QCD evolution of TMDs: what works?

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Goal: find a universal formalism which works for all the world data on SIDIS, DY, W/Z, e+e-
Why QCD evolution is needed

- Experiments are operated in different energy and kinematic regions, to make reliable predictions, one has to take into account these differences
  - $Q$ is different: $Q \sim 1 - 3$ GeV in SIDIS, $Q \sim 10$ GeV at $e^+e^-$, $Q \sim 4 - 90$ GeV for DY, $W/Z$
  - Also $\sqrt{s}$ dependence is important

- We use the energy evolution equation for the relevant parton distribution functions (PDFs) or fragmentation function (FFs) to account for the kinematic differences

Qiu-Zhang 1999
QCD evolution: meaning

- What is QCD evolution of TMDs anyway?
  - Evolution = include important perturbative corrections
  - One of the well-known examples is the DGLAP evolution of collinear PDFs, which lead to the scaling violation observed in inclusive DIS process
  - What it does is to resum the so-called single logarithms in the higher order perturbative calculations

\[
F_2^\text{em} \sim \log(x)
\]

\[
Q^2 \rightarrow \mu^2
\]

\[
\left( \alpha_s \ln \frac{Q^2}{\mu^2} \right)^n
\]
QCD evolution: TMDs

- TMD factorization works in the situation where there are two observed momenta in the process, such as SIDIS, DY, W/Z production and in the kinematic region where $Q \gg q_T$.
- Evolution again = include important perturbative corrections.
- What it does is to resum the so-called double logarithms in the higher order perturbative corrections.
- For SIDIS: $q_T$ is the transverse momentum of the final-state hadron.

\[ (\alpha_s \ln^2 \frac{Q^2}{q_T^2})^n \]
Many approaches for TMD evolution

- **Collins-Soper-Sterman (CSS) resummation framework**
  - Collins-Soper-Sterman 1985
  - ResBos: C.P. Yuan, P. Nadolsky
  - Qiu-Zhang 1999, Vogelsang ...

- **New Collins approach**
  - Aybat-Rogers 2011,
  - Aybat-Collins-Rogers-Qiu, 2012
  - Aybat-Prokudin-Rogers 2012

- **Soft Collinear Effective Theory (SCET)**
  - Echevarria-Idilbi-Schafer-Scimemi 2012

They are all consistent with each other perturbatively. However, they could have very different phenomenological predictions.
What the evolution looks like?

- We have a TMD distribution $F(x, k_\perp; Q)$ measured at a scale $Q$
  - It is easy to deal in the Fourier transformed space
    \[
    F(x, b; Q) = \int d^2 k_\perp e^{-i k_\perp \cdot b} F(x, k_\perp; Q)
    \]

- Standard CSS formalism tells us it evolves from an initial scale
  \[
  \mu_b = c/b, \quad c = 2e^{-\gamma_E} \sim O(1)
  \]
  \[
  F(x, b; Q) = F(x, b; c/b) \exp \left\{ -\int_{c/b}^{Q} \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\}
  \]
  \[
  A = \sum_{n=1} A^{(n)} \left( \frac{\alpha_s}{\pi} \right)^n, \quad B = \sum_{n=1} B^{(n)} \left( \frac{\alpha_s}{\pi} \right)^n
  \]
  \[
  A^{(1)} = C_F
  \]
  \[
  A^{(2)} = \frac{C_F}{2} \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R n_f \right]
  \]
  \[
  B^{(1)} = -\frac{3}{2} C_F
  \]
Connection to other approaches: new Collins

- Derive new Collins evolution from CSS

\[
F(x, b; Q_f) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\}
\]

\[
F(x, b; Q_i) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^{Q_i} \frac{d\mu}{\mu} \left( A \ln \frac{Q_i^2}{\mu^2} + B \right) \right\}
\]

\[
F(x, b; Q_f) = F(x, b; Q_i) \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left( \frac{Q_f^2}{Q_i^2} \right) - \int_{c/b}^{Q_i} \frac{d\mu}{\mu} A
\]

- This is the same as in SCET
Connection to other approaches: Sun-Yuan

- To derive Sun-Yuan evolution kernel, choose lowest order $A^{(1)}$, $B^{(1)}$

\[ I(b, Q_i, Q_f) = \left( \frac{Q_f^2}{Q_i^2} \right) \int_{c/b}^{Q_i} \frac{d\mu}{\mu} C_F \frac{\alpha_s(\mu)}{\pi} \]

\[ = \text{Exp} \left[ \ln \left( \frac{Q_f^2}{Q_i^2} \right) \int_{c/b}^{Q_i} \frac{d\mu}{\mu} C_F \frac{\alpha_s(\mu)}{\pi} \right] \]

\[ \ln \left( \frac{Q_f^2}{Q_i^2} \right) = 2 \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \]

