are discussed in Sec. IV F. The wall of the vacuum chamber just downstream of the target consists of a 0.18-mm-thick, stainless steel window. A 1.5-mm-thick scintillator (“F” in Fig. 3), immediately following this window, provides a start signal for a time-of-flight measurement for particle identification, and eliminates events originating in the beam pipe downstream of the F detector. The E detector is divided into eight segments. The trigger for processing an event is a coincidence between the F detector and at least two segments of the E detector. A veto issued by the last scintillator in the stack (“V” in Fig. 3) removes events where at least one particle is not stopped in either the E or the K scintillator, and thus are not from pion production.

Concurrent with the acquisition of $pp \rightarrow pp \pi^0$ events, $pp$ elastic scattering is observed near $\theta_{lab} = 45^\circ$ by four scintillators (labeled “S” in Fig. 3). For elastic scattering events a coincidence between two opposite detectors is required. Particles reaching the S detectors traverse the first set of wire chambers (“WC1” in Fig. 3). A coplanarity condition and the known angle between the two protons provide a clean selection of $pp$ elastic events.

**B. Acquired data**

The experiment has been conducted in two phases. In the first (called “run A”) the beam polarization was vertical (along or opposite the y axis) and the target polarization was alternated in 2 s intervals between four directions (along or opposite the x axis or the y axis). Thus, data were accumulated with eight combinations of beam and target polarization ($P_n, Q_m$), namely, $(\pm P_y, \pm Q_x)$ and $(\pm P_y, \pm Q_x)$. Run A, which took place in the fall of 1997, was thus limited to observables that are accessible with only transverse polarization.

In the second phase (called “run B”), spin rotators were employed to generate nonvertical beam polarization (see Sec. III A 1). In this case, the beam polarization was a sum of three components ($P_y, P_y, P_y$), and the target polarization was alternated between the six directions $\pm Q_z, \pm Q_z$, and $\pm Q_z$, giving rise to 12 different spin states ($\pm P_y, \pm Q_z, \pm Q_z$), and ($\pm P_y, \pm Q_z, \pm Q_z$). Run B was carried out in the spring and fall of 1998. All possible analyzing powers and spin correlation coefficients were measured.

During both runs data were acquired at the beam energies 325, 350, 375, and 400 MeV. The respective integrated luminosities, together with the values for beam and target polarization, are listed in Table III.

**C. Measured yields**

1. **Selecting the $pp \rightarrow pp \pi^0$ events**

Events of interest are selected off line by requiring that both particles be identified as protons, that their wire chamber tracks be consistent with the patterns of responding segments in the various scintillator arrays, and that the origin of the event be in the target region. For each event the mass of the third, unobserved particle is calculated from the four-momenta of the two protons. An example of a missing mass spectrum is shown in Fig. 4. To accept an event, its missing mass has to be close to the mass of a neutral pion.

The amount of background under the pion mass peak varies with bombarding energy but is never larger than 10%. This background is caused by reactions of protons with the aluminum cell walls and with impurities in the target gas. Monte Carlo studies show that only reactions with three or more protons in the final state contribute significantly while...
(p,2p) reactions are unimportant. The shape of the background is determined from a separate measurement where the hydrogen in the target cell is replaced by N$_2$. This measurement results in a missing-mass spectrum that closely matches the one observed with a hydrogen target, except for the $\pi^0$ peak, and is therefore used to subtract the background under the pion peak.

The kinematics of the event is transformed to the center-of-mass system, and the angles $\theta_p$, $\varphi_p$, $\theta_q$, and $\varphi_q$, as well as the energy-sharing parameter $\epsilon$ are calculated. For each accepted event, these parameters, together with information on the direction of the beam and target polarization at the time of the event, are stored for further processing.

2. Spin-dependent yields

We define the “yield” to be the number of events in a certain region $\Delta \xi$ of phase space, defined by conditions on the five kinematic variables $\xi$ of the final state. There is one such yield $Y_{m,n}(\xi)$, for each combination $(m,n)$ of beam and target polarization. For run A there are 8 and for run B 12 such combinations. The yields in different spin states are always background corrected and normalized such that they correspond to equal accumulated luminosity in every spin state. This normalization compensated differences of a few percent in the luminosity with different beam polarization. The integrated luminosity was determined from a concurrent measurement of pp elastic scattering (see next section).

3. Monitoring beam and target polarization and the luminosity

Concurrent with the measurement of pion production, elastic pp scattering is observed by a dedicated set of four detectors that covers the angular region near $\theta_{lab}=45^\circ$. For these angles, the pp scattering spin correlation coefficients $A_\Delta$ and $A_\Lambda$ are quite large and well known [15]. This provides a sensitive on-line monitor for the products $P, Q_x, P, Q_y, P, Q_z$ of all three beam polarization components and the target polarization $Q=Q_x=Q_y=Q_z$. Note that the pp elastic scattering analyzing powers near $\theta_{lab}=45^\circ$ are small, so that the individual values for $P$ and $Q$ are not well determined from this measurement; however, these numbers are not needed for the subsequent analysis. From the pp scattering yield, averaged over azimuth and from the known cross section, we also deduce the integrated luminosity accumulated with each of the combinations of beam and target polarization. The relative luminosities are used to normalize the pion production yields in different spin states to equal integrated luminosity.

D. Asymmetries

From the spin-dependent yields, three different asymmetries can be calculated. The first, $S_p$, is the beam polarization asymmetry. It is obtained from the difference in the yields with positive and negative beam polarization, summed over all target polarization directions $j$:

$$S_p = \frac{\sum_{m=x,y,z} (Y_+ Q_m - Y_- Q_m)}{\sum_{m=x,y,z} (Y_+ Q_m + Y_- Q_m)}.$$  (14a)

Since each target orientation occurs with both signs, this effectively corresponds to an unpolarized target. The sum in the denominator is an average over both beam and target polarization direction, and thus represents the spin-averaged yield. Note that for run B the beam polarization is not along one of the coordinate axes and the asymmetry $S_p$ contains contributions from all the three polarization components.

The three target polarization asymmetries for the target polarization directions $m = x, y$ or $z$ are given by

$$S_{Q_m} = \frac{\sum_{n=+, -} (Y_n + Q_m - Y_n - Q_m)}{\sum_{n=+, -} (Y_n + Q_m + Y_n - Q_m)},$$  (14b)

where the sum over $n$ provides the average over the beam polarization direction.

Finally, the three spin correlation asymmetries, again with the target polarization in the $m = x, y$, or $z$ directions, are given by

$$S_{P,Q_m} = \frac{(Y_+ Q_m - Y_- Q_m) - (Y_+ - Q_m + Y_+ Q_m)}{(Y_+ Q_m + Y_- Q_m) + (Y_+ - Q_m + Y_+ Q_m)}.$$  (14c)

These asymmetries will be needed as a function of some of the kinematic variables $\xi$ while integrating over the others. For instance, if we want to know the asymmetries as a function of $\theta_q$ and $\varphi_q$, we sort the events into bins that divide the full range of $\theta_q$ and $\varphi_q$ to obtain the yields $Y_{n,m}(\theta_q, \varphi_q)$ while ignoring the other kinematic variables. If the detector acceptance is 100%, ignoring a kinematic variable is equivalent to integrating over that variable. Corrections due to incomplete detector acceptance are discussed in Sec. IV F. The asymmetries $S_p, S_{Q_m}$, and $S_{P,Q_m}$ of Eq. (14) form the basis for deducing the observables as described in Secs. IV B and IV C.

IV. RESULTS

A. Exploring the five-dimensional phase space

The dependence of each polarization observable on five kinematic variables contains a wealth of detailed information about the reaction, but it also presents the difficulty of ordering and accessing this information. In the present case we benefit from the limited number of amplitudes, which permits us to determine the functional dependence of the observables on the angles $\theta_q, \varphi_q, \theta_p, \varphi_p$ [Eq. (11)]. Based on this knowledge we now develop a procedure for extracting polarization information from the data in a systematic and complete way.
Inspecting Eq. (11), we note that the azimuthal functions \( \Phi_k(\varphi_q, \varphi_p) \) that occur are one of the following: \( \varphi_q, \varphi_p, \varphi_p + \varphi_q, 2\varphi_p - \varphi_q, 2\varphi_q - \varphi_p, \) or \( \varphi_p - \varphi_q \). Assume that we evaluate the asymmetries versus one of these functions \( \Phi_k \) \((k=1, \ldots, 6)\) by sorting the events into bins of constant \( \Phi_k \). This is equivalent to an integral over azimuth with the condition \( \Phi_k = \text{const} \), and eliminates one of the two azimuthal degrees of freedom. The implied integration retains only terms in Eqs. (11) that either contain \( \Phi_k \) or do not depend on azimuth at all. To further reduce the remaining terms, we evaluate observables as a function of one of the polar angles \( \theta \) \((\theta_p \text{ or } \theta_q)\), while integrating over the other one by ignoring it. Thus, for each of the polarization observables listed in Eq. (8), we have the choice of six azimuthal functions \( \Phi_k \) and two polar angles. The resulting set of observables that are now functions of a single variable \((\theta_p \text{ or } \theta_q)\) represents completely the effect of polarized collision partners on the angular variables. For now, we ignore the dependence on the energy-sharing parameter \( \epsilon \), and integrate over this quantity as well. The dependence on \( \epsilon \) will be discussed separately in Sec. IV.E.

**B. \( A_{ij}, A_{\Sigma}, A_{zz}, A_{\Delta}, \text{ and } A_{xz} \)**

The spin-dependent cross sections \( \sigma_0A_{ij}, \sigma_0A_{\Sigma}, \sigma_0A_{zz}, \sigma_0A_{\Delta}, \text{ and } \sigma_0A_{xz} \) contain only terms that are either azimuth independent or proportional to \( \cos \Phi_k \) or \( \cos 2\Phi_k \) where \( \Phi_k \) is one of five azimuthal dependences. Let us define the polarization observable \( A_{ij}^p(\theta_q) \) \([A_{ij}^p(\theta_p)]\) as that part of the observable \( A_{ij} \) that remains when integrating over \( \theta_p \) \([\theta_q]\) and over \( \varphi_q \) and \( \varphi_p \) with the constraint \( \Phi_k = 0 \). Of course, we still distinguish contributions with \( \cos \Phi_k \) from those with \( \cos 2\Phi_k \), since we have knowledge of the full \( \Phi_k \) distribution. In this definition, the particular \( \Phi_k \) selected is used as a superscript as a reminder that \( \Phi_k \) is used to isolate the corresponding term; it no longer appears in the functional dependence of the observable. As an example, the transverse beam analyzing power that would be measured when observing just the pion, in the present notation, would be \( A_{zz}^p(\theta_q) \). Using this definition, we end up with the following observables:

\[
A_{\Sigma}(\theta_q), \quad A_{\Sigma}(\theta_p): \quad A_{zz}(\theta_q): \quad A_{zz}(\theta_p),
\]

\[
A_{y0}^p(\theta_q), \quad A_{y0}^p(\theta_p), \quad A_{\Delta}^p(\theta_q), \quad A_{\Delta}^p(\theta_p),
\]

These 25 independent observables are extracted from the data as follows. First, we sort the events into bins for the selected polar angle \( \theta = \theta_p \) or \( \theta_q \) and azimuth function \( \Phi_k \) to obtain the asymmetries \( S_p(\theta, \Phi_k), \quad S_{Q_p}(\theta, \Phi_k), \quad \text{and } \quad S_{P, Q_p}(\theta, \Phi_k) \) in Eq. (14). Next, we insert the spin-dependent cross section, Eq. (4), into the expression for the asymmetries. For instance, for the beam asymmetry [Eq. (14a)] this results in

\[
S_p(\theta, \Phi_k) = A_{y0}^p(\theta)P_s \cos \Phi_k - P_x \sin \Phi_k, \quad \text{(16a)}
\]

\[
S_{Q_p}(\theta, \Phi_k) = A_{y0}^p(\theta)Q \sin \Phi_k, \quad \text{(16b)}
\]

\[
S_{P, Q_p}(\theta, \Phi_k) = A_{y0}^p(\theta)Q \cos \Phi_k. \quad \text{(16c)}
\]

The \( \Phi_k \) distributions of the asymmetries on the left are measured. Since Eqs. (16) constrain the ratios \( P_s/Q \) and \( P_x/Q \), knowing just the products \( P_sQ \) and \( P_xQ \) (see Sec. III C 3) is sufficient to extract \( A_{y0}^p(\theta) \).

In a similar fashion, the spin correlation observables are extracted; note that the observables \( A_{\Sigma} \) and \( A_{zz} \) have no azimuthal dependence, except for the terms containing \( \Delta \varphi = \varphi_p - \varphi_q \) which will be discussed separately in the next section.
Some of the 25 observables that are determined in this manner are displayed in Figs. 5–9. Figures 5 and 6 show the spin correlation coefficients \( A_S(\theta) \) and \( A_{zz}(\theta) \) as a function of \( \theta_q \) and \( \theta_p \), respectively, for all four bombarding energies. Figure 7 shows the analyzing power \( A_y(\theta) \) and the two spin correlation coefficients \( A_{xz}(\theta) \) and \( A_{Dx}(\theta) \) that would be measured if only the pion were observed, i.e., if the direction of the relative pp momentum is ignored. Similarly, Fig. 8 shows these observables for the case where the pion direction is ignored. In Fig. 9, some of the remaining possible observables are shown at 375 MeV, the energy with the best statistics. The errors shown in these figures are from counting statistics only. The solid curve is obtained from Eq. (11) with the coefficients in Table IV, taking into account the restricted acceptance of the detector system, while the dashed curve results when a detector with 100\% acceptance is assumed. The only significant effect of the restricted acceptance occurs with the observables \( A_S \) and \( A_{zz} \). The dotted curves are theoretical calculations that will be discussed later.

Fig. 5. The observables \( A_S(\theta) \) and \( A_{zz}(\theta) \) as a function of bombarding energy. The dashed curve is obtained with the coefficients of Table IV inserted into Eqs. (11). The solid line is the same but takes into account the real acceptance of the detector (see Sec. IV F). The current status of the theory is illustrated by the dotted line (see Sec. V B).

Fig. 6. The observables \( A_S(\theta) \) and \( A_{zz}(\theta) \) as a function of bombarding energy. The curves are explained in the caption of Fig. 5.

Fig. 7. \( A_y(\theta) \), \( A_{xz}(\theta) \), and \( A_{Dx}(\theta) \) at all four bombarding energies. These observables are based on the direction of the \( \pi^0 \); i.e., the relative proton momentum is ignored. The curves are explained in the caption of Fig. 5.

Fig. 8 shows these observables for the case where the pion direction is ignored. In Fig. 9, some of the remaining possible observables are shown at 375 MeV, the energy with the best statistics. The errors shown in these figures are from counting statistics only. The solid curve is obtained from Eq. (11) with the coefficients in Table IV, taking into account the restricted acceptance of the detector system, while the dashed curve results when a detector with 100\% acceptance is assumed. The only significant effect of the restricted acceptance occurs with the observables \( A_S \) and \( A_{zz} \). The dotted curves are theoretical calculations that will be discussed later.

C. \( A_{z0} \) and \( A_{z1} \)

The longitudinal analyzing power \( A_{z0} \) and the combination \( A_{z1}=A_{3z}-A_{2z} \) of spin correlation coefficients are proportional to \( \sin \Delta \varphi \) or \( \sin 2\Delta \varphi \) [Eq. (11)], where \( \Delta \varphi=\varphi_p-\varphi_q \). Thus, these observables are invariant with respect to a rotation around the beam axis, and they vanish for \( \Delta \varphi=0 \) and \( \pi \), which is the case when the momenta of the three outgoing particles are coplanar. The vanishing of these observables in the case of a coplanar final state is a consequence of parity conservation. In fact, a measurement of \( A_{z0} \)
in a two-body final-state reaction (thus, in coplanar geometry), or in a total cross section, has been used as a tool to study the violation of parity conservation [16].

Recently, we have published a first analysis [17] of the longitudinal analyzing power $A_{zz}$ for $pp \rightarrow pp \pi^0$ in which we demonstrated that this observable can be quite large if noncoplanar final states are involved. Previous measurements of this observable are scarce: some indication of a large value of $A_{zz}$ was found [18] in another pion production reaction, $pn \rightarrow pp \pi^-$ at 443 MeV, while a measurement of $A_{zz}$ in the reaction $^3\text{H}(p,pn)$ at 9 MeV yielded values that are consistent with zero at the level of 0.003 [19].

In analogy with the previous section, we define the observables $A_{\Sigma 0}^1(\theta_p)$, $A_{\Sigma 2}^1(\theta_p)$, and $A_{\Sigma 2}^2(\theta_p)$ as $A_{\Sigma 0}(\xi)$ and $A_{\Sigma 2}(\xi)$, integrated over $\theta_p$, as well as integrated over azimuth with the condition $\Delta \varphi = \text{const}$ and evaluated at $\Delta \varphi = \pi/2$. This definition is suggested by Eqs. (11g) and (11h). Again, we can distinguish $A_{\Delta 0}^1(\theta_p)$ from $A_{\Delta 2}^1(\theta_p)$ because we know the full $\Delta \varphi$ distribution. Likewise, we define the $\Delta \varphi$ parts of $A_{\Sigma}$ and $A_{zz}$ as $A_{\Delta}^1$, $A_{\Delta 2}^1$, $A_{\Delta}^2$, and $A_{\Delta 2}^2$, in this case evaluated at $\Delta \varphi = 0$ [based on Eqs. (11c) and (11d)].

In order to extract $A_{\Sigma 0}$ and $A_{\Sigma 2}$ from the present data, we generate the asymmetries $S_p$, $S_Q$, and $S_{P,Q}$ as a function of $\Delta \varphi$. It is obvious that $A_{\Sigma 0}$, $A_{\Delta 0}$, and $A_{\Delta}$ do not contribute in this case, since they do not depend on $\Delta \varphi$. Ignoring for the moment a possible $\Delta \varphi$ dependence of the spin-averaged cross section, we obtain, for the asymmetries [analogous to Eqs. (16) and (17)],

\[ S_p(\theta, \Delta \varphi) = P_e A_{\Sigma 0}^1(\theta) \sin 2 \Delta \varphi, \]  
\[ S_Q(\theta, \Delta \varphi) = S_{Q_x}(\theta, \Delta \varphi) = 0, \]  
\[ S_{Q_y}(\theta, \Delta \varphi) = Q A_{\Sigma 0}^2(\pi - \theta) \sin \Delta \varphi + Q A_{\Sigma 2}^2(\pi - \theta) \sin 2 \Delta \varphi, \]  
\[ S_{P,Q_x}(\theta, \Delta \varphi) = \frac{1}{2} P e Q [A_{\Sigma}^1(\theta) + A_{\Delta}^1(\theta) \cos \Delta \varphi] \]  
\[ + A_{\Sigma 2}^2(\theta) \cos 2 \Delta \varphi - 1/2 A_{\Delta 2}^2(\theta) P e Q \sin \Delta \varphi, \]  
\[ S_{P,Q_y}(\theta, \Delta \varphi) = \frac{1}{2} P e Q [A_{\Sigma}^2(\theta) + A_{\Delta}^2(\theta) \cos \Delta \varphi] \]  
\[ + A_{\Sigma 2}^2(\theta) \cos 2 \Delta \varphi + 1/2 A_{\Delta 2}^2(\theta) P e Q \sin \Delta \varphi. \]
TABLE IV. Values at the four bombarding energies of the coefficients introduced in Eqs. (11). The derivation of these coefficients is discussed in Sec. IV D. All values have been normalized with the common factor $8 \pi^2/\sigma_{\text{tot}}$. These numbers parametrize all possible initial-state polarization observables of the reaction everywhere in phase space.