Ignore the scale-dependence in coupling constant

\[ \int_{c/b}^{Q_i} \frac{d\mu}{\mu} C_F \frac{\alpha_s(\mu)}{\pi} = C_F \frac{\alpha_s}{\pi} \int_{c/b}^{Q_i} \frac{d\mu}{\mu} = \frac{1}{2} C_F \frac{\alpha_s}{\pi} \ln \left( \frac{Q_i^2 b^2}{c^2} \right) \]

Now you “might” put the scale back into coupling constant

\[ I(b, Q_i, Q_f) = \text{Exp} \left[ C_F \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \frac{\alpha_s}{\pi} \ln \left( \frac{Q_i^2 b^2}{c^2} \right) \right] \]

- In general, Sun-Yuan could be different from new Collins evolution
  - perturbatively at $O(\alpha_s)$ they are exactly the same
  - should work fine for not-too-large $Q$ range
What’s the complication in QCD evolution?

- So far the evolution kernel is calculated in perturbation theory, so valid only for small b region:

\[
F(x, b; Q_f) = F(x, b; Q_i) \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left( \frac{Q_f^2}{Q_i^2} \right) - \int_{c/b}^{Q_i} \frac{d\mu}{\mu} A
\]

- Fourier transform back to the momentum space, one needs the whole b region (also large b): need some non-perturbative extrapolation

\[
F(x, k_\perp; Q) = \frac{1}{(2\pi)^2} \int d^2 b e^{i k_\perp \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^{\infty} db b J_0(k_\perp b) F(x, b; Q)
\]

- Widely used prescription (CSS):

\[
F(x, b; Q_f) = F(x, b; Q_i) R_{\text{pert}}(b_*, Q_i, Q_f) \times R_{\text{NP}}(b, Q_i, Q_f)
\]

\[
b_* = b/\sqrt{1 + (b/b_{\text{max}})^2}
\]
Choose some Gaussian form for TMDs at initial scale $Q_0$, then evolve to $W/Z$ scale, to see if it describes the pt distribution

- It does not (use a reasonable $b_{\text{max}}$). It always leads to a rather flat pt distribution: the integrand in $b$-space is almost a delta-function concentrated at $b=0$
- It will then lead to a rather flat pt distribution: curvature much smaller than data
Use conventional CSS formalism

- In the conventional CSS formalism, one further calculate TMD at c/b scale in terms of collinear PDFs

\[ F(x, b; Q) = F(x, b; c/b) \exp \left\{- \int_{c/b}^{Q} \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \]

- At LO, we have \( F(x, b; c/b) = F(x, \mu = c/b) \)

- Choose non-perturbative Sudakov function

\[ F(x, b; Q) = F(x, c/b_\ast) R^{pert}(Q, b_\ast) R^{NP}(Q, b) \]

\[ R^{NP}(Q, b) = \exp(-S^{NP}) \]

- Typical simplest form for unpolarized PDF and FF

\[ S^{NP}_{pdf} = b^2 \left[ g^{pdf}_1 + \frac{g_2}{2} \ln(Q/Q_0) \right] \]

\[ S^{NP}_{ff} = b^2 \left[ g^{ff}_1 z^2 + \frac{g_2}{2} \ln(Q/Q_0) \right] \]
Intuitive meaning of these parameters

- Let us understand these parameters

\[
S_{pdf}^{NP} = b^2 \left[ g_1^{pdf} + \frac{g_2}{2} \ln(Q/Q_0) \right] \quad S_{ff}^{NP} = b^2 \left[ g_1^{ff} / z^2 + \frac{g_2}{2} \ln(Q/Q_0) \right]
\]

- \( g_1^{pdf} = \langle k_{\perp}^2 \rangle / 4 \) intrinsic transverse momentum width for PDFs at scale Q0
- \( g_1^{ff} = \langle p_{\perp}^2 \rangle / 4 \) intrinsic transverse momentum width for FFs at scale Q0
- \( g_2 \) mimic the increase in the width observed by the experiments
  - large Q leads to more shower

- Sivers asymmetry is very sensitive to g2 (though the Drell-Yan unpolarized cross section is not)
  - Choose a wrong g2 leads to very different result
Tune the parameters to describe all data

- Now we will try to tune these parameters to describe all the world data for pt distribution for SIDIS, DY, W/Z at all energies
  - Let us choose $Q^2_0 = 2.4 \text{ GeV}^2$, i.e., the HERMES scale
  - At this scale, the intrinsic transverse momentum width is already extracted by different group: there are some freedom

$$\langle k^2_T \rangle = 0.25 - 0.44 \text{ GeV}^2 \quad \langle p^2_T \rangle = 0.15 - 0.2 \text{ GeV}^2$$

arXiv:1003.2190 & Torino
Finding a way to describe both SIDIS and DY/WZ

- Study unpolarized cross section, and pin-down $g_2$
  - Slightly adjust $g_2$ (within their fitted uncertainty) such that non-perturbative Sudakov can predict $\langle k_\perp^2 \rangle$ at HERMES
  - Once this is fixed, adjust $\langle p_T^2 \rangle$ such that it gives a good description of SIDIS