<table>
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<th>325 MeV</th>
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<td>Error</td>
<td>Value</td>
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<td>0.059</td>
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</tr>
<tr>
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</tr>
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<tr>
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<td>0.102</td>
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<tr>
<td>$H^{0}_5$</td>
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<td>$H^{0}_7$</td>
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<tr>
<td>$H^{0}_1$</td>
<td>-0.030</td>
<td>0.135</td>
<td>0.093</td>
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</table>
These asymmetries, integrated over polar angle, are shown in Fig. 10. Here, $S_P$ and $S_Q$ reflect the beam and target analyzing powers $A_{zz}$ and $A_{0z}$, which are related by Eq. (7). The quantities $S_{Qx}$ and $S_{Qy}$ are consistent with zero, as expected.

Evaluating the asymmetries as a function of $u_q$, thus integrating over $u_p$, we extract the $u_q$ distributions of the observables by fitting with the respective functions of $Dw$. In this way we obtain the observables

$$A_{zz}(\theta_q), \quad A_{zz}(\theta_q) (\Delta \varphi = \pi/2),$$

$$A_{zz}(\theta_q), \quad A_{zz}(\theta_q) (\Delta \varphi = 0),$$

$$A_{zz}(\theta_q), \quad A_{zz}(\theta_q) (\Delta \varphi = 0).$$

The part of $A_{zz}$ that scales with $\sin 2 \Delta \varphi$ [see Eq. (11g)] was found to be consistent with zero. It is clear from Eqs. (11g) and (11h) that the $\theta_p$ dependence does not contain independent information. Thus, from the $\Delta \varphi$-dependent asymmetries we extract six additional observables. They are shown in Fig. 11 for the measurements with better statistics at 375 and 400 MeV.

**D. Parametrization of the data**

The expansion into functions of the angles $\theta_q$, $\varphi_q$, $\theta_p$, $\varphi_p$ [Eq. (11)] allows one to calculate all polarization observables at any point in phase space, provided the expansion coefficients $E, F, G, \ldots$ are known. These coefficients thus represent a parametrization of all our measurements and constitute the central result of this experiment. The values for the coefficients, normalized by a common factor $8 \pi^2/\sigma_{tot}$, are listed in Table IV. Note that the common factor cancels when calculating a polarization observable $A_{ij}$ by dividing the spin-dependent cross section $\sigma_{ij}$ by the spin-averaged cross section $\sigma_{tot}$.

The task of determining the values of the coefficients of Eqs. (11) is simplified by the fact that a given polarization observable from the list in Eqs. (15) and (19) depends on only a few coefficients. For instance, the observable $A_{zz}(\theta_p)$ depends on $G_1^{y0}, (H_1^{y0}+I^0)$, and $(H_1^{y0}+I)$, and $A_{zz}(\theta_p)$ depends on $G_2^{y0}, G_2^{x0},$ and $(H_2^{y0}+F_2+K)$. However, the quality of the data, especially at the lower two energies, is

**FIG. 10.** The asymmetries versus $\Delta \varphi = \varphi_p - \varphi_q$ at 375 MeV bombarding energy. Integrated over both polar angles, the curves represent a fit to the $\Delta \varphi$ distribution according to Eq. (18).

**FIG. 11.** Polar angle $\theta_q$ dependence of the observables that depend on $\Delta \varphi = \varphi_p - \varphi_q$, as discussed in Sec. IV C, at the two bombarding energies with the best statistics. The curves are explained in the caption of Fig. 5.
not sufficient to fit the coefficients to the data without any constraining assumptions. In the following, we describe these assumptions and a step-by-step procedure to determine the coefficients of Eq. (11).

In the first step, we address the coefficients $E$, $F_1$, $H_0^{10}$, $H_0^{\Sigma}$, and $H_0^{zz}$. The corresponding terms in Eq. (11) do not depend on angle but represent different final states ($S_s$)$^2$, ($Ps$)$^2$, and ($Pp$)$^2$ (see Table II). The relative weight of the ($Ps$)$^2$ final state follows from the spin-dependent total cross section [Eq. (13)], but the relative contributions of the ($S_s$)$^2$ and ($Pp$)$^2$ final states can only be determined since they depend on energy $\epsilon$ differently. This is explained in more detail in Sec. IV E. Using that result, we set the coefficient $H_0^{10}$ equal to $\sigma_0(Pp)/\sigma_{tot}$, the relative contribution of the ($Pp$)$^2$ final state. Having fixed the ($Pp$)$^2$ strength, the coefficients $E$, $F_1$, $H_0^{10}$, $H_0^{\Sigma}$, and $H_0^{zz}$ follow from Eqs. (12), with the values of the spin-dependent total cross sections $\Delta \sigma_{EL}/\sigma_{tot}$ and $\Delta \sigma_{EL}/\sigma_{tot}$, which have been deduced from the total, spin-dependent yields as listed in Table V.

Next, we turn to the coefficients that multiply the terms with $(3 \cos^2 \theta - 1)$ in $\sigma_0$, $\sigma_p A_z$, and $\sigma_p A_{zz}$ [Eqs. (11a), (11c), (11d)]. Those coefficients are $H_0^{10}$, $H_1^{\Sigma}$, $H_1^{zz}$, $H_0^{10}$, $H_2^{\Sigma}$, and $F_2$, two of which can be eliminated by Eq. (12d). The $S_s$Sd and $S_s$Ds interference terms, $I$ and $K$ may be lumped with the corresponding $H_k^{ij}$ terms with Eq. (12d) still satisfied. Since calculating the observables $\sigma_0 A_z/\sigma_0$ involves a ratio of similar functions, the statistical accuracy of the present data is insufficient to distinguish these coefficients separately for each bombarding energy. Instead, we impose an energy dependence on the coefficients by setting $H_k^{ij}(\eta) = H_k^{ij} \eta^k/\sigma_{tot}(\eta)$ and $F_2(\eta) = F_2(\eta) \eta^4/\sigma_{tot}(\eta)$. The justification for this assumption is given in the next section, and the values for $\sigma_{tot}(\eta)$ are those listed in Ref. [11] and in Table V. Thus, we fit five variables to the angular distributions $A_z(\theta_p)$, $A_z(\theta_q)$, $A_{zz}(\theta_p)$, and $A_{zz}(\theta_q)$ at all four energies simultaneously. The fit is shown as a solid line in Figs. 5 and 6; the $\chi^2$ per degree of freedom is 1.5.

Next, we determine the coefficients $H_0^{10}$, $H_1^{\Sigma}$, $H_1^{zz}$, and $G_{\Xi}$ that appear with terms that contain $\Delta \varphi$. Again, Eq. (12d) constrains the $H_k^{ij}$. The corresponding observables have been discussed in Sec. IV C. We again impose a bombarding energy dependence of the $H$ coefficients as described in the preceding paragraph and set $G^{ij}(\eta) = G^{ij} \eta^k/\sigma_{tot}(\eta)$. The remaining seven variables are then fit to the angular distributions $A_z(\theta_p)$, $A_z^{\Delta \varphi}(\theta_p)$, $A_z^{\Delta \varphi}(\theta_q)$, $A_z^{\Delta \varphi}(\theta_q)$, $A_z^{\Delta \varphi}(\theta_p)$, and $A_z^{\Delta \varphi}(\theta_q)$ at all four energies simultaneously. The fit is shown as a solid line in Fig. 11; the $\chi^2$ per degree of freedom is 1.6.

With the angular dependence of the spin-averaged cross section now known, the remaining coefficients are determined by fitting the corresponding observables without any constraint on their energy dependence. The errors are obtained by propagating the statistical errors of the measurements.

Note that the observables [Eqs. (15) and (19)] are integrated over either $\theta_p$ or $\theta_q$ and thus do not constrain the coefficients $H_0^{10}$, $H_1^{\Sigma}$, $H_1^{zz}$, $H_2^{10}$, and $H_2^{\Xi}$.

The values of the coefficients in Table IV have been obtained from the data by taking into account the incomplete acceptance of the detector (for more detail, see Sec. IV F). The resulting parametrization of the data is shown as a solid line in Figs. 5–9. Using the same coefficients, but pretending that the detector accepts all of phase space, leads to the dashed line. This illustrates the smallness of the effect of incomplete detector acceptance.

We note that the coefficients $I$ and $K$ that represent interfering $S_s$Sd and $S_s$Ds amplitudes always occur in a sum with an $H_k^{ij}$ coefficient. These sums become a single parameter in the analysis. Thus, the present analysis provides no information on the importance of these terms.

Equations (11) contain a total of 49 coefficients. Of these, we determine 44 from the data (see Table IV). Among these, there are six known relations [Eqs. (12a), (12d)], resulting in 38 numbers determined. On the other hand, the coefficients are (known) functions of the amplitudes listed in Table I. Ignoring contributions from $S_d$ and $D_s$ amplitudes, there are 12 amplitudes. Since there is no interference between amplitudes with $s_f = 0$ and $s_f = 1$, there are two free phases, and, in principle 22 real numbers should be sufficient to completely describe the data. The parametrization presented here [Eq. (11) and Table IV] has some redundancy; i.e., there are relations between the parameters [in addition to those in Eq. (12)]. These relations will be revealed in the course of the amplitude analysis which is planned for the future.

### E. Energy dependence

#### 1. Definitions and kinematics relation

A complete description of the final-state kinematics, apart from the four angles $\theta_p$, $\varphi_p$, $\varphi_q$, $\varphi_q$, must include an en-
ergy variable that specifies the sharing of the available kinetic energy between the pion and the NN pair. There is only one such variable since the total energy of the system, \( \sqrt{s} \), is determined by the bombarding energy. For instance, if \( q \) is the magnitude of the pion center-of-mass momentum, the proton momentum in the NN rest system is given by

\[
p = \frac{1}{2} \sqrt{s_{12} - 4m_p^2} = q_{\text{max}} \sqrt{1 - (q/q_{\text{max}})^2},
\]

(20)

where \( s_{12} = s - 2\sqrt{s(q^2 + m^2)} + m^2 \) is the square of the energy of the NN subsystem. The second part of Eq. (20) is the corresponding nonrelativistic expression, which is a good approximation near threshold. Here, \( q_{\text{max}} \) [Eq. (1)] is the largest possible pion momentum, which is realized when the two protons are at rest relative to each other \( (p=0) \). In the following, we use the energy-sharing variable, the kinetic energy \( \epsilon \) in the NN subsystem given by

\[
\epsilon = \sqrt{s_{12} - 2m_p},
\]

(21)

which ranges from \( \epsilon = 0 \) \( (q = q_{\text{max}}) \) to \( \epsilon_{\text{max}} = \sqrt{s} - 2m_p - m_q \) (when \( q = 0 \)). The value for \( \epsilon_{\text{max}} \) is determined by the bombarding energy, or \( \eta \) [Eq. (2)], as good a approximation as possible in the initial and final states is only an approximation (for more on this topic, see Ref. [2]).

For a limited energy range, the partial-wave coefficients \( E, F, \cdots \) for the energies of this experiment. Using Eqs. (20) and (21), \( p \) and \( q \) may be expressed in terms of \( \epsilon \).

2. Leading contributions to the energy dependence

For a limited energy range, the dynamics of pion production is often considered energy independent. The strong energy dependence of the observables near threshold is then due to a number of known factors, as discussed in the following.

The first energy dependence is due to the phase space volume \( dp(\epsilon) \). Nonrelativistically the phase space volume is proportional to \( q(\epsilon)p(\epsilon)d\epsilon \). The second energy-dependent factor arises from the radial wave functions for the pion and the NN pair. Close to threshold, the momenta \( q \) and \( p \), and thus the arguments of these wave functions, are small, and one can use their limiting form to obtain the factor \( q^{8}p^{8}r^{12} \), where \( l_q \) and \( l_p \) are the respective angular momenta. It is this factor that makes it possible to use the energy dependence of the reaction to make statements about partial-wave contributions, but one must keep in mind that the simple power law is an approximation, strictly true only for \( p \rightarrow 0 \) or \( q \rightarrow 0 \).

The third energy-dependent factor arises from distortion in the entrance and exit channel. By far the strongest energy dependence is due to the final-state interaction (FSI) between two nucleons in a relative \( S \) state. Watson showed [20] that the FSI energy dependence of the cross section can be separated as a factor \( f(\epsilon) \) that follows from the NN phase shifts at energy \( \epsilon \). One method to calculate \( f(\epsilon) \) is by representing the \( S \)-wave phase shift by an effective-range expansion. Since the two nucleons carry charge, Coulomb repulsion has to be incorporated into the effective-range expansion [21]. In the present work, this procedure is adopted for calculations that involve FSIs. Other authors have used a fit to a phenomenological representation of the NN interaction to obtain \( f(\epsilon) \) [10].

When integrating over the energy-sharing parameter \( \epsilon \) one obtains, via the upper limit \( \epsilon_{\text{max}} \), a dependence on bombarding energy, or \( \eta \). Thus, close to threshold, where only the \( Ss \) wave contributes, the shape of total \( pp \rightarrow pp \pi^0 \) cross section as a function of bombarding energy should be determined by the phase space and FSI, an expectation that is borne out by the data [5]. However, in order to reproduce the measured proton angular distributions, one has to use a value \( \sim 1.5 \text{ fm} \) for the scattering length (see Ref. [5]). This is significantly larger than the accepted, Coulomb-uncorrected value for the \( pp \) scattering length of \( a_{pp} = 7.82 \pm 0.01 \text{ fm} \) [22]. This indicates clearly that factorizing the FSI of the protons and neglecting all other distortions in the initial and final states is only an approximation.

In Eq. (11), the partial-wave coefficients \( E, F, \cdots \) are given

\[
\begin{align*}
E^2 & = 0, \\
F^2 & = 0, \\
G^2 & = 0, \\
H^2 & = 0, \\
I & = 0, \\
K & = 0,
\end{align*}
\]

and may be integrated over \( \epsilon \). This integration is independent of the angular variables since \( \epsilon \) ranges from 0 to \( \epsilon_{\text{max}} \) for any choice of angles.

To reveal the explicit energy dependence of these coefficients, we separate off the probability \( w_L(\epsilon) \) with which a given \( \epsilon \) occurs where \( L \) denotes the set of four final-state angular momenta, \( l_p, l_q, l_p', l_q' \), that occur in the bilinear sums of amplitudes,

\[
w_L(\epsilon)d\epsilon = \zeta q(\epsilon)^{l_q + l_p'} p(\epsilon)^{l_q + l_p} f_L(\epsilon) d\epsilon,
\]

(22)

where the normalization \( \zeta \) ensures that \( \int w_L(\epsilon)d\epsilon = 1 \). The final-state factor is given by \( f_L(\epsilon) = f(\epsilon) \) if both \( l_p \) and \( l_p' \) are zero, by \( f_L(\epsilon) = \sqrt{f(\epsilon)} \) if either \( l_p \) or \( l_p' \) is zero, and by \( f_L(\epsilon) = 1 \) in all other cases. The \( \epsilon \) dependence for partial waves with various angular momenta is given in Table II. The three functions \( w_{L}(\epsilon), w_{L'}(\epsilon), \) and \( w_{L L'}(\epsilon) \) represent \( (Ss)^2, (Ps)^2, \) and \( (PP)^2 \) partial waves. For a bombarding energy of 375 MeV, these three functions are displayed as solid curves in Fig. 12. Note that \( w_{L}(\epsilon) \) clearly shows an enhancement for small \( \epsilon \), caused by the final-state interaction. In general, the weight functions \( w_L(\epsilon) \) depend on the detector acceptance, since in the laboratory the momenta of the two protons do depend on \( \epsilon \). This is illustrated in Fig. 12 by Monte Carlo–generated histograms that show the effect of a \( 5^o \) central hole in the detector coverage. The consequences of incomplete detector acceptance are discussed further in Sec. IV F.

As briefly noted, the dependence of the amplitudes on \( \epsilon \) implies a dependence on bombarding energy, or \( \eta \), because the upper limit \( \epsilon_{\text{max}} \) of the integration over \( \epsilon \) depends on \( \eta \). In the absence of FSIs, and with the nonrelativistic expression for the phase volume and for \( p(\epsilon) \) [Eq. (20)], the integration of Eq. (22) is analytic and a simple power law results. From this, we expect the partial-wave coefficients \( F, G, \) and \( H \) to be proportional to \( \eta^5/\sigma_{tot}(\eta), \eta^7/\sigma_{tot}(\eta), \) and \( \eta^8/\sigma_{tot}(\eta) \), respectively. Such a simple dependence on bombarding energy is not expected for the coefficients \( E, I, \) and \( K \), since these are affected by the FSI.

3. Dependence of \( A_x \) and \( A_y \) on the energy-sharing parameter

Some of the coefficients in Eq. (11) cannot be distinguished from each other based on the angular distributions.
known functions of \( e \) previously the angular distributions. A similar method has been applied; these coefficients are not accessible separately by a study of \( A_{zz} \).

However, their individual values can still be assessed, using the fact that they depend differently on the energy parameter \( e \). In this section, we explain how this can be done.

When we integrate the spin-dependent cross sections of Eq. (11) over all angles, only \( \sigma_0(e) \), \( \sigma_0(e)A_{zz}(e) \), and \( \sigma_0(e)A_{zz}(e) \) remain which in turn depend on four coefficients \( E \), \( F_1 \), \( H_{00} \), \( H_{0}^{zz} \), and \( H_{0}^{zz} \), where \( H_{00}^{zz} = H_{0}^{zz} - H_{0}^{zz} \). Note that these coefficients when normalized by \( 8 \pi^2 \sigma_{\text{tot}} \) are related to the partial-wave total cross sections \( \sigma(l_p, l_q) \) by \( \sigma(Ss)/\sigma_{\text{tot}} = E \), \( \sigma(Ps)/\sigma_{\text{tot}} = F_1 \), and \( \sigma(Pp)/\sigma_{\text{tot}} = H_{00}^{zz} \).

The present notation is related to that used in Ref. [11] by \( 2\sigma(Pp)/\sigma_{\text{tot}} = H_{0}^{zz} \). The two observables \( A_{zz}(e) \) and \( A_{zz}(e) \) in terms of the partial-wave coefficients are now given by

\[
A_{zz}(e) = \frac{2(E \cdot w_E - F_1 \cdot w_F + H_{00}^{zz} \cdot w_H)}{E \cdot w_E + F_1 \cdot w_F + H_{00}^{zz} \cdot w_H},
\]

(23a)

\[
A_{zz}(e) = \frac{-E \cdot w_E - F_1 \cdot w_F + (H_{00}^{zz} - H_{0}^{zz}) \cdot w_H}{E \cdot w_E + F_1 \cdot w_F + H_{00}^{zz} \cdot w_H}.
\]

(23b)

In these equations, the probabilities \( w_E \), \( w_F \), and \( w_H \) are known functions of \( e \) that differ from each other (see Fig. 12). Thus, it is possible to determine the coefficients \( E \), \( F_1 \), \( H_{00}^{zz} \), and \( H_{0}^{zz} \) from a fit to the measured \( A_{zz}(e) \) and \( A_{zz}(e) \). These coefficients are not accessible separately by a study of the angular distributions. A similar method has been applied previously [10] to the spin-averaged total cross section as a function of \( e \).

From the set of good events we determine \( A_{zz}(e) \) and \( A_{zz}(e) \) following the same procedure as described in Sec. IV B, except that the argument \( \theta_q \) (or \( \theta_p \)) is replaced by the energy-sharing parameter \( e \). The result is shown in Fig. 13 for all four bombarding energies. The solid curves are obtained from Eq. (23) with weight functions \( w_L \) that take into account the acceptance of the detector. The coefficients in Eq. (23) were forced to depend on bombarding energy as \( F_1(e) = F_1^{00} e^{00} / \sigma_{\text{tot}}(e) \), \( H_{00}^{zz}(e) = H_{00}^{zz} e^{00} / \sigma_{\text{tot}}(e) \), and \( H_{0}^{zz}(e) = H_{0}^{zz} e^{00} / \sigma_{\text{tot}}(e) \). At \( T = 325 \) MeV an accurate value for the total cross section exists \( \sigma_{\text{tot}} = 7.70 \pm 0.26 \mu \text{b} [5] \). However, at higher energies, data are few and of poor quality. For the present purpose we use for \( \sigma_{\text{tot}}(e) \) a smooth approximation to the world’s data (see Ref. [11] and Table V). Assuming that there are no other partial waves, we have \( E = 1 - F_1 - H_{00}^{zz} \). Therefore, only three energy-independent parameters are adjusted. The \( \chi^2 \) of the best fit per degree of freedom is 1.8, which leads us to suspect that the limitations of the simple energy dependence adopted here may be noticeable, especially at the higher energies. The resulting partial-wave contributions to the total cross section are shown in Fig. 14. The error bars are obtained by repeating the fit by varying the values assumed for \( \sigma_{\text{tot}} \) or by using

![FIG. 13. Dependence of \( A_{zz} \) and \( A_{zz} \), integrated over both polar angles, on the energy-sharing parameter \( e/\epsilon_{\text{max}} \). The solid lines represent a three-parameter fit to the data at all four energies simultaneously; see Sec. IV E 3.](image)

![FIG. 14. Contribution of the three possible final-state angular momenta to the total cross section. The dashed and solid lines represent the expected \( \eta^1 (\eta^2) \) dependence of the \( \text{Ps (Pp)} \) partial-wave cross section, while the dotted line indicates the remainder, which represents the \( \text{Ss} \) partial-wave cross section.](image)
weight functions \( w_L \) calculated directly from Eq. (22), as would be appropriate for a detector with 100% acceptance. The dashed line in Fig. 14 represents the expected \( \eta^6 \) dependence of the \( P_s \) partial cross section, \( \sigma(P_s) = F_1 \), and the solid line corresponds to the imposed \( \eta^6 \) dependence of \( \sigma(P_p) = H_{00} \), while the dotted line indicates the remainder, given by \( E = 1 - F_1 - H_{00} \), which represents the \( S_s \) partial-wave cross section.

4. Dependence of observables on bombarding energy

As pointed out at the end of Sec. IV E 1, based on the phase space, angular momentum dependence of the wave functions, and FSI, we expect that the partial-wave coefficients \( F, G, \) and \( H \) times the total cross section \( \sigma_{\text{tot}}(\eta) \) are proportional to \( \eta^6, \eta^7, \) and \( \eta^8 \), respectively. We have also explained that the integration over \( \epsilon \) is independent of the angular variables. Thus, each of the coefficients in Eq. (11) that does not contain a \( NN S \) state \( (F, G, \text{and } H \text{ coefficients}) \) is expected to obey such a power law. In order to test this expectation, we have to multiply the values for the coefficients in Table IV by the total cross section \( \sigma_{\text{tot}}(\eta) \) at the corresponding energy. For \( \sigma_{\text{tot}}(\eta) \) we use a smooth approximation to the world’s data, as explained in the previous section. The resulting \( \eta \) dependence of some of the coefficients in Table IV that have been obtained without constraining their energy dependence is shown in Fig. 15. The two lines shown in the figure correspond to the best fit with an \( \eta^6 \) or \( \eta^8 \) dependence. As can be seen, the simple power-law \( \eta \) dependence of the coefficients is at least qualitatively correct. This is also true for the coefficient \( (H_{11} + K/2) \), which could in principle contain a contribution from a \( D_s \) amplitude. The observation that the \( G \) and \( H \) coefficients obey the power law that is expected from the ‘‘trivial’’ energy-dependent factors confirms a similar finding based on partial-wave contributions to the spin-dependent total cross sections [11].

F. Systematic uncertainties and corrections

1. Corrections for a nonideal detector

For a number of reasons, the apparatus does not register all the generated \( pp \to pp \pi^0 \) events. The main loss of events occurs because the detector system has a hole in the center to allow for the 3-cm-diam beam pipe just downstream of the target. Seen from the center of the target, this hole subtends a cone with about 5° opening angle. Between 25% (at 325 MeV) and 10% (at 400 MeV) of all events have at least one proton that falls into this cone. At 400 MeV a few percent of the events miss the detector on the outside, and about 3% contain a proton that is energetic enough to fire the veto detector. In about 2% of the events, both protons strike the same segment of the E detector, and therefore do not trigger the detector. The efficiency of an individual wire chamber plane is between 93% and 95%, but since only three planes have to respond for a valid event, only about 8% of all events are lost because of this. All of these effects combined amount to a loss of events between 30% and 22% for the energies from 325 MeV to 400 MeV. A Monte Carlo simulation of the detailed detector performance was used to determine these numbers. Reactions in the scintillators might lower the proton energy measured by the K and E scintillators, leading to a tail of the \( \pi^0 \) peak in the missing-mass spectrum (Fig. 4), placing some good events outside the accepted mass range. However, there is no evidence for a significant tail in the mass spectrum.

The correction of the data presented in this paper for the losses discussed above turns out to be small. This is because polarization observables are a ratio of yields measured with and without polarization. If the fraction of lost events is the same in both cases, there is no net correction. For this reason, there is no correction for the data in a given volume element \( d\Omega_x d\Omega_y d\epsilon \) of the five-dimensional phase space. Thus, corrections arise only when integrating over some region of the phase space.

Acceptance corrections are estimated as follows. Let us denote by \( \alpha(\xi) \) the detector acceptance at a given point \( \xi \) in phase space. Since the corresponding event is either seen or not seen, \( \alpha(\xi) \) has a value of 1 or 0. In five-dimensional phase space the transition from \( \alpha = 0 \) to \( \alpha = 1 \) occurs at well-defined boundaries. However, when one integrates over several variables, the dependence of \( \alpha \) on the remaining variables is smoothed out, and this is another reason for the smallness of the acceptance corrections. Since the functional dependence of the observables on all five variables \( \theta_q, \varphi_q, \theta_p, \varphi_p, \) and \( \epsilon \) is known, we can carry out the integration over kinematic variables, weighting the integrand with \( \alpha(\xi) \) and thus taking into account the real detector acceptance. These integrals are evaluated numerically using the Monte Carlo method for each of the partial waves in Table II and for each of the trigonometric functions of the kinematic vari-
ables. For comparison, setting \( \alpha = 1 \), independent of \( \xi \), yields the result for a detector with 100% acceptance. The effect of incomplete acceptance on the angular distributions is illustrated in Figs. 5–9. The solid curve is obtained from Eqs. (11) and the coefficients in Table IV using the true detector acceptance, while the dashed line results when 100% detector acceptance is assumed. As can be seen the effects are very small.

The acceptance corrections for the total cross sections, Eq. (12), involve the integrals over the entire phase space for three partial waves with the final states \( S_S, P_S \) and \( P_P \), corresponding to \( E, F_1 \), and \( H \) in Eq. (12). Again, if the fractional loss for all three partial waves were the same, there would be no correction. However, as can be seen from Fig. 12, the \( S_S \) partial wave is affected more strongly by losses in the central hole than the other two partial waves. In order to evaluate the correction for \( \Delta \sigma_T / \sigma_{tot} \) and \( \Delta \sigma_L / \sigma_{tot} \) the relative strength of the three partial waves is taken as shown in Fig. 14. The resulting corrections are listed in Table V. They are slightly different than those used in Ref. [11] because more has since been learned about the relative importance of the three contributing partial waves.

2. Other systematic effects

The dead time of the data acquisition system was measured for each of the different spin states of beam and target. The dead time is a few percent and differences between spin states are less than 10\(^{-3}\). Thus, dead time effects can be neglected.

The reconstruction of the pion polar angle \( \theta_q \) depends sensitively on the absolute energy calibration of the E and K scintillators, since the pion has to account for the remaining momentum. However, because of the identity of the collision partners, the spin-averaged cross section has to be symmetric around \( \theta_q = 90^\circ \). This condition has been used as one of the criteria in determining the energy calibration of the scintillators.

Finally, one has to worry about the resolution of the detector system as a whole for the cms angles \( \theta_p, \phi_p, \theta_q \), and \( \phi_q \). This has been studied with a Monte Carlo simulation of the response of the detector system. The generated events were processed by the same code that was used to analyze the data. For all four angles, the difference between the reconstructed angle and the “true” angle (as chosen initially by the Monte Carlo simulation) falls into a distribution which is very nearly a Gaussian, centered on zero within the widths of the distributions. We identify the angular resolution with the \( \sigma \) of this Gaussian in each case. These distributions vary somewhat with beam energy and are widest for the lowest-energy data. Therefore, we here report the \( \sigma \) of the Gaussian fit to each distribution at 325 MeV beam energy. The results are \( \sigma = 3.0^\circ \) for \( \theta_p \), 1.5\(^\circ\) for \( \phi_p \), 8.0\(^\circ\) for \( \theta_q \), and 6.0\(^\circ\) for \( \phi_q \). The \( \sigma \) corresponding to the \( \cos(\theta_p) \) distribution of errors is 0.04, and for \( \cos(\theta_q) \) it is 0.12. There is no correlation observed between the errors in the reconstructed \( p \) and \( q \) vectors. Clearly, this resolution is sufficient to resolve the harmonic content of the angular distributions in this experiment.

V. COMPARISON WITH THEORY

A. Current status of the theory of \( NN \rightarrow NN\pi \)

The advent of new data due to the three technical advances mentioned in Sec. I was answered by theoretical developments. The first measurements triggered a study of quantum number selection rules, of the role of the final-state interaction, and of nucleon excited states, and led to a theory of pion production in analogy with quantum electrodynamics. The availability of kinematically complete cross section data led to the application of effective chiral Lagrangians, of soft pion techniques, and models with coupled channels, and the recent precise cross section data close to threshold obtained at storage rings stimulated the construction of meson exchange models, and a study of the short-range part of the \( NN \) interaction as well as the role of chiral symmetry in the interpretation of pion production. A review of the development of the theory of \( \pi NN \) systems, prior to 1990 is given in Ref. [24].

We now recognize the fact that the reaction \( pp \rightarrow pp\pi^0 \) near threshold is sensitive to short-range exchange mechanisms in the two-nucleon system, because the main pion exchange term is prohibited by isospin conservation. Soon after the first accurate total cross section measurement with an electron-cooled beam [5], it was realized [25,26] that pion production on a single nucleon underestimates the empirical cross section by about a factor of 5. Lee and Riska proposed [27] that this shortfall of the theoretical cross section might be explained by the omission of pair diagrams with an exchanged heavy meson (\( \sigma, \omega \)). This was confirmed quantitatively [28]. Subsequently, the role of residual, virtual pion exchange was found to be not necessarily small [29,30]. However, at this time the role of pion rescattering is still controversial, especially since field theoretical models and chiral perturbation theory [31,32] disagree on the sign of the pion exchange amplitude. On the other hand, the importance of heavy-meson exchange also has been questioned [33]. Additional short-range mechanisms have been studied as well, including transition couplings between different exchanged mesons [34] and the role of the \( \Delta(1232) \) isobar [26,32,30] and the \( S_{11} \) and \( D_{13} \) nucleon resonances [35]. An interpretation of the reaction on the basis of approximately conserved chiral symmetry [36,37,31,32] has, so far, not been able to reproduce the cross section close to threshold. Fully relativistic calculations have been carried out in a covariant one-boson exchange model with parameters fitted to the amplitudes of elastic \( NN \) scattering [38,39].

B. Theory and polarization observables

The impressive theoretical effort during the past decade that is summarized in the preceding section has been mostly devoted to a study of the lowest partial wave. Since, as we have seen, the energy dependence of that partial wave is well described by “trivial” factors, this means that, so far, only its strength, i.e., a single experimental number, has been confronted with theory. Some of the models mentioned in the preceding section naturally include higher partial waves and thus would be able to predict polarization observables. How-
ever, at this time, such calculations have only been carried out by groups at Osaka [40] and at Jülich [30,41,42].

Pion production in the Jülich model [43] includes direct production, s- and p-wave pion rescattering, an intermediate \( \Delta (1232) \) nucleon excited state, and a contribution from pair diagrams. The latter carries an adjustable parameter; it is taken to represent those short-range mechanisms that are not explicitly included in the model. Final-state angular momenta up to 2 are included. The prediction of the Jülich model for some of the observables presented in this paper is shown as a dotted line in Figs. 5–8. It is fair to say that there is little similarity between theoretical estimates and the data. We hope that the theoretical community views this disagreement as a challenge.

Finally, we point out that the experimental information now available offers the possibility to discuss individual reaction amplitudes, and that a comparison with theory should take place on this level. Such a study is currently in progress.

VI. SUMMARY AND CONCLUSIONS

We have studied the reaction \( pp \rightarrow pp \pi^0 \), kinematically complete, with a polarized beam and a polarized target. The experiment relies on the advantages offered by the use of an internal target in a storage ring. The experiment has been carried out at four bombarding energies between 325 and 400 MeV. In this energy range the Ss partial wave ceases to be dominant, and higher partial waves become important (see Fig. 14).

Throughout the present energy region, the number of significant partial amplitudes is still small (at most 12). Under these conditions, it is feasible to expand the observables into a complete set of angular functions. The expansion coefficients are determined from the data. This results in a parametrization of the findings of this experiment and allows one to calculate any analyzing power or correlation coefficient for any configuration of the three-body final state. We include as an appendix the necessary framework to discuss polarization observables in a reaction with polarized spin-1/2 collision partners and a three-body final state.

From a formal partial-wave analysis we learn that the amplitudes can be arranged into the two groups \((Ss, Sd, Ss)\) and \((Ps, Pp)\), and only amplitudes within one group can interfere with each other. We also see that in the coefficients of the angular distributions, terms that represent the interference between \((SsSd)\) and \((SdSs)\) amplitudes, always occur in a sum with a term that contains only \(Pp\) waves. These sums then become a single parameter in the analysis. Thus the contribution from \(Sd\) and \(Ds\) partial waves cannot be deduced from the angular distribution and must rely on a study of the energy dependence. However, we find no evidence that terms that contain \(Sd\) and \(Ds\) partial waves depart in their energy dependence from what is expected for the competing \(Pp\) wave alone.

The formalism presented in this paper shows that it is possible to calculate the observables from the partial-wave amplitudes directly. Embedding this calculation into a fitting procedure would allow one to discuss the constraints on individual amplitudes that follow from the present measurement. Such an amplitude analysis is currently in progress.

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APPENDIX: PARTIAL-WAVE FORMALISM

1. Expansion of the reaction amplitude

We present here the details of the partial-wave formalism which was employed to determine the form of the angular distributions of the cross section and polarization observables, Eqs. (11). The main difficulty for reactions such as \( pp \rightarrow pp \pi^0 \) is to understand how a partial-wave expansion can be carried out for situations in which the final state has three particles.

We work in the c.m. frame and adopt coordinates \( \mathbf{r} \) and \( \mathbf{p} \) conjugate to the momenta \( \mathbf{p} \) and \( \mathbf{q} \) of Fig. 2. The symbol \( \Psi \) represents the full wave function of the system that evolves from the \( pp \) initial state, and we wish to focus on the components of \( \Psi \) which correspond to some three-body channel \( \beta \). We know from Ref. [44] that for reactions leading to three-body final states, the outgoing wave in the asymptotic region is of the form

\[
\Psi_\beta(\mathbf{r}, \mathbf{p}) \rightarrow \frac{e^{i \mathbf{r} \cdot \mathbf{p}}}{R_\beta} f_\beta(\mathbf{p}, \mathbf{q}; k_i),
\]

where \( k_i \) is the initial momentum. The quantities \( \xi \) and \( R_\beta \) are given by

\[
\xi^2 = 2 \sqrt{\mu_1 \mu_2 E_\beta / \hbar^2},
\]

and

\[
R_\beta = (\mu_1 r^2 + \mu_2 p^2) / \sqrt{\mu_1 \mu_2},
\]

where \( \mu_1 \) and \( \mu_2 \) are the reduced masses associated with the coordinates \( \mathbf{r} \) and \( \mathbf{p} \), and \( E_\beta \) is the available kinetic energy in the final state.

If the particles have spin, we may construct a wave function with spin projections \( \sigma_a \) and \( \sigma_b \) for the two particles in the initial state, and the full wave function \( \Psi \) that evolves from this initial state will contain outgoing waves with various final-state spin projections \( \sigma_1, \sigma_2, \) and \( \sigma_3 \). It follows that the reaction amplitudes \( f_\beta \) must carry all five spin labels. Isospin projection quantum numbers may be incorporated in a similar way.
Formal expressions for the reaction amplitudes can be obtained by employing a three-body Green’s function [45] in conjunction with a Lippmann-Schwinger-like equation (see Ref. [44]). The result for the asymptotic wave function in channel $\beta$ is

$$\Psi_\beta(r, p) = \int \frac{2}{\pi \xi} e^{-i\hat{R}_\beta} \frac{E_{p\mu_1} \mu_2}{R^5} \psi_f |V_\beta| \Psi, \quad \text{(A4)}$$

where $V_\beta$ is some kind of interaction potential and

$$\psi_f = e^{ikr} r^\nu \rho \phi_1 \phi_2 \phi_3. \quad \text{(A5)}$$

In the last formula the $\phi_i$'s are the internal wave functions of the particles in the final state. For $pp \rightarrow pp \pi^0$ these are just spin and isospin wave functions. The matrix element in Eq. (A4) implies integration over all coordinates of the problem, and the actual dependence of $\Psi_\beta$ on $r$ and $p$ is contained in the $e^{-i\hat{R}_\beta} R^{5/2}$ factor. The formula for the reaction amplitude can simply be read off from Eq. (A4) with the help of Eq. (A1).

To obtain a partial-wave expansion of $f_\beta$ we need to expand both $\Psi$ and the outgoing plane waves in terms of angular momentum eigenfunctions. One begins by dividing $\Psi$ into two parts,

$$\Psi = \psi_i + \Phi, \quad \text{(A6)}$$

where $\psi_i$ is the unscattered incident plane wave and $\Phi$ is everything else. For $\psi_i$ we write

$$\psi_i = \chi^a_a \chi^b_b \eta^a_a \eta^b_b e^{ikr}, \quad \text{(A7)}$$

where the $\chi$'s and $\eta$'s are spin and isospin wave functions, respectively.