\[ g_2 = 0.184 \pm 0.18 \]

\[ a_2 = 0.19 \text{ GeV}^2 \]

\[ b_{\text{max}} = 1.5 \text{ GeV}^{-1} \]

\[ \langle k_\perp^2 \rangle = 0.38 \text{ GeV}^2 \]

\[ \langle p_T^2 \rangle = 0.19 \text{ GeV}^2 \]
This actually works

Description of W/Z data at Tevatron and LHC
Drell-Yan lepton pair production

$E_d^3 p (\text{pb}/\text{GeV}^2)$

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Multiplicity distribution in SIDIS 1
Comparison with COMPASS data

![Graphs showing multiplicity distribution in SIDIS with COMPASS data.](image)

FIG. 2. The comparison with the COMPASS data (deuteron target) at $\langle Q^2 \rangle = 7.57$ GeV$^2$ and $\langle x_B \rangle = 0.093$. The data points from top to bottom correspond to different $z$ region: [0.2, 0.25], [0.25, 0.3], [0.3, 0.35], [0.35, 0.4], [0.4, 0.5], [0.5, 0.6], [0.6, 0.7], and [0.7, 0.8].
Multiplicity distribution in SIDIS 2

- Comparison with HERMES data

![Graphs showing multiplicity distribution in SIDIS 2 with HERMES data for different $p_T$ values.](image)

**FIG. 1.** The comparison with the HERMES data (proton target) [6]. The data points from top to bottom correspond to different $z$ region: $[0.2, 0.3]$, $[0.3, 0.4]$, $[0.4, 0.6]$, and $[0.6, 0.8]$. 
Sivers effect

- Now let us try to use the same formalism to describe Sivers effect

\[
F(x, b; Q) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^{Q} \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\}
\]

- Now \( F(x, b; Q) \) is given by

\[
f_{1T}^{q(\alpha)}(x, b; Q) = \frac{1}{M} \int d^2 k_\perp e^{-i k_\perp \cdot b} k_\perp^\alpha f_{1T}^{\perp q}(x, k_\perp^2; Q)
\]

- The perturbative expansion gives Qiu-Sterman function

\[
f_{1T}^{q(\alpha)}(x, b; \mu = c/b) = \left( \frac{-ib^\alpha}{2} \right) T_{q,F}(x, x, \mu = c/b)
\]
Fitting parameters

- Similar form for non-perturbative Sudakov factor (note: g2 is spin-independent, so use the same g2)

\[ S_{NP}^{sivers}(b, Q) = b^2 \left[ g_1^{sivers} + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right] \quad g_1^{sivers} = \frac{(k_{s\perp})_1}{4} \]

- Intrinsic kt-width for Sivers has to be fitted
- x-dependence has to be fitted

- Qiu-Sterman function

\[ T_q,F(x, x, \mu) = N_q \frac{(\alpha_q + \beta_q)(\alpha_q + \beta_q)}{\alpha_q \beta_q} x^{\alpha_q} (1 - x)^{\beta_q} f_{q/A}(x, \mu) \]

- Total parameters (11):

\[ \langle k_{s\perp}^2 \rangle, N_u, N_d, N_{\bar{u}}, N_{\bar{d}}, N_s, N_{\bar{s}}, \alpha_u, \alpha_d, \alpha_{sea}, \beta \]
Fitted results

- COMPASS proton
HERMES Kaons

HERMES Proton

\[ A_{\text{UT}}^{\pi^0} \]

\[ A_{\text{UT}}^{\pi^-} \]

\[ A_{\text{UT}}^{\pi^+} \]

\[ A_{\text{UT}}^{\pi^+} \]

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Fitted Qiu-Sterman function

- $\chi^2/d.o.f = 1.3$: slightly larger than the usual Gaussian fit. Feel more confident when extrapolated to the whole $Q$ range.
Some predictions for asymmetries of DY and W

- At 510 GeV RHIC energy (DY: pt [0,1], Q [4,9]  W: pt [0,3] GeV)
Summary

- Perturbatively, the QCD evolution kernel for TMDs are the same in all existing approaches.

- The difficult on the QCD evolution comes from pinning down the non-perturbative part, which has to be fitted from experimental data.

- We find some simple non-perturbative form, which can describe all the data on SIDIS, DY, W/Z production.

- Use the same non-perturbative form, we extract the Sivers function and predict the asymmetry for DY and W production at RHIC energy.
Summary

- Perturbatively, the QCD evolution kernel for TMDs are the same in all existing approaches.

- The difficult on the QCD evolution comes from pinning down the non-perturbative part, which has to be fitted from experimental data.

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Thank you
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- spin physics
- small-x
- SCET
- lattice QCD
- pQCD
- EIC physics
- ......

http://www.jlab.org/conferences/qcd2014/