For the angular momentum expansion we choose basis states that are simultaneous eigenfunctions of the initial total spin $s_{J}$, orbital angular momentum $l$, total angular momentum $J$, and total isospin $t$, with the coupling orders $[(s_{a}, s_{b})s_{J}, l]J$ and $(t_{a}, t_{b})t$. We use the symbol $\nu$ to denote initial state quantum numbers $J$, $l$, $s_{J}$, and $t$. Then, by employing standard angular momentum identities (see, for example, Ref. [46]) we obtain

$$\psi_i = 4 \pi \sum_{\nu} \sum_{M, \lambda, \sigma_{t}, \tau} \langle s_{a} \sigma_{a}, s_{b} \sigma_{b} | s_{J} \sigma_{I} \rangle \langle s_{l} \sigma_{l}, l \lambda | JM \rangle \times \langle t_{a} \tau_{a}, t_{b} \tau_{b} | t \tau \rangle j_{l}(k \rho), \quad \text{(A8)}$$

where $\nu$ is the angular momentum/isospin function:

$$\nu_{\nu}^{M, \lambda} = \sum_{\sigma_{a}, \sigma_{b}, \sigma_{t}, \lambda, \tau} \langle s_{a} \sigma_{a}, s_{b} \sigma_{b} | s_{J} \sigma_{I} \rangle \langle s_{l} \sigma_{l}, l \lambda | JM \rangle \times \langle t_{a} \tau_{a}, t_{b} \tau_{b} | t \tau \rangle j_{l}(k \rho). \quad \text{(A9)}$$

One can easily argue that the full wave function $\Psi$ must have the same basic angular momentum structure as $\psi_i$. To see this we write the Bessel function $j_{l}$ in terms of spherical Hankel functions so that $\psi_i$ becomes a sum of ingoing and outgoing spherical waves, each having well-defined quantum numbers. For example, the ingoing wave in a given angular momentum channel will have the asymptotic form

$$X_{\nu}^{(in)} = \left( \frac{1}{2ik \rho} \right) e^{-i(k \rho - \lambda \pi/2)} \nu_{\nu}^{M, \lambda}. \quad \text{(A10)}$$

We then assume that whatever interactions are present conserve total angular momentum and total isospin. These interactions affect the outgoing waves but do not alter the ingoing wave, and so it follows that the full wave function will be of the form

$$\Psi = 4 \pi \sum_{\nu} \sum_{M, \lambda, \sigma, \tau} \langle s_{a} \sigma_{a}, s_{b} \sigma_{b} | s_{J} \sigma_{I} \rangle \langle s_{l} \sigma_{l}, l \lambda | JM \rangle \times \langle t_{a} \tau_{a}, t_{b} \tau_{b} | t \tau \rangle \Phi_{\nu}^{M, \lambda} \eta_{\nu}^{M, \lambda}(\hat{k}), \quad \text{(A11)}$$

where $\Phi_{\nu}^{M, \lambda}$ is the wave function that evolves from $\nu_{\nu}^{(in)}$. Although the exact form of $\Phi_{\nu}$ may not be known, by our assumptions it must be an eigenfunction of $J$, $M$, $t$, and $\tau$. The formula in Eq. (A11) is our working equation for the expansion of $\Psi$. The three-body final state wave function given in Eq. (A5) must also be expanded in terms of angular momentum eigenfunctions. For now we keep the discussion general and allow all three particles to have nonzero spin. Symbolically, the coupling order we adopt is $[(s_{a}, s_{b})s_{J}, l]J$ for the angular momenta and $[(t_{a}, t_{b})t_{f}, t_{J}]t_{f}$ for the isospins. The corresponding angular momentum/isospin functions are

$$\nu_{\nu}^{M, \lambda} = \sum_{\sigma_{a}, \sigma_{b}, \sigma_{t}, \lambda_{p}, \lambda_{q}} \langle s_{a} \sigma_{a}, s_{b} \sigma_{b} | s_{J} \sigma_{I} \rangle \langle s_{l} \sigma_{l}, l \lambda | JM \rangle \times \langle t_{a} \tau_{a}, t_{b} \tau_{b} | t \tau \rangle j_{l}(k \rho) y_{\nu}^{M, \lambda}(\hat{p}) \nu_{\nu}^{M, \lambda}(\hat{q}) X_{\nu}^{M, \lambda} X_{\nu}^{M, \lambda} \eta_{\nu}^{M, \lambda} \eta_{\nu}^{M, \lambda} \eta_{\nu}^{M, \lambda}, \quad \text{(A12)}$$

where in this context $\beta$ is shorthand for the final-state quantum numbers $l_{p}$, $l_{q}$, $j_{p}$, $j_{q}$, $J_{s}$, $s_{f}$, $t_{f}$, and $t_{f}$. The expansion of $\psi_f$ in terms of the $\nu$ functions is
\[ \psi_f = (4\pi)^2 \sum_{\beta} \sum_{M', \sigma_f, \tau_f, \lambda_q} \langle s_1 \sigma_1, s_2 \sigma_2 | s_f \sigma_f, l_p \lambda_p | m \rangle \langle s_3 \sigma_3, l_q \lambda_q | j' m' \rangle \langle j m, j' m' | \lambda' \rangle (t_1 \tau_1, t_2 \tau_2 | t_f \tau_f) \]
\[ \times (t_f \tau_f, t_5 \tau_5 | t' \tau') j_f (p r) j_s (q \rho) Y_{p_q}^{\lambda_p} (\hat{p}) Y_{q_q}^{\lambda_q} (\hat{q}) Y_{\lambda}^{\lambda'}. \]

We may now obtain the partial wave expansion of \( f_\beta \) by substituting Eqs. (A11) and (A13) into Eq. (A4). The result is

\[ f_{\sigma_a, \sigma_b}^{\sigma_1, \sigma_2, \sigma_3} = \int \frac{2}{\sqrt{\xi}} \frac{E_\beta \mu_1 \mu_2}{\hbar^4} \sum_{s_1, \sigma_1, \sigma_2, \tau_f, \lambda_q} \cdot \langle s_1 \sigma_1, s_2 \sigma_2 | s_f \sigma_f, l_p \lambda_p | m \rangle \langle s_3 \sigma_3, s_3 \sigma_3 | j m, j' m' \rangle \langle j m, j' m' | \lambda' \rangle (t_1 \tau_1, t_2 \tau_2 | t_f \tau_f) \]
\[ \times (t_f \tau_f, t_5 \tau_5 | t' \tau') j_f (p r) j_s (q \rho) Y_{p_q}^{\lambda_p} (\hat{p}) Y_{q_q}^{\lambda_q} (\hat{q}). \]

At this point we can simplify the result by assuming that the interaction potential \( V_p \) is a rotational scalar in both ordinary and isospin space. It follows that the matrix elements are nonzero only for \( \{ J, M, \tau, \lambda \} \rightarrow \{ J', M', t', \lambda' \} \). Furthermore, we know from the Wigner-Eckhart theorem that, for a given set of quantum numbers \( \nu \) and \( \beta \), the matrix elements are independent of both \( M \) and \( \tau \). With this in mind we adopt the shorthand notation

\[ U_a (\epsilon) = \sqrt{2J + 1} \int (j_f (p r) j_s (q \rho) Y_{p_q}^{\lambda_p} (\hat{p}) Y_{q_q}^{\lambda_q} (\hat{q}), \]

where, as in Eq. (10), \( a \) is shorthand for the full set of initial- and final-state quantum numbers. We see from Eq. (A15) that the matrix element \( U_a \) depends explicitly on the momentum parameters \( p \) and \( q \). These parameters are constrained by the requirement that the total kinetic energy in the final state must be \( E_\beta \), and therefore \( U \) is effectively a function of the energy sharing parameter \( \epsilon \).

To obtain our final expression for the reaction amplitude we adopt the coordinate frame of Fig. 1, in which the \( z \) axis is along \( k_i \). The result is

\[ f_{\sigma_a, \sigma_b}^{\sigma_1, \sigma_2, \sigma_3} = \frac{8i \sqrt{2}}{\sqrt{\xi}} \frac{E_\beta \mu_1 \mu_2}{\hbar^4} \sum_{s_1, \sigma_1, \sigma_2, \tau_f, \lambda_q} \cdot \langle s_1 \sigma_1, s_2 \sigma_2 | s_f \sigma_f, l_p \lambda_p | m \rangle \langle s_3 \sigma_3, s_3 \sigma_3 | j m, j' m' \rangle \langle j m, j' m' | \lambda' \rangle (t_1 \tau_1, t_2 \tau_2 | t_f \tau_f) \]
\[ \times (t_f \tau_f, t_5 \tau_5 | t' \tau') j_f (p r) j_s (q \rho) Y_{p_q}^{\lambda_p} (\hat{p}) Y_{q_q}^{\lambda_q} (\hat{q}). \]

Equation (A16) simplifies considerably if we specialize for \( pp \rightarrow pp \sigma^0 \). In this case the isospin Clebsch-Gordan coefficients become constant numerical factors. In addition \( s_3 \) is zero and \( l_q = j_q \). The result is

\[ f_{\sigma_a, \sigma_b}^{\sigma_1, \sigma_2, \sigma_3} = \frac{8i \sqrt{2}}{\sqrt{\xi}} \frac{E_\beta \mu_1 \mu_2}{\hbar^4} \sum_{s_1, \sigma_1, \sigma_2, \tau_f, \lambda_q} \cdot \langle s_1 \sigma_1, s_2 \sigma_2 | s_f \sigma_f, l_p \lambda_p | m \rangle \langle s_3 \sigma_3, s_3 \sigma_3 | j m, j' m' \rangle \langle j m, j' m' | \lambda' \rangle (t_1 \tau_1, t_2 \tau_2 | t_f \tau_f) \]
\[ \times (t_f \tau_f, t_5 \tau_5 | t' \tau') U_a (\epsilon) Y_{p_q}^{\lambda_p} (\hat{p}) Y_{q_q}^{\lambda_q} (\hat{q}). \]

2. Cross section and polarization observables

In most respects, the procedure for obtaining the observables from the reaction amplitude is the same as for reactions with two-body final states. In particular one can show that the fivefold differential cross section for a three-body final state is proportional to \( f_\beta^2 \) (averaged over initial spin states and summed over final spin states), where the proportionality constant involves only kinematic factors. For our purposes it is useful to introduce a “reaction matrix” \( M \) directly proportional to \( f \), with normalization chosen in such a way that the spin-dependent partial cross section \( \Delta \sigma \) for reactions leading from initial state \( \sigma_a, \sigma_b \) to final state \( \sigma_1, \sigma_2 \), with \( \hat{p} \) and \( \hat{q} \) in the intervals \( \Delta \Omega_p \) and \( \Delta \Omega_q \), and with the energy-sharing parameter \( \epsilon \) in the interval \( \Delta \epsilon \) is given by

\[ \sigma \text{ vs. } \epsilon \text{ for } pp \text{ reaction} \]
\[
\Delta \sigma = |M_{\sigma_1 \sigma_2}^\sigma|^2 2 \Delta \Omega_p \Delta \Omega_q. \quad (A18)
\]

For the case in which \( \epsilon \) is taken to be the pp relative kinetic energy [as in Eq. (21)] the result for \( M \) is

\[
M_{\sigma_a \sigma_b}^{\sigma_1 \sigma_2} = 8i \left( \frac{\mu_1 \mu_2 \hbar^2}{v_J} \right)^{\frac{1}{2}} \sum_{\lambda_p, \sigma_f, \lambda_q} \left( \frac{2J + 1}{2J + 1} \right)^{\frac{1}{2}} \notag
\times (s_a \sigma_a, s_f \sigma_f | s_b \sigma_b, 10 | JM) \notag
\times (s_1 \sigma_1, s_2 \sigma_2 | s_f \sigma_f, 1_p \lambda_p | jm) \notag
\times (jm, 1_p \lambda_q | JM) U_{\sigma}^\lambda_m(\vec{p}) Y_{1_p}^{\lambda_q}(\vec{q}), \quad (A19)
\]

where \( v_J \) is the relative velocity in the initial state.

The differential cross section and polarization observables may now be obtained directly from the reaction matrix \( M \). In general, the observables \( O \) are found by taking the trace of a matrix product, i.e.,

\[
O = \text{Tr}[M M^\dagger], \quad (A20)
\]

where \( T \) is the appropriate operator. To obtain the unpolarized cross section, the partial cross sections of Eq. (A18) are to be summed over final states and averaged over initial states with the result

\[
\sigma_0 = \frac{1}{(2s_a + 1)(2s_b + 1)} \text{Tr}[M M^\dagger]. \quad (A21)
\]

The polarization observables are obtained by using the appropriate spin operators for \( T \) in Eq. (A20). For the analyzing powers the operators we want are the Pauli matrices, and the result is

\[
\sigma_0 A_{10} = \frac{1}{(2s_a + 1)(2s_b + 1)} \text{Tr}[M \sigma_0 M^\dagger], \quad (A22)
\]

where the subscript \( i \) can be \( x, y, \) or \( z \). In a similar way, the spin correlation parameters are obtained by using for \( T \) the direct product of the Pauli matrices for beam and target particles:

\[
\sigma_0 A_{ij} = \frac{1}{(2s_a + 1)(2s_b + 1)} \text{Tr}[M \sigma_i^{(b)} \otimes \sigma_j^{(t)} M^\dagger]. \quad (A23)
\]

Obtaining the partial-wave expansions is simplified considerably if one introduces spherical tensor spin operators to use in place of the Cartesian spin operators that appear in Eqs. (A22) and (A23). The new operators transform under rotations like the spherical harmonics and are defined, for each particle, by the equations

\[
\tau_{00} = I, \notag \\
\tau_{10} = \sigma_z, \quad (A24)
\]

where \( I \) is the 2\times2 unit matrix. Associated with these operators, there is corresponding set of ‘‘spherical tensor’’ polarization observables [47]

\[
T_{k_1 q_1 k_2 q_2} = \frac{1}{(2s_a + 1)(2s_b + 1)} \text{Tr}[M \tau_{k_1 q_1}^{(b)} \otimes \tau_{k_2 q_2}^{(t)} M^\dagger], \quad (A25)
\]

From the definitions given above, it is straightforward to find simple relationships between the Cartesian analyzing powers and spin correlation coefficients and the spherical tensor observables. The relevant formulas are

\[
\sigma_0 = T_{00,00}, \notag \\
\sigma_0 A_{10} = - \sqrt{2} \text{Im}[T_{11,00}], \notag \\
\sigma_0 A_{z0} = T_{10,00}, \notag \\
\sigma_0 A_{zz} = T_{10,10}, \notag \\
\sigma_0 A_x = - 2 \text{Re}[T_{11,1}], \notag \\
\sigma_0 A_{z} = 2 \text{Re}[T_{11,1}], \notag \\
\sigma_0 A_{zz} = - 2 \text{Im}[T_{11,1}]. \quad (A26)
\]

The introduction of the spherical tensor spin operators leads to a compact, general formula for the partial-wave expansion of the observables. The simplification comes from the fact that the spin operators of Eq. (A24) can be represented in angular momentum language:

\[
\langle \sigma | \tau_{k q} | \sigma' \rangle = (-1)^{s - s'} \sqrt{2s + 1} \langle s \sigma, s - \sigma' | k q \rangle. \quad (A27)
\]

To obtain the partial-wave expansion formula we now substitute this expression, along with Eq. (A19) for \( M \), into Eq. (A25). The angular dependence of the observables is expressed as an expansion in terms of bipolar harmonics:

\[
B_{L_p, L_q;}^A ;\ell; \ell'(p, q) = \sum_{\Lambda_p, \Lambda_q} \langle L_p \Lambda_p, L_q \Lambda_q ; L Q \rangle Y_{L_p}^{\Lambda_p}(\hat{p}) Y_{L_q}^{\Lambda_q}(\hat{q}). \quad (A28)
\]

After carrying out an angular momentum reduction that eliminates the sums over the magnetic quantum numbers we obtain the result

\[
T_{k_1 q_1 k_2 q_2} = \frac{1}{(2s_a + 1)(2s_b + 1)} \left( \frac{16 \mu_1 \mu_2 p q}{v_i \pi \hbar^4} \right) \notag \\
\times \sum_{L_p, L_q, \ell, \ell'} \sum_{a, a'} \left( C_{L_p, L_q, \ell}^{a, a'; \ell} \right) U_{a}(e) U_{a'}^{*}(e) \notag \\
\times B_{L_p, L_q;}^Q ;\ell; \ell'(p, q), \quad (A29)
\]

\[064002-22\]
where the label $\kappa$ is shorthand for the indices $k_1,q_1,k_2,q_2$ and where $Q = q_1 + q_2$.

Equation (A29) represents our central result for the partial-wave expansion of the cross section and polarization observables. Each observable has a set of allowed angular dependences, $B_{L_p,l_q;\hat{p},\hat{q}}$, and the factor inside the square brackets gives the expansion coefficient. Each of these coefficients is a sum of terms involving an angular momentum coupling coefficient $C$ and a bilinear product of matrix elements $U_{\alpha}$. The selection rules that determine which partial-wave combinations contribute to a given angular function are contained in the $C$ coefficients.

The angular momentum coefficients are given by the following expression:

\[
C_{L_p,l_q;\hat{p},\hat{q}}^{s_a,s_b;\kappa} = (-)^{s_a+s_b+|l_q|} \sum_{l_i} \left[ (2s_a+1)(2s_b+1)(2k_1+1)(2k_2+1)(2K+1) \times (2l_1+1)(2l_1'+1)(2l_q+1)(2l_q'+1)(2j+1)(2j'+1)(2J+1) \right]^{1/2} \langle 0,l_1|l_0 \rangle \langle l_0,k_1,k_2|0 \rangle \times \langle 0,KQ|LQ \rangle \times \langle l_1,l_1'|0 \rangle \langle l_q,l_q'|0 \rangle \langle k_1,k_2|KQ \rangle W(j,s_j,L_p,l_q,l_1,l_1',j,s_j,J,L) \times \left\{ s_i^j,J,J'; \right\}
\]

This equation differs from the analogous formula given in Ref. [9] in two respects. First of all the Clebsch-Gordan coefficient $\langle 0,l_1|l_0 \rangle$ was inadvertently omitted in Ref. [9]. Second, we have changed the coupling order for the angular momenta in the initial state [see Eqs. (A8) and (A9)] and this results in additional phase factors in $C$.

Although the expression given in Eq. (A30) is fairly complex, the coefficients are easily evaluated since computer codes for calculating the Clebsch-Gordan, Racah, and 9j symbols are readily available.

The expansion formulas given in Eq. (A30) are obtained most readily by substituting Eq. (A28) into Eq. (A29) to obtain

\[
T_{k_1,q_1,k_2,q_2} = \frac{1}{(2s_a+1)(2s_b+1)} \left[ 16 \mu_1 \mu_2 p q \right] \times \sum_{l_p,l_q;\hat{p},\hat{q}} \left( \sum_{a,a'} X_{L_p,l_q;\hat{p},\hat{q};\mu}^{a,a';\kappa} U_{\alpha}(\hat{p}) U_{\alpha'}^{*}(\hat{q}) \right) \times Y_{L_p}^{\mu}(\hat{p}) Y_{L_q}^{\mu*}(\hat{q});
\]

where the coefficients $X$ are given by

\[
X_{L_p,l_q;\hat{p},\hat{q};\mu}^{a,a';\kappa} = \sum_{L} \langle L_p\mu_L,q_q\mu_L|LQ \rangle C_{L_p,l_q;\mu}^{a,a';\kappa}.
\]

Equations (11) are then obtained by using Eq. (A31) in conjunction with Eqs. (A30) and (A32) assuming that only the partial waves of Table I contribute and that terms quadratic in $\delta_{\hat{p}}$ or $\delta_{\hat{q}}$ are negligible. In general, one finds that only a few distinct angular functions are allowed for each observable. The constraints, which arise from conservation laws and the antisymmetrization requirements, can be seen by inspecting Eq. (A30).

The first constraint comes from the $\delta_{s_j,s_j'}$ factor. For $pp$ → $pp'p''$, $s_f$ is the $pp$ total spin quantum number. Since we have only antisymmetric $pp$ states, the conclusion is that there will be no interference between even $l_p$ and odd $l_p$ partial waves.

The next constraint is on the allowed values of $L_p$. This constraint comes from the Clebsch-Gordan coefficients $\langle l_1,l_1'|0 \rangle\langle l_q,l_q'|0 \rangle$ which requires that $l_p + l_q$ be even. There are analogous constraints on $L_q$. Thus, for example, interference between $PS$ and $PP$ may give rise to angular distributions with $L_p=0$ and 2 and with $L_q=1$. For the conditions we assume, the angular distributions involve no spherical harmonics of degree greater than $L=2$.

One can easily demonstrate from Eq. (A30) that $X$ coefficients are either symmetric or antisymmetric under the interchange of $\alpha$ and $\alpha'$:

\[
X_{L_p,l_q;\mu}^{a,a';\kappa} = (-)^{k_1+k_2} X_{L_p,l_q;\mu}^{a',a;\kappa}.
\]

This means that the unpolarized cross section and the spin correlation parameters depend only on $\text{Re}[U_{\alpha}U_{\alpha'}^{*}]$ whereas the analyzing powers depend only on $\text{Im}[U_{\alpha}U_{\alpha'}^{*}]$. One consequence is that the factor inside the square brackets in Eq. (A29) is either purely real or purely imaginary. From this it follows that a given observable will depend either on $\text{Re}[Y_{L_p}^{\mu}(\hat{p}) Y_{L_q}^{\mu*}(\hat{q})]$ or on $\text{Im}[Y_{L_p}^{\mu}(\hat{p}) Y_{L_q}^{\mu*}(\hat{q})]$, and as a
result the \( \phi \) dependences of the allowed angular distributions are relatively simple. In particular we see that \( \sigma_0 A_{20} \) and \( \sigma_0 A_2 \) (both of which have \( Q=0 \)) go as \( \sin[\mu(\phi_p-\phi_q)] \), while the remaining observables go as \( \cos[\mu(\phi_p-\phi_q)] + Q \phi_q \).

The formalism outlined in this appendix leads to a number of additional useful results that are described in the main text and in other publications.


[45] It appears that the Green’s function given in Eq. (1.38) of Ref. [44] contains a typographical error. The dependence on the masses should be \( M_1 M_2 \) rather than \( M_1 M_2^{1/2} \). See E. Gerjuoy, Ann. Phys. (N.Y.) **5**, 58 (1958).


Observation of a large longitudinal analyzing power in a nuclear reaction

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Abstract

We have measured the longitudinal analyzing power \( A_L \) of the \( pp \rightarrow pp\pi^0 \) reaction at 375 MeV bombarding energy. We find that for certain angle combinations of the outgoing particles the observed \( A_L \) is as large as 0.3, demonstrating that sizeable longitudinal analyzing powers in reactions with multi-particle final states are possible. This result has implications for \( pp \) parity violation experiments above the pion threshold. The observed \( A_L \) is dominated by the interference between \( s \) and \( p \) wave pions in conjunction with nucleon-nucleon \( P \) waves in the final state. © 2000 Elsevier Science B.V. All rights reserved.

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Experience shows that measurements of polarization observables are important in studies of reaction mechanisms in nuclear and particle physics. In the simplest of such experiments one measures the analyzing power \( A_y \) by observing the change in the reaction cross section as one switches between spin up and spin down with a beam polarized perpendicular to the reaction plane.

Analogous observables obtained with the beam polarized either along the beam direction (\( A_z \)) or perpendicular to the beam direction with the polarization vector lying in the reaction plane (\( A_x \)), are not ordinarily measured since, if parity is conserved,
these analyzing powers are assumed to be identically zero. In fact, a measurement of the longitudinal analyzing power $A_z$ in, for example, p-p elastic scattering provides a means for detecting parity violation in nuclear interactions (see for example Ref. [1]). Observed values for the parity-violating longitudinal analyzing power are of the order of $A_z \approx 10^{-7}$, i.e., very small.

It is often not understood that the $A_z = 0$ parity constraint does not apply to reactions in which the final state contains more than two particles and at least two of them are detected in non-coplanar kinematics. In this case $A_z$ may differ from zero. Since the allowed angular patterns for this observable are rather complex, one might expect that measurements of $A_z$ would provide a critical test of any reaction model.

In a previous attempt to measure $A_z$ for a three-body final state reaction in non-coplanar kinematics [2], the reaction $^3$H(p,pp)n was studied with a longitudinally polarized 9 MeV proton beam. The reported $A_z$ is consistent with zero at the level of 0.003. In this Letter, we present new results for $A_z$ for the reaction $pp \rightarrow pp\pi^0$ at $T_p = 375$ MeV, and demonstrate that for this case $A_z$ is large.

The longitudinal analyzing power $A_z$ is measured by observing the reaction yields $Y_+$ and $Y_-$ with beam polarization parallel or anti-parallel to the beam momentum (z-axis),

$$A_z = \frac{1}{P_z} \frac{Y_+ - Y_-}{Y_+ + Y_-}, \quad (1)$$

where $P_z$ is the beam polarization. The yield from a reaction with more than two particles in the final state depends on the experimental arrangement. Consider, for instance, an experiment where two particles are detected in coincidence. We denote the direction of the two particles by $(\theta_1, \varphi_1)$ and $(\theta_2, \varphi_2)$ where $\theta$ is the polar angle with respect to the z-axis and the azimuth $\varphi$ is measured relative to the $x$-axis.

The $x$-axis we fix so that it points horizontally to the left of the beam and the $y$-axis, pointing up, completes the right-handed Cartesian coordinate frame.

The invariance of physical laws under spatial inversion is treated formally by the parity operation which reverses polar vectors (coordinates, momenta) but does not affect axial vectors (spins, cross products of polar vectors). As shown in Ref. [3], as a consequence of parity conservation, polarization observables in nuclear reactions either remain the same or change sign when the final state is reflected on the $x$–$z$-plane. In particular, for the longitudinal analyzing power, parity conservation requires that

$$A_z(\varphi_1, \varphi_2) = -A_z(-\varphi_1, -\varphi_2). \quad (2)$$

Since the initial state is invariant against a rotation around the $z$-axis, we can set $\varphi_1 = 0$ without loss of generality. It is then easy to see that for coplanar detected particles (when $\varphi_2 = 0$ or $\pi$) Eq. (2) requires that $A_z = 0$. It is also clear that $A_z$ vanishes if only one particle is detected.

In the present experiment we studied the reaction $pp \rightarrow pp\pi^0$ at 375 MeV bombarding energy. The measurement yields the momenta of both outgoing protons, $b_1$ and $b_2$, over most of the available phase space. From the center-of-mass values of $b_1$ and $b_2$ we deduce the kinetic energy $\epsilon$ of the two protons in their rest frame, and the canonical momenta $p = \frac{2}{\sqrt{\epsilon}}(b_1 - b_2)$ (the relative momentum between the two protons) and $q = -b_1 - b_2$ (the pion momentum), with the corresponding angles $\hat{\rho} = (\theta_\rho, \varphi_\rho)$ and $\hat{\varphi} = (\theta_\varphi, \varphi_\varphi)$. The identity of two of the outgoing particles restricts the range of $\theta_\rho$ since we can always number the two protons such that $\theta_\rho$ is between $0^\circ$ and $90^\circ$. Furthermore, the rotational symmetry around the $z$-axis has the effect that only one azimuthal angle, $\Delta \varphi = \varphi_\rho - \varphi_\varphi$, is relevant. Thus, the kinematics of the final state is described by four numbers, $\theta_\rho, \theta_\varphi, \Delta \varphi, \epsilon$, all of which are known for each event. In the following we ignore the energy parameter $\epsilon$ which means that the reaction amplitudes are integrated over $\epsilon$. Their $\epsilon$ dependence is well described by phase space, an angular momentum factor and, if the outgoing nucleons are in an S-state, by the final-state interaction between them [4].

In Ref. [5] it has been shown how polarization observables in reactions with a three-body final state can be expanded into partial waves of given angular momentum quantum numbers. The angular momentum in the final state shall be denoted by $l_\rho/l_\varphi$ for the pp system and $l_\pi$ for the pion. Using spectroscopic notation, we label the final state accordingly as $(l_\rho/l_\varphi) = (Ss), (Ps), (Pp)$. For beam energies below
400 MeV, these are all the angular momentum states that contribute measurably to $A_\mathrm{c}$ ([S]p is forbidden for this reaction, and the partial waves [D]s and [D]d can only be significant when interfering with the [S]s wave, but such terms do not contribute to $A_\mathrm{c}$). Because there are only a few angular momentum states, a partial-wave expansion yields a simple expression for $A_\mathrm{c}$ in terms of the four kinematics variables, $\theta_p, \theta_q, \Delta \varphi$, and $\varepsilon$,

$$\sigma (\theta_p, \theta_q, \Delta \varphi, \varepsilon) \cdot A_{A_\mathrm{c}}(\theta_p, \theta_q, \Delta \varphi, \varepsilon) = \pm B_{A_\mathrm{c}}^{(1)}(\varepsilon) \sin \theta_p \cos \theta_q \sin \theta_q \sin \Delta \varphi$$

$$+ B_{A_\mathrm{c}}^{(2)}(\varepsilon) \sin \theta_p \cos \theta_q \sin \theta_q \cos \theta_q \sin \Delta \varphi$$

$$+ B_{A_\mathrm{c}}^{(3)}(\varepsilon) \sin^2 \theta_p \sin^2 \theta_q \sin 2 \Delta \varphi .$$

Here, $\sigma$ is the differential cross section that would be observed with unpolarized collision partners. The $\pm$ and $-$ sign in the first term refers to the analyzing power measured with a polarized beam ($A_{A_\mathrm{c}}^b$) or with a polarized target ($A_{A_\mathrm{c}}^t$), respectively. Because in our case the colliding particles are identical, the beam and target analyzing powers are related. This can be seen by rotating the experiment by 180° around the $z$-axis. This exchanges beam and target and reverses the polarization. It then follows that

$$A_{A_\mathrm{c}}^t(\theta_p, \theta_q, \Delta \varphi) = -A_{A_\mathrm{c}}^b(\pi - \theta_p, \pi - \theta_q, -\Delta \varphi)$$

which is consistent with Eq. 3.

For technical reasons, the beam polarization $P = (P_x, P_y, P_z)$ could not be oriented completely in the beam direction, in fact the longitudinal and vertical polarization components, $P_x$ and $P_y$, were about of the same magnitude and there was also a small sideways component, $P_z$. Data were acquired with the beam polarization in the $P$ direction, as well as opposite to it ($-P$). The target polarization, on the other hand, could be oriented either in the $x$-y- or $z$-direction without affecting its magnitude $Q$. Data were acquired with both signs for each of these directions, $\pm Q, \pm Q$, and $\pm Q$. The value of the product of beam and target polarization, $P_zQ = 0.265 \pm 0.004$, was deduced from elastic pp scattering, using known spin correlation coefficients [8]. Elastic pp scattering near $\theta_{lab} = 45^\circ$ was measured concurrently with pion production. Since $A_{A_\mathrm{c}}^b$ and $A_{A_\mathrm{c}}^t$ are related (see Eq. (3)), only the product $P_zQ$ affects our results, however, it might be of interest that the individual values that make $A_{A_\mathrm{c}}^b$ and $A_{A_\mathrm{c}}^t$ consistent are $P_z = 0.44$ and $Q = 0.60$. 

We note that Eq. (3) indeed satisfies the parity conservation condition of Eq. (2), and that it is invariant against a rotation of the coordinate system around the $z$-axis, as expected. Eq. (3) is also invariant against the exchange of the two observed protons which is affected by replacing $\varphi_p$ by $(\varphi_p + \pi)$, and $\theta_p$ by $(\pi - \theta_p)$.

This pp $\rightarrow$ pp$\pi^0$ measurement was carried out with the Indiana Cooler storage ring. The experimental setup is described in Ref. [6]. The directions of the two outgoing protons were observed in coincidence by a set of four wire chambers, and their energies were deduced from the light from a stack of scintillators in which they were stopped. A thin scintillator just downstream of the target provided a start signal for a time-of-flight measurement from which the particles were identified as protons. From the two measured four-vectors, the center-of-mass angles $\theta_p, \varphi_p, \theta_q$, and $\varphi_q$ were calculated, and the mass $m_3$ of the third particle was deduced. Events of interest were selected by constraining $m_3$ to a region near the pion mass peak. The background under the peak was less than 10%. The shape of the background under the peak was determined from a separate measurement with an N$_\mathrm{A}$ target and used to correct the raw data. A more detailed description of the measurement can be found in Refs. [6,7].

The experiment was carried out with a polarized proton beam on a polarized hydrogen storage-cell target. This makes it possible to measure both the beam and the target analyzing power, $A_{A_\mathrm{c}}^b$ and $A_{A_\mathrm{c}}^t$. For technical reasons, the beam polarization $P = (P_x, P_y, P_z)$ could not be oriented completely in the beam direction, in fact the longitudinal and vertical polarization components, $P_x$ and $P_y$, were about of the same magnitude and there was also a small sideways component, $P_z$. Data were acquired with the beam polarization in the $P$ direction, as well as opposite to it ($-P$). The target polarization, on the other hand, could be oriented either in the $x$-y- or $z$-direction without affecting its magnitude $Q$. Data were acquired with both signs for each of these directions, $\pm Q, \pm Q$, and $\pm Q$. The value of the product of beam and target polarization, $P_zQ = 0.265 \pm 0.004$, was deduced from elastic pp scattering, using known spin correlation coefficients [8]. Elastic pp scattering near $\theta_{lab} = 45^\circ$ was measured concurrently with pion production. Since $A_{A_\mathrm{c}}^b$ and $A_{A_\mathrm{c}}^t$ are related (see Eq. (3)), only the product $P_zQ$ affects our results, however, it might be of interest that the individual values that make $A_{A_\mathrm{c}}^b$ and $A_{A_\mathrm{c}}^t$ consistent are $P_z = 0.44$ and $Q = 0.60$. 

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Since the statistics of the present experiment do not allow a study of a four-fold differential cross section, we have ignored the energy variable $s$ and the polar angles $\theta_p, \theta_q$ keeping track only of $\Delta \varphi$ for each event. This is equivalent to integrating the observables over the energy $s$, and over $\theta_p, \theta_q$ ranging from $0^\circ$ to $90^\circ$ and $0^\circ$ to $180^\circ$. The resulting analyzing powers as a function of $\Delta \varphi$ are displayed in Fig. 1. The upper left panel shows the beam analyzing power $A_b^z$, obtained by evaluating Eq. (1) with yields measured with $+P$ and $-P$, while averaging over the target polarization. The upper right panel shows the target analyzing power $A_t^z$ obtained by averaging over the beam polarization. The bottom row shows the target analyzing powers $A_t^\perp$ and $A_t^\parallel$, measured with transverse target polarization. The latter two quantities, as a function of the rotationally symmetric $\Delta \varphi$ are expected to vanish. This observation proves that the beam analyzing power, which is measured with a mixture of transverse and longitudinal polarization components, is sensitive only to the longitudinal component, $P_z$.

The curves in the upper part of Fig. 1 represent a fit, using an expression which is derived from Eq. (3), noting that the second term in Eq. (3) vanishes when integrating over the polar angles,

$$A_i^{z/\perp}(\Delta \varphi) = \frac{\pm b_i \sin \Delta \varphi + b_{i2} \sin 2 \Delta \varphi}{1 + b_0 \cos 2 \Delta \varphi}. \tag{5}$$

The unpolarized cross section $\sigma$ depends on $\Delta \varphi$ in a way that follows from the partial-wave expansion mentioned earlier. In addition, the blind spot in the center of the detector (because of the circulating beam) causes a very similar $\Delta \varphi$ dependence of the acceptance. The denominator in Eq. (5) takes both these effects into account. The best-fit values of the three parameters with their statistical errors are $b_1 = -0.162 \pm 0.012$, $b_3 = -0.013 \pm 0.009$ and $b_0 = -0.17 \pm 0.11$. The $\chi^2$ per degree of freedom is 0.98. The resulting value for $b_0$ is relatively small compared to unity. In addition, the term with $b_0$ peaks where the numerator of Eq. (5) vanishes and thus the values of $b_1$ and $b_3$ are insensitive to a variation of $b_0$. We note that $b_3$ is much smaller that $b_1$. The same is true for the contribution by the second term in Eq. (3). This was determined by analyzing the data for pion polar angles $\theta_p, \theta_q$ in the forward and backward hemisphere separately. Thus, $A_t$ in this reaction is dominated by $b_1$, i.e., the first term (with) $B_2^{(1)}$ in Eq. (3).

In order to be able to calculate $A_t$ for any choice of $\theta_p, \theta_q$ and $\Delta \varphi$, we must relate the parameters $b_k$ of Eq. (5) to the coefficients $B_2^{(1)}$ in Eq. (3). Since the unpolarized cross section $\sigma$ is dominated by its isotropic part, we can set $\sigma \approx \sigma_{\text{iso}}/4 \pi$. The integration over $\theta_p$ and $\theta_q$ can then be carried out, and we obtain, retaining only the dominant term with $B_2^{(1)}$,

$$A_t^{z/\perp}(\theta_p, \theta_q, \Delta \varphi) \approx \pm (0.619 \pm 0.046) \times \sin \theta_p \cos \theta_q \sin \theta_q \sin \Delta \varphi. \tag{6}$$

where the largest possible value of the product of trigonometric functions equals $\frac{1}{2}$. The detector setup covers all of the phase space, except a hole, centered on the $z$-axis which subsects a cone of about $5^\circ$ opening angle. The effect of this missing part of the acceptance was estimated, using the the dependence of the observables on angle given in Eq. (5), and found to be negligible on the level of the present statistical uncertainty.
From the present experiment we draw three main conclusions:

First, up until now, one might have assumed that the inherent symmetry of the longitudinal analyzing power would constrain the reaction dynamics such that $A_z$ becomes small in general. By measuring a clearly identified and sizeable longitudinal analyzing power in the reaction $pp \to pp\pi^0$ we have shown that this assumption is not justified.

Second, the present result has obvious implications for studies of parity violation in $pp$ collisions at energies above the pion production threshold. As an example, such an experiment at 450 MeV is in progress at TRIUMF [9]. At this energy, the $pp \to pp\pi^0$ reaction contributes about 1% to the total cross section, and the $pp \to pn\pi^+$ reaction (for which there is some indication [10] that it also exhibits a large $A_z$) contributes about 6%. If in such an experiment the detection system is sensitive to more than one of the final-state particles and is not perfectly symmetric around the beam axis there is a possibility that a parity-conserving $A_z$ may contribute to the measured signal. Thus, at higher energies, measurements of parity violation must deal with a new source for systematic errors that is absent below the pion production threshold.

Third, it has been pointed out [11] that $A_z \neq 0$ requires that the final state description contains an axial vector (in our case, $\vec{p} \times \vec{q}$). This fact might make the observable $A_z$ sensitive to specific terms in the transition operator. Thus, measurements of $A_z$ constitute an important test of possible models for pion production in NN collisions. It is also noteworthy that the dominant contribution to $A_z$, arises from an interference between $|Ps\rangle$ and $|Pp\rangle$ waves, and that any terms with $|Pp\rangle$ waves alone seem to be unimportant.

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References

Measurement of spin correlation coefficients in $\vec{p}\vec{p}\rightarrow d\pi^+$


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The spin correlation coefficient combinations $A_{xx}+A_{yy}$ and $A_{xx}-A_{yy}$, the spin correlation coefficients $A_{xz}$ and $A_{zy}$, and the analyzing power were measured for $\vec{p}\vec{p}\rightarrow d\pi^+$ between center-of-mass angles $25^\circ\leq\theta\leq65^\circ$ at beam energies of 350.5, 375.0, and 400.0 MeV. The experiment was carried out with a polarized internal target and a stored, polarized beam. Nonvertical beam polarization needed for the measurement of $A_{xz}$ was obtained by the use of solenoidal spin rotators. Near threshold, only a few partial waves contribute, and pion $s$ and $p$ waves dominate with a possible small admixture of $d$ waves. Certain combinations of the observables reported here are a direct measure of these $d$ waves. The $d$-wave contributions are found to be negligible even at 400.0 MeV.

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I. INTRODUCTION

Over the past 50 years $NN\rightarrow NN\pi$ reactions have received considerable interest. Of those, $pp\rightarrow d\pi^+$ is probably the one which has been most extensively studied. This is because it is experimentally much easier to identify a two-particle final state. Most older measurements of this reaction are concentrated at higher energies where production via the $\Delta$ resonance dominates. With the advent of storage ring technology and internal targets the energy regime closer to threshold has become accessible. The first $NN\rightarrow NN\pi$ measurements close to threshold were restricted to cross section and analyzing power measurements [1–8], since polarized internal targets were not yet available. Measurements of spin correlation coefficients close to threshold became feasible only recently with the availability of windowless and pure polarized targets internal to storage rings [9–11].

At energies well above threshold a number of measurements of spin correlation coefficients in $pp\rightarrow d\pi^+$ exist. Of these, the ones closest to threshold are measurements of $A_{xx}$ at 401 and 425 MeV [12] which have been performed using an external beam and a polarized target.

A parametrization in terms of partial wave amplitudes of the $pp\rightarrow d\pi^+$ data from threshold to 580 MeV was carried out by Bugg 15 years ago [13]. A more recent, updated partial-wave analysis is maintained by the Virginia group [14]. With only the cross section and the analyzing power as input, the number of free parameters is usually lowered by theoretical input such as a constraint on the phases of the amplitudes which is provided by Watson’s theorem [15]. In this respect, a measurement of spin correlation parameters represents crucial new information because one can relate these observables to certain combination of amplitudes without any model assumptions.

Close to threshold, $s$ and $p$ wave amplitudes in the pion channel with a possible small admixture of $d$ waves, are sufficient to parametrize the data. In the following, we will demonstrate that combinations of the spin correlation observables presented here are directly sensitive to the strength of these $d$ waves.

II. EXPERIMENTAL ARRANGEMENT

In this paper we report measurements of spin correlation coefficients in $\vec{p}\vec{p}\rightarrow d\pi^+$ at 350.5, 375.0, and 400.0 MeV at center-of-mass angles between $25^\circ$ and $65^\circ$. The experiment was carried out at the Indiana Cooler with the PINTEX setup. PINTEX is located in the A region of the Indiana Cooler. In this location the dispersion almost vanishes and the horizontal and vertical betatron functions are small, al-
allowing the use of a narrow target cell. The target setup consists of an atomic beam source [16] which injects polarized hydrogen atoms into the storage cell.

Vertically polarized protons from the cyclotron were stack-injected into the ring at 197 MeV, reaching an orbiting current of several 100 $\mu$A within a few minutes. The beam was then accelerated. After typically 10 min of data taking, the remaining beam was discarded, and the cycle was repeated.

The target and detector used for this experiment are the same as described in [9], and a detailed account of the apparatus can be found in [17]. The internal polarized target consisted of an open-ended 25 cm long storage cell of 12 mm diameter and 25 $\mu$m wall thickness. The cell is coated with teflon to avoid depolarization of atoms colliding with the wall. During data taking, the target polarization $\vec{Q}$ is changed every 2 s pointing in sequence, up or down ($\pm y$), left or right ($\pm x$), and along or opposite to the beam direction ($\pm z$). The magnitude of the polarization was typically $|\vec{Q}| \sim 0.75$ and is the same within $\pm 0.005$ for all orientations [18,19].

The detector arrangement consists of a stack of scintillators and wire chambers, covering a forward cone between polar angles of $5^\circ$ and $30^\circ$. From the time of flight and the relative energy deposited in the layers of the detector, the outgoing charged particles are identified as pions, protons, or deuterons. The system detector was optimized for an experiment to study the spin dependence in $pp \rightarrow pp\pi^0$ and $pp \rightarrow pn\pi^+$ near threshold [9,10]. The $pp \rightarrow d\pi^+$ data presented here are a by-product of that experiment.

Date were taken with vertical as well as longitudinal beam polarization. To achieve nonvertical beam polarization the proton spin was precessed by two spin-rotating solenoids located in nonadjacent sections of the six-sided Cooler. The vertical and longitudinal components of the beam polarization $\vec{P}$ at the target are about equal, with a small sideways component, while its magnitude was typically $|\vec{P}| \sim 0.6$. Since the solenoid fields are fixed in strength, the exact polarization direction depends on beam energy after acceleration. In alternating measurement cycles, the sign of the beam polarization is reversed. More details on the preparation of nonvertical beam polarization in a storage ring can be found in Ref. [19].

III. DATA ACQUISITION AND PROCESSING

For each beam polarization direction, data are acquired for all 12 possible polarization combinations of beam ($+, -$) and target ($\pm x, \pm y, \pm z$). The event trigger is such that two charged particles are detected in coincidence. Then, a minimum $\chi^2$ fit to the hits in each of the four wire chamber planes is performed in order to determine how well the event conforms with a physical two-prong event that originates in the target. Events with $\chi^2 < 5$ were included in the final data sample. The information from the $\chi^2$ fit allows us to determine the polar and azimuthal angles of both charged particles. In the case of a two-particle final state the two particles are coplanar and the expected difference between the two azimuthal angles is $\Delta \phi = 180^\circ$. Events between $150^\circ < \Delta \phi < 210^\circ$ are accepted for the final analysis. The polar angles of the pion and the deuteron are correlated such that the deuteron, because of its mass, exits at angles close to the beam whereas pion laboratory angles as large as $180^\circ$ are kinematically allowed. At energies below 350.5 MeV the limited acceptance of the detector setup is caused by deuterons traveling through the central hole of the detector stack, whereas at higher energies the angular coverage is more and more restricted because of $\pi^+$ missing the acceptance of $30^\circ$. Events within $1^\circ$ of the predicted correlation of the $\pi^+d$ polar scattering angles were included in the final analysis. Since there is a kinematically allowed maximum deuteron angle which depends on the beam energy, only those two-prong events were included in the final data sample for which the deuteron reaction angle was $\approx 7^\circ$, $8^\circ$, and $9^\circ$ at 350.5, 375.0, and 400.0 MeV, respectively. The correlation between the polar angles of the pion and the deuteron and the coplanarity condition uniquely determine the $pp \rightarrow d\pi^+$ reaction channel. Therefore, the inclusion of particle identification gates in the analysis did not change the spin correlation coefficients by more than a fraction of an error bar. Consequently, no particle identification gates were used in the final analysis.

Since an open-ended storage cell is used, the target is distributed along the beam ($z$) axis. The resulting target density is roughly triangular extending from $z = -12$ cm to $z = +12$ cm with $z=0$ being the location where the polarized atoms are injected. The angular acceptance of the detector depends on the origin of the event along the $z$ axis. Specifically, the smallest detectable angle increases towards the upstream end of the cell. It was only possible to detect deuterons which originated predominantly from the upstream part of the target. Since the kinematically allowed maximum deuteron angle increases with energy, vertex coordinates between $z = -12$ cm and 4, 6, 8 cm at 350.5, 375.0, and 400.0 MeV, respectively, were accepted into the data sample. This way, background events originating from the downstream cell walls were suppressed.

The known spin correlation coefficients of proton-proton elastic scattering [20] are used to monitor beam and target polarization, concurrently with the acquisition of $pp \rightarrow d\pi^+$ events. To this end, coincidences between two protons exiting near $\theta_{lab} = 45^\circ$ are detected by two pairs of scintillators placed behind the first wire chamber at azimuthal angles $\pm 45^\circ$ and $\pm 135^\circ$. From this measurement, the products $P_yQ_y$ and $P_yQ_z$ of beam and target polarization are deduced. Values for the products of beam and target polarization can be found in Table I of Ref. [11].

Background arises from reactions in the walls of the target cell and from outgassing of the teflon coating [17]. The contribution arising from a $\approx 1\%$ impurity in the target gas is negligible. Background, which is not rejected by the software cuts described above, manifests itself as a broad distribution underlying the peak at $\Delta \phi = 180^\circ$. A measurement with a nitrogen target matches the shape of the $\Delta \phi$ distribution seen with the H target, except for the peak at $180^\circ$. This shape is used to subtract the background under the $\Delta \phi$ peak. Within statistics, the background shows no spin dependence.
and is independent of the polar angle. The subtracted background ranges from 2% to 5%.

**IV. DETERMINATION OF $A_y$, $A_{xx}$, $A_{yy}$, AND $A_{xz}$**

The analyzing power and spin correlation coefficients were determined using the method of diagonal scaling. Previously, we used diagonal scaling to analyze a series of experiments to measure spin correlation coefficients in $pp$ elastic scattering and a detailed description of the formalism can be found in Ref. 21.

Since we only measure the product of beam and target polarization $P \cdot Q$, we cannot normalize beam and target analyzing power independently. Because of the identity of the colliding particles, beam and target analyzing power as a function of center-of-mass angle are related such that $A_y^b(\theta) = -A_y^t(180° - \theta)$, in particular $A_y^b(90°) = -A_y^t(90°)$. For comparison with theory we use the quantity $\sqrt{(A_y^b(\theta)A_y^t(\theta))}$ for which the absolute normalization depends on $P \cdot Q$ and therefore is known. The final data with their statistical errors are shown in Fig. 1.

**V. PARTIAL WAVES**

When one restricts the angular momentum of the pion to $l_\pi \leq 2$, there are seven partial waves possible, each corresponding to a given initial and final state with all angular momentum quantum numbers given. Using the usual nomenclature [22], these amplitudes are

- $a_0: ^1S_0 \to ^3S_1$, $l_\pi = 1$,
- $a_1: ^3P_1 \to ^3S_1$, $l_\pi = 0$,

Very close to threshold, only $s$-wave pions are produced, and a single amplitude ($a_1$) is sufficient to describe the reaction. Slightly above threshold, the $p$-wave $a_2$ becomes significant, while the other $p$ wave ($a_0$) remains small and is often neglected entirely. Among the $d$-wave amplitudes, at, for instance, 400 MeV, $a_6$ is the largest, followed by $a_4$, $a_3$, and $a_5$ with a factor of 20 between $a_3$ and $a_6$. This is known indirectly from partial-wave analyses [13,14] of angular distributions of cross section and analyzing power. The spin correlation data presented here offer a more direct study of the $d$-wave strength in $pp \to d\pi^+$. In order to demonstrate this, we need to express the observables in terms of the seven partial-wave amplitudes [Eq. (1)]. This has previously been done for the cross section and the analyzing power [23]. For the spin correlation coefficients, we find analogously

\[
\frac{d\sigma}{d\Omega} = \frac{1}{16\pi} \left[ a_0^2 + a_1^2 + a_2^2 + C_1 + (a_2^2 - 2\sqrt{2}\text{Re}(a_0a_2^*)\right]
+ B_1 + C_2P_2^0(\cos \theta) + C_4P_4^0(\cos \theta),
\]

for $j$ and $m_j = \pm 1$ and $\gamma = \pm 1$.

![Figure 1](attachment:image.png)

**FIG. 1.** Spin observables for $\vec{p}p \to d\pi^+$. Beam and target analyzing power are taken at $\theta$ and $180° - \theta$, respectively. The open triangles are the $A_{zz}$ measurement at 401 MeV of Ref. [12]. The solid and dashed lines are the SAID SP96 and SP93 solutions, respectively. The dotted line represents zero.
\[
\frac{d\sigma}{d\Omega}(A_{xx} + A_{yy}) = \frac{1}{16\pi} \left[-2a_0^2 - 2a_2^2 + C_4 \right. \\
+ \left. (-2a_2^2 + 4\sqrt{2} \text{Re}(a_0a_2^{*})) + C_5 \right] P_2^0(\cos \theta) + C_6 P_4^0(\cos \theta), \tag{3}
\]

\[
\frac{d\sigma}{d\Omega}(A_{zz}) = \frac{1}{16\pi} \left[-a_2^2 + a_1^2 - a_2^2 + C_1 - C_4 \right. \\
+ \left. (-a_2^2 + 2\sqrt{2} \text{Re}(a_0a_2^{*})) + B_1 + C_2 - C_3 \right] \\
\times P_2^0(\cos \theta) + (C_3 - C_6) P_4^0(\cos \theta), \tag{4}
\]

\[
\frac{d\sigma}{d\Omega}(A_{xx} - A_{yy}) = \frac{1}{16\pi} \left[(B_2 + C_1) P_2^0(\cos \theta) + C_6 P_4^0(\cos \theta) \right]. \tag{5}
\]

with \(d\sigma/d\Omega\) being the unpolarized differential cross section. The observables in Eqs. (2)–(5) are functions of the center-of-mass reaction angle, \(\theta\), and the functions \(P_n^m(\cos \theta)\) are the usual Legendre polynomials. In the above equations, the terms \(B_n\) are caused by interference between \(s\) and \(d\) waves and are given by a sum of terms \(\text{Re}(a_ia_k^{*})(i=3,4,5,6)\), weighted by numerical factors which follow from angular momentum coupling. The terms \(C_n\) contain \(d\) waves only, i.e., a sum of \(a_2^2\) and \(\text{Re}(a_i a_k^{*})(i,k=3,4,5,6)\), again weighted by the appropriate numerical factors. A derivation of Eqs. (2)–(5) can be found in Ref. [24].

From Eq. (5) one sees that the combination \(A_{xx} - A_{yy}\) vanishes at all angles when there are no \(d\) waves. A departure from this behavior would be caused by an interference between \(s\) and \(d\) waves. It is also easy to see from Eqs. (2)–(4) that for all reaction angles, the following holds:

\[
A_{zz} - A_{xx} - A_{yy} = 1 - \delta,
\]

where \(\delta\) contains only \(C_n\) terms, i.e., only \(d\)-wave amplitudes.

Thus, both quantities, \(A_{xx} - A_{yy}\) and \(A_{zz} - A_{xx} - A_{yy} - 1\) are a direct measure of \(d\)-wave contributions. Clearly, \(A_{xx} - A_{yy}\) provides a more sensitive test because here \(d\) waves interfere with the dominant \(a_1\) amplitude.

VI. FINAL DATA AND COMPARISON WITH THEORY

We compare our data to the two newest, published phase shift solutions, namely SP96 the published partial-wave analysis of the VPI group [25]; range of validity 0–500 MeV, where the quoted energy is the laboratory energy of the pion in \(\pi^- d \to pp\); and BU93 the published partial-wave analysis of the VPI group and Bugg [26]; range of validity 9–256 MeV.

Numerical values for both phase shift analyses have been obtained from the SAID interactive program [14]. As can be seen from Fig. 1, our data are in good agreement with both phase shift solutions, although the agreement with SP96 (overall \(\chi^2 = 0.90\)) is better than with BU93 (overall \(\chi^2 = 1.29\)). As a function of energy we have \(\chi^2 = 0.90, 0.87, 0.99\) (SP96), and \(\chi^2 = 1.64, 0.88, 1.50\) (BU93) at 350.5, 375.0, and 400.0 MeV, respectively. This indicates that the energy dependence of SP96 near threshold is slightly closer to the data. Also shown in Fig. 1 are the \(A_{zz}\) data of Ref. [12] which agree with our data.

The quantity \(A_{xx} - A_{yy}\) provides a sensitive and direct test of \(d\)-wave contributions in \(\pi^- d \to d \pi^+\). Our data are consistent with negligible \(d\)-wave contributions even at 400 MeV (see Fig. 1).

VII. SUMMARY

In summary, we have measured the analyzing power and spin correlation coefficients in \(\pi^- d \to d \pi^+\). The spin correlation coefficients allow a direct determination of \(d\)-wave contributions. In particular, \(A_{xx} - A_{yy}\) provides a sensitive test because here \(d\) waves interfere with the dominant \(a_1\) amplitude. Our data are consistent with negligible \(d\)-wave contributions even at 400 MeV. In addition, our data are well described by a partial wave solution (SP96) of the Virginia group.

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APPENDIX

Since Mandl and Regge [22] introduced their nomenclature to describe the reaction \(\pi^- d \to d \pi^+\), several different notations for the observables have been in use. We have been adhering to the notation of the Madison Convention [27] for the polarization observables, i.e., analyzing powers and spin correlation coefficients are labeled \(A_i\) and \(A_j\) where \(i,j = x,y,z\). The positive \(z\) axis is along the direction of momentum of the incident particle. In order to facilitate easy reading and comparison with the literature, we relate our observables to the notation of Niskanen [28]. The following equations have been obtained using Table I of Ref. [28]:

\[
\frac{\sigma_{0x}}{\sigma_{00}} = \cos \phi A_{ix}^b, \tag{A1}
\]

\[
\frac{\sigma_{0y}}{\sigma_{00}} = -\sin \phi A_{iy}^c. \tag{A2}
\]
\[ \frac{\sigma_{yy}}{\sigma_{00}} = \sin^2 \phi A_{xx} + \cos^2 \phi A_{yy}, \quad (A3) \]
\[ \frac{\sigma_{yx}}{\sigma_{00}} = \sin \phi \cos \phi (A_{xx} - A_{yy}), \quad (A4) \]
\[ \frac{\sigma_{yz}}{\sigma_{00}} = \sin \phi A_{xz}, \quad (A5) \]

Here, \( \sigma_{00} \) is the unpolarized differential cross section and \( \phi \) is the azimuthal angle of the pion. Furthermore, Niskanen uses \( P_B \) and \( P_T \) for beam and target polarization where we use \( P \) and \( Q \), respectively.

Angular distribution of the longitudinal $pp$ spin correlation parameter $A_{zz}$ at 197.4 MeV

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A polarized proton beam with a large longitudinal polarization component of $0.545 \pm 0.005$ (96% of the total polarization) was prepared in a storage ring (IUCF–Cooler). This was achieved by means of spin precession solenoids in two of the six straight sections of the ring. A polarized hydrogen storage cell target internal to the ring was used to measure the longitudinal spin correlation coefficient $A_{zz}$ in $pp$ elastic scattering over the laboratory angular range $5.5^\circ - 43.5^\circ$ ($\theta_{c.m.} = 11.5^\circ - 90^\circ$) with statistical errors of typically 0.025. The absolute normalization was determined to an accuracy of 2.0% by use of the identity $A_{zz} - A_x - A_y = 1$ at $\theta_{c.m.} = 90^\circ$. The identity also allows a reduction of the scale factor uncertainty of the previously published analyzing powers and spin correlation coefficients. The results are compared to recent $pp$ partial wave analyses and $NN$ potential models.

PACS number(s): 24.70.+s, 13.88.+e, 13.75.Cs, 25.40.Cm

I. INTRODUCTION

In recent papers we reported measurements of analyzing power $A_y$ and spin correlation parameters $A_{xx}$, $A_{yy}$, and $A_{zz}$ in $pp$ elastic scattering at eight energies between 197.4 and 448.9 MeV [1,2]. The remaining independent spin correlation parameter $A_{zz}$ can only be measured with both beam and target polarized in the beam direction (longitudinal). Here, we report on the development of longitudinal beam polarization in the proton storage ring (‘‘Cooler’’) at the Indiana University Cyclotron Facility. This polarized 197.4 MeV beam was used in conjunction with a polarized hydrogen storage cell target [3] to measure the spin correlation parameter $A_{zz}$ in $pp$ elastic scattering as a function of laboratory scattering angles between $5.5^\circ$ and $43.5^\circ$. Two spin precession solenoids were introduced into the storage ring to prepare longitudinal beam polarization at the location of the polarized hydrogen target.

The experimental apparatus and methods, including analysis and study of systematic effects, are very similar to those described in Ref. [1], and thus will not be discussed in detail. Measurements of $pp$ elastic scattering were taken with vertical, horizontal, and longitudinal target polarization. The measurements with longitudinal target polarization allow determination of the product $P_z Q_z A_{zz}$, longitudinal beam polarization $P_z$, and spin correlation parameter $A_{zz}$.

The measurements with horizontal and vertical target polarization are used to determine the product of longitudinal beam polarization $P_z$ and transverse target polarizations $Q_x$ and $Q_y$. This makes use of the spin correlation parameter $A_{zz} = A_{yz}$, which is known from our previous measurement at the same beam energy [1]. Under the assumption that the target polarization for the three different holding field orientations is the same, this determines the product $P_z Q_z$, which is needed to extract the angular distribution of the spin correlation parameter $A_{zz}$.

The absolute normalization of our previous spin correlation data, as well as the normalization of the present $A_{zz}$ measurement ultimately depend on a measurement of the analyzing power $A_y$ in $pp$ scattering at 183.1 MeV [4]. An interesting check of the absolute normalization is offered by the model independent relationship $A_{yy} - A_{xx} - A_{zz} = 1$ [5] at $\theta_{c.m.} = 90^\circ$. Here this relation is exploited to check the correctness of the previous calibration and to improve its absolute normalization accuracy.

The preparation of longitudinal beam polarization will be discussed in Sec. II. Section III contains an overview of the experimental apparatus. The extraction of the spin correlation parameter $A_{zz}$ from the measured yields is discussed in Sec. IV. Section V presents the final absolute calibration of the $pp$ spin correlation parameter. A short discussion of corrections and systematic effects is given in Sec. VI. The results for the angular distribution of $A_{zz}$ and a comparison to theoretical predictions is given in Sec. VII. This is followed by the conclusion in Sec. VIII.

II. LONGITUDINAL BEAM POLARIZATION

A. Polarization of a stored proton beam

The polarization of an ensemble of particles with spin $1/2$ is described by a vector $\vec{P}$, which is parallel to the sum $\vec{\mu}$ of

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the magnetic moments of all particles in the ensemble. Spin-
1/2 beam polarization in a storage ring is thus fully deter-
mined by the polarization of the injected beam and the mo-
tion of the magnetic moments of the stored particles.

As a proton progresses along the closed orbit, its magnetic
moment precesses around the prevailing magnetic field di-
rection. The general, relativistic equation of the motion of
the direction of a magnetic dipole travelling through electro-
magnetic fields is known as the BMT equation [6]. From this
equation we can derive the action of the two basic field ele-
ments which we need for the present purpose:

The first element we need to understand is the vertical ($y$)
field of a bending magnet which deflects the beam in the
horizontal ($x$-$z$) plane by an angle $\theta$, while it precesses
the magnetic moment around the longitudinal ($z$) direction
by an angle $\xi_B(\theta)$, where

$$\xi_B(\theta) = (g - 1) \theta \gamma = 1.792 \ 847 \ 39(6) \ \theta \gamma.$$  

(2.1)

Here, $\gamma$ is the usual relativistic kinematic parameter, and $g$ is
the $g$ factor of the proton.

The second field element is a solenoid with an integrated
field $B = f B \ dz$ along the beam direction which precesses
the magnetic moment around the longitudinal ($z$) direction
by an angle $\xi_s(B)$, where

$$\xi_s(B) = \frac{c g B}{m \beta \gamma} = 0.89235 \frac{B}{\beta \gamma}.$$  

(2.2)

Here, $c$ is the speed of light in m/s, $m$ the proton mass in
eV/c$^2$, $\beta$ the usual relativistic kinematic parameter, and $B$
the longitudinal field integral in Tesla meters (Tm).

We now study a particle which completes a single turn
around the ring, starting and ending at a point $s^*$ somewhere
on the stored orbit. As a consequence of the precession of
the magnetic moment by the magnetic elements in the ring lat-
tice, there will be a ‘‘one-turn’’ net rotation $R(s^*)$ between
the initial and final direction of the moment. This rotation
can be calculated easily by concatenating the individual ro-
tations around the vertical $\{R_z[\xi_B(\theta)]\}$ and longitudi-
nal $\{R_z[\xi_s(B)]\}$ directions due to bends and solenoids, starting
at $s^*$ and proceeding against the beam direction [see Eq.
(2.3)]. These rotations may be described by $3 \times 3$ matrices,
but in practice it is more elegant and more convenient to
adapt the spinor formalism from quantum mechanics in
which rotations are expressed as complex $2 \times 2$ matrices [7].

The one-turn rotation $R(s^*)$ is characterized by a rotation
axis $\hat{n}(s^*)$, and a rotation angle $\Psi$ which is independent of
the choice of $s^*$. The unit vector $\hat{n}(s^*)$ which is given by the
eigenvector of $R(s^*)$ is called the ‘‘spin closed orbit,’’ and
$\Psi/2\pi$ is known as the ‘‘spin tune.’’ It is obvious that the
component of $\hat{\mu}$ (and thus the polarization vector $\hat{P}$) which
is parallel to $\hat{n}(s^*)$ is preserved. The component perpendicular
to $\hat{n}(s^*)$ precesses around it and over many orbits averages
to zero. Thus, the direction of the beam polarization is
given by the spin closed orbit.

Normally, in a storage ring the spin closed orbit is vertical
for all $s^*$, since the effect from transverse focusing fields
averages to zero. To carry out an experiment with longitudi-
nal beam polarization one has to provide spin rotators in the
ring lattice that cause the spin closed orbit at the target
$\hat{n}(s_{\text{target}})$ to point along the beam direction. For energies
below a few GeV, it is best to use solenoid fields to rotate the
spin. In the next section we describe how this was done in
the IUCF Cooler.

**B. Preparation of longitudinal beam polarization**

The IUCF Cooler storage ring is a six-sided synchrotron
with a polarized hydrogen target in the $A$ region straight
section, as shown in Fig. 1. This figure also shows the two
spin-rotation solenoids in the $C$ and $T$ region which were
used to prepare longitudinal polarization at the target. The
placement and strength of the solenoid fields is governed by
the task of achieving the desired spin closed orbit, but in
practice is also constrained by space requirements and by the
fact that solenoids also focus the beam and thus have an
impact on the ring optics.

**The $C$-region solenoid.** The electron beam which is used
for phase space cooling is transversely confined by a sole-
noi d field. In normal operation, the effect of this field on
the spin closed orbit is compensated by two additional sole-
noids with opposite field, immediately upstream and down-
stream of the cooling region. For the present experiment we
operate these compensating solenoids with reversed current
such that the field direction is the same for all three sole-
noids. In this mode, a longitudinal field integral of $B_C$
$=0.877$ Tm is achieved, limited by the maximum allowed
power dissipation in the solenoids.

**The $T$-region solenoid.** A superconducting solenoid was
placed in the $T$ region (see Fig. 1). The coil of this solenoid
has an inner diameter of 17.5 cm and a length of 30 cm. The
insertion length of the device is 58 cm with a clear bore of
10.8 cm. The field integral of this magnet is 1.10 Tm.

With these elements present in the ring, one obtains for
the one-turn rotation starting at the target

![Image](https://via.placeholder.com/150)

**FIG. 1.** The magnet lattice of the IUCF Cooler. The target is
located in the $A$ region. Bending magnets are marked with the bend-
ing angle in degrees. The spin precession solenoids in the $T$ and $C$
region are shown to illustrate their position with respect to the
bending magnets.
\begin{align*}
R(s_{\text{target}}) &= R_z[\xi_B(123^\circ)]R_z[\xi_C(B_c)]R_z[\xi_B(117^\circ)] \\
R_z[\xi_C(B_c)]R_y[\xi_B(120^\circ)],
\end{align*}

where the deflection angles 123°, 117°, and 120° are the net deflections between A, T, and C regions (see Fig. 1). Evaluating the eigenvector of this matrix, normalized to 1, yields the spin closed orbit at the target \( n(s_{\text{target}}) = (0.250, 0.125, 0.960) \), where the three numbers denote horizontal (x), vertical (y), and longitudinal (z) components. The fact that the polarization is not purely longitudinal is caused by the limit on the thermal load of the C solenoids. The field integral required for longitudinal beam polarization at the target is roughly 1.1 Tm in each solenoid. As will be discussed in Sec. IV, the angle of 16.3° of the polarization direction with the beam direction is taken into account in the analysis of the data. Aside from a small reduction in statistical accuracy (compared to pure longitudinal polarization), the measurement of the spin correlation parameter \( A_{zz} \) is not affected.

The spin closed orbit at the injection point can be evaluated analogously. The one-turn rotation in this case is given by

\begin{align*}
R(s_{\text{injection}}) &= R_z[\xi_B(60^\circ)]R_z[\xi_C(B_c)]R_y[\xi_B(243^\circ)] \\
R_z[\xi_C(B_c)]R_y[\xi_B(57^\circ)].
\end{align*}

The corresponding spin closed orbit follows as \( \tilde{n}(s_{\text{injection}}) = (0.252, 0.953, 0.157) \). Its direction is almost vertical. Injection of vertically polarized beam loses about 5% of the injected polarization, but eliminates the technical complication of having to make use of precession solenoids in the beam line from the cyclotron to the Cooler.

III. EXPERIMENTAL ARRANGEMENT AND EVENT IDENTIFICATION

The experiment was carried out in the IUCF Cooler storage ring at the Indiana University Cyclotron Facility. The polarized target is located in the A region of the ring, which has low dispersion and small β function and thus is best suited for internal storage cell targets. Polarized hydrogen atoms for the target are produced by an atomic beam source. The polarized atomic beam is injected into a T-shaped, thin walled storage cell, located on the axis of the storage ring. The orientation of the target polarization is defined by three sets of guide field coils, and can be changed in less than 10 ms between the longitudinal (z), vertical (y), and horizontal (x) direction.

Figure 2 shows a three-dimensional representation of the detector setup used to detect coincidences between two protons from \( pp \) elastic scattering in the target. Two types of \( pp \) elastic scattering events are detected. Type I events, covering an angular range of \( \theta_{\text{lab}} = 5° \), are detected as coincidence between forward scintillators and recoil detectors. The forward scintillators are two plastic scintillators (E and K) and recoil detectors are eight silicon micro strip detectors (R1–8) mounted at azimuthal angles ±45° and ±135° around the storage cell. Position and angle information for the event is provided by two wire chambers (XY and UV) for the forward scattered proton, and the strip position for the recoil.

Type II events, covering the angular range of \( \theta_{\text{lab}} = 30° \), are detected as coincidence between two of four scintillators (S1–4), mounted between the two wire chambers at azimuthal angles ±45° and ±135°. For these events, both protons pass the first wire chamber (XY), allowing reconstruction of angles and origin of the event.

Both event types are subjected to a kinematic fit to determine scattering angle \( \theta \), azimuthal angle \( \phi \), and vertex position z assuming the event originates on the beam axis and follows \( pp \) elastic scattering kinematics (see Ref. [1]). To avoid sensitivity to the physical boundaries of roughly ±20° around the nominal azimuthal center positions (±45°, ±135°) of the recoil (R1–8) and scintillation detectors (S1–S4), only events within ±18.5° are accepted. For event type I an additional cut on the correlation between energy loss in the recoil detectors and scattering angle is applied (see Ref. [1]).

For a more detailed description of target, detector system and event selection the reader is referred to Ref. [1]. The measurement was organized in cycles consisting of 3 min injection of polarized beam at 197.4 MeV and 3 min data taking. At the end of a cycle, the beam remaining in the ring was discarded, and the next cycle begins with injection of new beam. Approximately every 30 min, the polarization direction of the injected beam was reversed at the ion source.

The data acquisition was subdivided into 12 s subcycles, in which the target polarization direction was cycled in 2 s intervals through the 6 possible states \((±x, ±y, ±z)\). The current of the stored beam ranged from 50 to 150 µA with beam lifetimes of 2000–3000 s. A total of approximately \( 3 \times 10^6 \) \( pp \) elastic scattering events in 12 spin combinations were acquired in 6 days.

IV. DETERMINATION OF THE SPIN CORRELATION PARAMETER \( A_{zz} \)

For all orientations of the target holding field (x, y, or z), yields \( Y_{44}(\theta) \) are measured as a function of scattering angle. The experiment uses four different ranges of azimuthal
angles $\theta_i$ centered at $\phi=\pm45^\circ$ and $\pm135^\circ$ and four different combination of beam and target polarization $k$ ($++,+-,-+,--$). Thus for each orientation of the target guide field, the yields $Y_{ik}$ are represented by a 4x4 matrix. These yields can be related to the $pp$ elastic scattering cross section by factors that contain detector efficiencies on one hand, and luminosities (target thickness, number of incident protons) for the different beam and target polarization combinations on the other hand. Multiplication of the rows $i$ of $Y_{ik}$ by suitable efficiency factors $e_i$ and multiplication of the columns by luminosity factors $\lambda_j$ yields a matrix $X_{ik}=e_iY_{ik}\lambda_j$. Efficiency factors compensate for differences in the detector efficiencies, while luminosity factors normalize the luminosities such that for unpolarized beam and target $X_{ik}^{unpol} = 1$ for all $i,k$. The $X_{ik}$ are related to the cross section by $X_{ik}=\sigma_{ik}/\sigma_0$ where $\sigma_0$ is the unpolarized differential cross section and $\sigma_{ik}$ the spin dependent cross section for the specific beam polarization $P=(P_x, P_y, P_z)$ and target polarization $Q=(Q_x, Q_y, Q_z)$. The method used to determine the $X_{ik}$ from the measured yields $Y_{ik}$ is known as diagonal scaling and described in detail in Ref. [8]. For each scattering angle $\theta$, the experiment yields 48 values of $X_{ik}$ (6 target spin directions, two beam spin directions, four azimuthal angles), which are used to determine experimental quantities of the form (polarization) $\times$ (analyzing power $A_x$), and (beam polarization) $\times$ (target polarization) $\times$ (spin correlation parameters $A_{ik}$) (see Ref. [8]). All data are simultaneously analyzed as described in Ref. [1], allowing for possible differences in polarization when the sign of beam and target polarization is reversed, as well as small deviations of the target polarizations from the ideal orientation.

However, for the purpose of illustration, we discuss here the case of longitudinal beam polarization with negligible transverse components ($P_x=P_y=0$). For purely longitudinal beam polarization, the $X_{ik}$ are given by

$$X_{ik} = 1 + A_x(Q_x \sin \phi + Q_y \cos \phi) + A_{xz} P_z(Q_x \sin \phi + Q_y \cos \phi) + A_{zz} P_z Q_z.$$

(4.1)

Measurements with longitudinal beam polarization $P_z$ and longitudinal target polarization $Q_z$ (with $Q_x=Q_y=0$) determine the angular distribution $A_{zz}(\theta)$ within a scale factor given by $P_z Q_z$.

Neither $P_z$ nor $Q_z$ can be measured directly since the longitudinal analyzing power $A_z$ vanishes by parity conservation. The method to determine $P_z$ and $Q_z$ used here is based on the assumption (discussed below) that the target polarization $Q$ is independent of orientation of the target spin ($Q_x=Q_y=Q_z=0$). Part of this assumption ($Q_z=0$) has been explicitly verified in Ref. [3]. Since $A_{zz}$ is known from previous measurements, $P_z Q_x$ and $P_z Q_y$ can be determined from the measurements with transverse target polarization, which yield values of $A_{zz}(\theta)P_z Q_x$ and $A_{zz}(\theta)P_z Q_y$. Using the known values of $A_{zz}$ from Ref. [1] as input, the product $k_{x(y)}=P_z Q_{x(y)}$ of beam and target polarization is varied to minimize the $\chi^2$ between the present results for $P_z Q_x A_{zz}(\theta)$ or $P_z Q_y A_{zz}(\theta)$ and the scaled $A_{zz}$:

$$\chi^2 = \sum_\theta \left[ \frac{(P_z Q_{x(y)} A_{zz} - k_{x(y)} A_{zz}^{\text{ref}})^2}{\delta(P_z Q_{x(y)} A_{zz})^2 + (k_{x(y)} \delta A_{zz}^{\text{ref}})^2} \right].$$

(4.2)

Since the analysis uses data over a wide range of angles (5.5° to 43.5°), the final results are of high statistical accuracy. Best agreement with $A_{zz}^{\text{ref}}$ is obtained for $P_z Q_x = 0.4267 \pm 0.0051$ for target polarization along $x$ and $P_z Q_y = 0.4225 \pm 0.0055$ when the target polarization is along $y$. The weighted mean $P_z Q = 0.4248 \pm 0.0037$ (or 0.9% relative uncertainty) was used to determine $A_{zz}(\theta)$. The absolute calibration of the resulting $A_{zz}(\theta)$, which ultimately depends on the $A_z$ calibration point [4] which was used in the determination of $A_{zz}^{\text{ref}}$, has an overall uncertainty of 2.66%. This uncertainty can be reduced further as will be explained in Sec. V. The final results for $A_{zz}$ are shown in Fig. 3.

The absolute normalization of $A_{zz}(\theta)$ depends on the assumption that $PQ$ with target polarization along $z$ is the same as for target polarization along $x$ or $y$. The beam polarization $P$ can be assumed independent of the ($<$1 mT) guide field over the target since the polarization lifetime of the stored beam is very long ($>1$ h [9]) compared to the rapid (6 s) sequence of target polarization states. In an earlier experiment with transverse beam polarization, independence of $P$ on guide field direction was confirmed by direct measurement to better than 0.5% (see Table 1 in Ref. [1]).

We now discuss the assumption that the magnitude of the target polarization does not depend on orientation. Change in the target polarization direction is accomplished by changing the guide field in the target region, which is provided by three sets of coils external to the vacuum system [3]. Information on the uniformity and accuracy of the guide field direction over the target, and effects of the guide field on the proton closed orbit is given in Ref. [3]. The absolute value of the polarization of the gas target is independent of the orientation and sign of the guide field because between the exit of the last sixpole magnet of the atomic beam source and the target cell the spin of the atoms follow the magnetic field direction adiabatically: from an initially inhomogeneous field in the sixpole, as the atoms travel into the homogeneous field.
at the target, the magnetic moments follow the field direction, no matter what the orientation of the guide field is. The condition of adiabaticity requires that along the trajectory of the atoms the field direction changes experienced by the atoms are slow compared to the Larmor precession rate of the atom in that field. That this condition is easily met is evident from the many polarized ion sources based on the atomic beam method, that provide large polarization without making special provisions to assure adiabaticity. However, if there is a point between atomic beam source and target, where the magnetic field is both small in magnitude (low Larmor frequency) and changing rapidly in field direction, loss of polarization arises. This loss may depend on the direction of the guide field over the target, because fringe fields of the guide field coils may under some conditions nearly cancel the ambient field. Consequently, careful field measurements were made along the atomic beam axis for each of the six different target guide field conditions. The rate of change in the field direction compared to the Larmor precession was found to be less than $5 \times 10^{-6}$ and satisfies the adiabaticity condition of being $\ll 1$.

The second concern is the possibility that the transition unit [medium-field transition (MFT)], which is used in the atomic beam source to select a single hyperfine state of hydrogen atoms, may be affected by the fringe field of the guide field coils. While it is found that the resonance region shifts slightly when the guide field is reversed, the resonance range is wide enough (see Ref. [10]) that a working point exists for which the transition works properly for all guide field orientations.

Finally, the expectation that the product of beam and target polarization is independent of guide field can be checked directly for guide fields along $x$ and $y$. No statistically significant difference has been observed, neither in this experiment, nor in previous experiments with transverse beam polarization [1,3].

V. ABSOLUTE NORMALIZATION

The absolute normalization of the present $A_{zz}$ data and the previously reported values of $A_{xx}$, $A_{yy}$, and $A_{xz}$ [1,3] all depend on the $A_c$ calibration point reported in Ref. [4]. The calibration can be checked and the accuracy of the calibration can be improved by use of the identity

$$A_{yy} - A_{xx} - A_{zz} = 1, \quad (5.1)$$

which applies to spin correlation coefficients in elastic scattering of spin 1/2 particles at a center of mass angle of 90°. The relation follows directly from symmetry relations between the five helicity amplitudes at that scattering angle [5].

To improve the statistical accuracy of the experimental value of $S = A_{yy} - A_{xx} - A_{zz}$ at $\theta_{c.m.} = 90°$ ($\theta_{lab} = 43.57°$), a polynomial fit of the angular distribution of $A_{yy} - A_{xx} - A_{zz}$ was performed in the vicinity of $\theta_{lab} = 43.57°$. Because the spin correlation coefficients are symmetric around $\theta_{c.m.} = 90°$, the angular distribution has an extremum at the corresponding laboratory angle of $\theta_{lab} = 43.57°$ so that only even terms with extrema at this laboratory angle were used as fitting functions.

Figure 4 shows examples for parabolic fits and fits including a second and fourth order term. The extracted values of $S$ are insensitive to the number of data points used as long as the $\chi^2$ (degree of freedom) of the fits is close to its minimum (see bottom panels in Fig. 4). For 10–20 data points included in the fits, the extracted values vary by $\pm 0.005$, which is taken into account as an interpolation uncertainty.

To test the accuracy of the above procedure, simulated data were produced from predictions for the $A_{ik}$ from partial wave analyses for the laboratory angular range 23° to 43.5° (corresponding to 20 data points). The values for $S$ differed from the correct value $S = 1$ by less than $10^{-3}$.

The result for the sum $S$ is taken from the parabolic fit with 14 data points

$$S = (A_{yy}^{90°} - A_{xx}^{90°} - A_{zz}^{90°}) = 0.996 \pm 0.011, \quad (5.2)$$

where the uncertainty contains statistical and interpolation uncertainties added in quadrature. The final values for the angular distribution of $A_{zz}$ were determined by dividing the $A_{zz}$ obtained in Sec. IV, which were normalized to the $A_{xz}$ from [1] by $S = 0.996$ in order to satisfy the identity Eq. (5.1). The results are given in Table I.

The absolute normalization uncertainty of the spin correlation coefficients is affected by two factors: the 1.1% error in $S$ and the relative uncertainty in the determination of $P, Q$ which is given as 0.9% in Sec. IV. The error analysis must take into account that changing $P, Q$ by 0.9% requires a change in $A_{zz}$ and a change in $A_{yy} - A_{xx}$ if the identity Eq. (5.1) is to be maintained. Numerical calculations show that
TABLE I. Final results for the angular distribution of the spin correlation parameter $A_{zz}$, using the relation $A_{yy} - A_{xz} - A_{zz} = 1$ for the normalization. The absolute normalization uncertainty is 2.0%.

<table>
<thead>
<tr>
<th>$\theta_{\text{lab}}$ (deg)</th>
<th>$A_{zz}$</th>
<th>$\delta A_{zz}$</th>
<th>$\theta_{\text{lab}}$ (deg)</th>
<th>$A_{zz}$</th>
<th>$\delta A_{zz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
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<td>0.854</td>
<td>0.023</td>
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<tr>
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<td>0.027</td>
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<td>0.840</td>
<td>0.024</td>
</tr>
<tr>
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<td>0.018</td>
<td>37.5</td>
<td>0.913</td>
<td>0.021</td>
</tr>
<tr>
<td>18.5</td>
<td>0.497</td>
<td>0.018</td>
<td>38.5</td>
<td>0.928</td>
<td>0.020</td>
</tr>
<tr>
<td>19.5</td>
<td>0.554</td>
<td>0.018</td>
<td>39.5</td>
<td>0.855</td>
<td>0.020</td>
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<tr>
<td>20.5</td>
<td>0.612</td>
<td>0.018</td>
<td>40.5</td>
<td>0.906</td>
<td>0.019</td>
</tr>
<tr>
<td>21.5</td>
<td>0.663</td>
<td>0.019</td>
<td>41.5</td>
<td>0.924</td>
<td>0.019</td>
</tr>
<tr>
<td>22.5</td>
<td>0.689</td>
<td>0.019</td>
<td>42.5</td>
<td>0.883</td>
<td>0.018</td>
</tr>
<tr>
<td>23.5</td>
<td>0.734</td>
<td>0.020</td>
<td>43.5</td>
<td>0.890</td>
<td>0.024</td>
</tr>
<tr>
<td>24.5</td>
<td>0.786</td>
<td>0.021</td>
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</tbody>
</table>

...an increase of $P_xQ$ by 0.9% reduces $A_{zz}$ by 1.6% and increases $A_{yy} - A_{xz}$ by 0.8%. Adding the statistical error of $S$ in quadrature yields a final scale uncertainty of 2.0% for the $A_{zz}$ in Table I.

The present results suggest that the values of the $A_{ik}$ reported in Ref. [1] should be divided by $S = 0.996$ and should be assigned a scale uncertainty of 1.4%. Since the measurement of $A_y$ in Ref. [1] only involves either a beam or a target polarization, the values should be divided by $\sqrt{S} = 0.998$ and assigned a 0.7% uncertainty.

VI. CORRECTIONS

In this section a summary of small corrections applied to the final results will be given. As the methods used to determine these corrections are discussed elsewhere [1], only a brief overview is given here.

A. Deadtime

Deadtime of the data acquisition system is of concern because the total event rate changes by some 40% between parallel and antiparallel beam and target helicities. The fractional dead time was determined by using fast scalers to count the number of events presented to the data acquisition computer compared to the number of processed events. These scalers were read once a second. The loss rate is found to be a linear function of the rate of accepted events (Fig. 5). From the slope a deadtime per processed event of $232 \pm 7 \ \mu s$ is found. The average loss probability is about 3 and 5% for parallel and antiparallel beam and target spins, respectively. Before executing the polarization analysis, the yields are corrected for the number of lost events. The deadtime correction increases the value of $A_{zz}$ by about 0.014, or 2/3 of the statistical error. This is by far the largest correction that needed to be applied.

B. Finite $\theta$-bin correction

The angular distribution of the spin correlation coefficient $A_{zz}$ is reported at the center of 1° $\theta_{\text{lab}}$ angle bins and the entire analysis was executed with this binning. Since both cross section and polarization observables depend on angle, the values at the center of the bin may differ slightly from the measured mean over the bin.

The measured angular distributions of the spin correlation parameters $A_{zz}$ used for normalization and $A_{zz}$ are corrected for this effect. The correction to $A_{zz}$ is typically 0.001. For angles below 10°, where the acceptance of the detector system is angle dependent, the correction changes to about 0.006 and becomes comparable with the statistical error for the two smallest angle bins. The effect on the normalization was found to be less than 0.07% and is neglected, as the overall norm error is 2.0%.

C. Correction for nonuniform $\phi$ acceptance

In the polarization analysis we assume that the acceptance of the detector system as a function of the azimuthal angle $\phi$ is uniform. However, the data show that the $\phi$ acceptance depends slightly on the scattering angle $\theta_{\text{lab}}$ and is in general not uniform.

The term containing the spin correlation parameter $A_{zz}$ has no $\phi$ dependence, thus there is no effect on the angular distribution of $A_{zz}$. However, the $\phi$ dependence of the $A_{zz}$ term, required a correction of $-0.001$ to the absolute normalization of $A_{zz}$. This correction is well below the absolute error of the norm and the treatment as a small correction is justified.

D. Background

Although the storage cell wall is made of thin Teflon foils, it is still about $10^7$ times more massive than the polar-
TABLE II. Table with $\chi^2$ per datapoint for the comparison of the data to potential models and partial wave analyses. The second column contains the $\chi^2$ between the present $A_{zz}$ results and the predictions. Columns three and four contain the $\chi^2$ and scaling factor $k$, by which the $A_{zz}$ data need to be multiplied to yield the best agreement between prediction and data. Column five gives the overall $\chi^2$ per degree of freedom for $A_y$ and all spin correlation parameters ($A_{xx}, A_{yy}, A_{xz}, A_{yz}$). The $A_y$ and $A_{ik}$ are multiplied by factors $k_y$ and $k_{ik}$, respectively, adjusted for best agreement with each calculation. The data for $A_y$ and $A_{ik}$, $A_{xx}, A_{yy}, A_{xz}$, are from Ref. [1], to which the small correction described in Sec. V was applied.

<table>
<thead>
<tr>
<th>Prediction</th>
<th>$A_{zz}$ scaled $\chi^2$</th>
<th>$A_y$ and all $A_{ik}$ scaled $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV18</td>
<td>4.03</td>
<td>1.50</td>
</tr>
<tr>
<td>CD-Bonn</td>
<td>2.09</td>
<td>0.70</td>
</tr>
<tr>
<td>Paris80</td>
<td>3.27</td>
<td>1.32</td>
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<tr>
<td>Ni93</td>
<td>4.56</td>
<td>1.01</td>
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<tr>
<td>Ni97</td>
<td>2.00</td>
<td>0.72</td>
</tr>
<tr>
<td>SM94</td>
<td>2.24</td>
<td>0.77</td>
</tr>
<tr>
<td>W96</td>
<td>5.29</td>
<td>4.41</td>
</tr>
<tr>
<td>SM97</td>
<td>5.76</td>
<td>3.36</td>
</tr>
<tr>
<td>SP99</td>
<td>2.34</td>
<td>0.92</td>
</tr>
</tbody>
</table>

FIG. 6. Comparison between $A_{zz}$ and partial wave analyses (top) and predictions from potential models (bottom). The data and predictions are plotted as difference to the reference N93. The left two panels show predictions divided by the scaling factors giving best agreement with the predictions and the $A_{zz}$ data (column 4 of Table II). The right two panels use the factors $k_y$ from scaling $A_y$ and all $A_{ik}$ (column 6 of Table II).

were compared to current partial wave analyses (PWA) (Nijmegen group: Ni93 [11], Ni97 [12]; Virginia group: SM94, SM97 [13]) and to a number of potential model calculations (Reid93 [14,12], Argonne potential AV18 [15,16]. CD-Bonn [17,13], Paris80 [18,13]). In Table II, the columns labeled $\chi^2$ shows the quality of agreement for some of these calculations, as well as for the most recent VPI analysis SP99 [13], which already includes the current results for $A_{zz}$. Best agreement ($\chi^2$ per point = 2) is found for the Nijmegen PWA analyses (Ni93, Ni97) which is based on a fit to $NN$ data in the energy range 0–350 MeV, and the most recent VPI analyses (SM97, SP99), which analyzed data up to 2500 MeV. Similar quality of agreement is observed for the updated Reid potential (Reid93) constructed by the Nijmegen group.

The limit on the fractional background in the final data is found to be <0.5%. As the normalization is based on measurements with $x$ and $y$ orientation of the holding field, and the angular distribution for $A_{zz}$ is determined from the measurement with the $z$ orientation, only a dependence of the background on the holding field orientation would affect the results. Within the 0.1% statistical uncertainty in the determination of the fractional background no dependence on the holding field direction was found.

The effect of the background on the final results for $A_{zz}$ was estimated from a simulation of yields with and without added background, and the effect was found to be less than 10% of the statistical errors. No correction to the results was applied.

VII. COMPARISON TO THEORY

In Ref. [1], angular distributions of the analyzing power $A_y$ and three spin correlation coefficients ($A_{xx}, A_{yy}, A_{xz}$)
scale the calculations by the appropriate factors, since the scaled data points would be different for each calculation. The figure shows that the most recent phase shift analyses, as well as the Nijmegen version of the Reid potential, are in excellent agreement with the measurements, when the scale factors listed in Table II are applied.

The analysis was repeated to include the previous data [1] on \(A_x\) and \(A_{ik}\) at the same energy. In accordance with Sec. V, the published values of \(A_x\) and \(A_{ik}\) were divided by 0.998 and 0.996, respectively. The last two columns of Table II give the overall \(\chi^2\) per degree of freedom (\(\chi^2 / \nu\)) if the measured \(A_x\) and all \(A_{ik}\) are multiplied by factors \(k_x'\) and \(k_{ik}'\), respectively. Good overall agreement is found for the Reid93 potential (\(\chi^2 / \nu = 1.24\)) and for the most recent phase shift analyses Ni97 (\(\chi^2 / \nu = 1.12\)) and SP99 (\(\chi^2 / \nu = 1.51\)), where the new SP99 analysis by the GW/NPI group already took advantage of the present \(A_{zz}\) data. For SP99 in particular, the agreement with the present \(A_{zz}\) data is excellent. The agreement of these calculations with the \(A_{zz}\) reported here is shown on the right hand side of Fig. 6. According to Table II, the scale correction for the \(A_{ik}\) is around 2\% for Reid93 and Ni97 and SP99, which is compatible with the experimental scale uncertainty for the \(A_{ik}\) of [1] of 1.4\% and the \(A_{zz}\) of 2\% (Sec. V).

It should be mentioned that the above comparisons have the defect that the same scale factor was applied to \(A_{zz}\) and to the \(A_{ik}\) of [1], while indeed it was pointed out in Sec. V that the component of the scale uncertainty that arises from the \(A_{zz}\) comparison is different for \(A_{zz}\) and the other \(A_{ik}\).

VIII. CONCLUSIONS

A beam of polarized protons whose polarization in the target region is along the beam direction was developed at the IUCF Cooler synchrotron. Since the spin precesses in the bending magnets, stable longitudinal polarization in the target straight section required the introduction of solenoids. Limitations of the available solenoid strength caused a deviation from the ideal longitudinal polarization, but the remaining transverse beam polarization components are small and easily taken into account in the data analysis. We believe this is the first time that stable longitudinal beam polarization has been used for a nuclear physics measurement in a proton storage ring.

The 197.4 MeV beam was incident on a polarized H gas target, whose polarization was changed in 2 s intervals between six different orientations (\(\pm x, \pm y, \pm z\)) by changing a weak guide field over the target. Longitudinal target polarization allowed the determination of the spin correlation parameter \(A_{zz}\). Elastically scattered protons were detected in coincidence in silicon-strip recoil detectors and in scintillators and wire chambers in forward direction. The effect of background events was investigated and found negligible. The only significant correction was for deadtime losses, which are spin dependent because of count rate changes between parallel and antiparallel beam and target spins.

Measurements of \(A_{zz}\) were obtained for laboratory angles between 5.5° and 43.5° (\(\theta_{c.m.} = 11.5° – 90°\)) in 1° intervals with a statistical error of about 0.02. Except for a limited amount of data at 305 MeV [19] in a narrow angular range near \(\theta_{c.m.} = 90°\) this is the only \(A_{zz}\) data in \(pp\) scattering below the pion threshold.

The identity \(A_{yy} – A_{xx} = A_{zz} = 1\) at \(\theta_{c.m.} = 90°\) is exploited to check the absolute calibration of earlier spin correlation measurements by our group [1] and to provide an improved absolute calibration of the data. In order to relate the product \(P Q\) of beam and target polarization in the present experiment to the \(P Q\) calibration in the determination of \(A_{xx}\) and \(A_{yy}\) in Ref. [1], measurements here were taken at the same time with transverse target polarization. This allowed relating the calibration in the two experiments via the common measurement of the spin correlation parameter \(A_{zz}\). The result determined the absolute calibration of the present \(A_{zz}\) angular distribution to an accuracy of \(\pm 2.0\%\). The above identity also allows a recalibration of the absolute normalization for the results of Ref. [1]. The new calibration would multiply the \(A_y\) by 1.002 and the \(A_{ik}\) by 1.004, which is well within the uncertainties reported in Ref. [1].

In addition to the inherent interest in measuring \(A_{zz}\) to complete the entire set of independent spin correlation parameters in \(pp\) elastic scattering at 197.4 MeV, the measurements have particular significance since they strengthen the absolute polarization calibration over the entire energy range from 200 to 450 MeV [20], which was based on exporting the 200 MeV calibration to the higher energies.

ACKNOWLEDGMENTS

We are grateful for the efforts of the accelerator operation group at IUCF, in particular D. Friesel and T. Sloan. This work was supported in part by the National Science Foundation and the Department of Energy. One of us (F.R.) would also like to thank the Alexander von Humboldt Foundation for their generous support.


A Region Layout

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Target cell feed tube.</td>
</tr>
<tr>
<td>b</td>
<td>Spin separation sextupole magnets, used to select a polarization state for the target beam.</td>
</tr>
<tr>
<td>c</td>
<td>Medium field magnet. This magnet induces a hyperfine transition in the atomic beam which reduces the beam intensity but boosts the target polarization by a factor of two.</td>
</tr>
<tr>
<td>d</td>
<td>The target cell - 25 cm long and 10x10 mm in cross section, it confines the ABS beam and effectively increases the target thickness by a factor of 100.</td>
</tr>
<tr>
<td>e</td>
<td>Helmholtz coils used to align the target polarization in a user-selectable direction.</td>
</tr>
<tr>
<td>f</td>
<td>The Pintex detector stack.</td>
</tr>
</tbody>
</table>

Last modified: Mon May 13 20:10:59 1996
The Pintex detector system has several parts:

<table>
<thead>
<tr>
<th>Label</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1-R8</td>
<td>R1-R8 are silicon strip detectors, each with a surface area of 4x8 centimeters. Each detector has 28 position sensitive strips 2mm wide. These detectors also give an energy signal. They are used to provide a coincidence with a forward going elastically scattered proton.</td>
</tr>
<tr>
<td>S1-S4</td>
<td>S1-S4 are four plastic scintillators placed symmetrically around the Phi = 45,135,225 and 315 degree points. Each detector covers scattering angles from 30 to 60 degrees in the lab. This set of detectors is used in conjunction with the XY wire chamber to measure pp elastic scattering at lab angles near 45 degrees.</td>
</tr>
<tr>
<td>XY,UV</td>
<td>XY and UV are our wire chambers. The UV chamber is rotated 45 degrees relative to the XY chamber to resolve the &quot;left-right&quot; ambiguity of wire hits in a single chamber. These two chambers are used to reconstruct the flight path of reaction protons so that their scattering angles can be determined.</td>
</tr>
<tr>
<td>E,K</td>
<td>E and K are segmented plastic scintillators. They have two functions. First, they are used to measure the energy of the reaction protons and second they are used in the trigger to enforce an &quot;opposite side requirement&quot; on allowed events, meaning that an event must fire a silicon detector and one of the E/K segments on the opposite side of the beam. This requirement reduces background levels significantly.</td>
</tr>
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</table>

Last modified: Mon May 13 20:11:19 1996
Pintex Forward Detector Stack for CE44

Last modified: March 14 1998

http://www.iucf.indiana.edu/experiments/PINTEX/apparatus/detector_ce44.html[10/16/2013 2:33:46 PM]
the Pintex Silicon Barrel for recoil detection CE80

Last modified: January 27 2000
Target Polarization Switch Time

The plot shows the target polarization along the X axis as a function of time. The horizontal axis is time in milliseconds and the plot shows a single transition of the target polarization from the +X to the -X direction. The complete switch occurs in approximately 50 milliseconds.

Last modified: Mon May 13 20:09:24 1996
ABS Target Thickness

This plot shows the ABS target thickness over an extended running period. The vertical axis is the target thickness, the horizontal is running time in days. Two data series are shown - the Event 6 data is forward angle pp elastic scattering and the Event 9 data is 45 degree pp elastic scattering. Both data sets were measured concurrently. Over the run period the target thickness averaged 3.1E13 atoms per square centimeter.

Last modified: Mon May 13 20:10:00 1996
ABS Target Polarization

The plot demonstrates the stability of the ABS target polarization during a week long experimental run. During this running period the target polarization remained constant.

Last modified: Mon May 13 20:09:41 1996
some pictures of the polmol target featuring a target cell and a recombiner cell made from copper with a small gate valve to open or close the recombiner

polmol target in beam direction
polmol target in upstream direction
polmol target top view
polmol target: yet another view
polmol target: and yet another