TMD evolution of Collins asymmetries in e^+e^- annihilation

Peng Sun

LBNL

in collaboration with C. P. Yuan and F. Yuan

Outlines

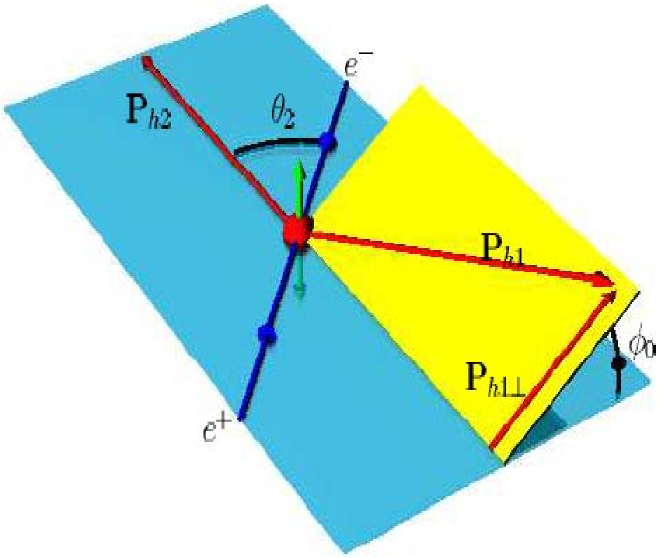
- Energy evolution in TMD factorization (e^+e^- annihilation) Sudakov factor

- Nonperturbative Sudakov factor fitting from Drell-Yan and SIDIS processes

- Collins asymmetry at BELLE and BABAR

- Summary

Collins asymmetries in e^+e^- annihilation



For the process

$$e^+ + e^- \to H_1 + H_2 + X$$

The cross section can be written as

$$\frac{d\sigma}{dz_{h1}dz_{h2}d^2P_{h\perp}d\theta} = \frac{2\pi N_c\alpha^2}{4Q^2}\left[\left(1+\cos^2\theta\right)Z_{uu} + \sin^2\theta\left(2\hat{e}_x^\alpha\hat{e}_x^\beta - g_\perp^{\alpha\beta}\right)Z_{collins}^{\alpha\beta}\right]$$

$\Downarrow$ $\cos(2\phi_0)$

Boer, Jakob, Mulders, 1998
Boer, 2001, 2009

Energy Evolution (Ji-Ma-Yuan TMD factorization)

At the small transverse momentum, the TMD factorization

It ignores all higher order of Q_t / Q

$$\widetilde{Z}_{uu} = D(z_1, b_\perp, \zeta_1; \mu) D(z_2, b_\perp, \zeta_2; \mu) H_{uu}^{e^+e^-}(Q;\mu) S(b_\perp, \rho; \mu) ,$$

$$\widetilde{Z}^{\alpha\beta}_{\rm collins} = \widetilde{H}_1^{\perp\alpha}(z_1, b_\perp, \zeta_1; \mu) \widetilde{H}_1^{\perp\beta}(z_2, b_\perp, \zeta_2; \mu) H^{e^+e^-}_{\rm collins}(Q;\mu) S(b_\perp, \rho; \mu)$$

$\zeta^2/\rho = Q^2 \times z^2$

Q_t

Fourier transformation

factorization scale

Z_{uu} and $Z_{collins}$ satisfy CSS evolution equation

$$\frac{\partial}{\partial \ln Q^2} \widetilde{Z}_{uu}(Q;b) = (K(b,\mu) + G(Q,\mu)) \, \widetilde{Z}_{uu}(Q;b)$$

At one-loop order

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{b^2\mu^2}{c_0^2} \qquad G(Q,\mu) = -\frac{\alpha_s C_F}{\pi} \left(\ln \frac{Q^2}{\mu^2} - \frac{3}{2} \right)$$

Substituting the above result into the evolution equation, and taking into account the running effects in K(b, μ)

CSS ⇒ $\text{Exp}\left[\int_{c_0/b}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \left[A \ln \frac{Q^2}{\bar{\mu}^2} + B \right] \right]$

$$\widetilde{Z}_{uu}(Q;b) = e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}^{e^+e^-}(Q,b)} \Sigma_q D_q(z_1, C_0/b_*) D_{\bar{q}}(z_2, C_0/b_*) ,$$

$C_0 = 2\, e^{-\gamma} \approx 1$

$$\widetilde{Z}^{\alpha\beta}_{\text{collins}}(Q;b) = \left(\frac{-ib^\alpha_\perp}{2}\right)\left(\frac{-ib^\beta_\perp}{2}\right) e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{\text{collins}}^{e^+e^-}(Q,b)} \Sigma_q \hat{H}_{1q}(z_{h1}, C_0/b_*) \hat{H}_{1\bar{q}}(z_{h2}, C_0/b_*)$$

$$e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{\text{collins}}^{e^+e^-}(Q,b)}$$

→ NP part

perturbative part:

$$\mathcal{S}_{pert}(Q,b_*) = \int_{c_0/b_*}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \left[A \ln \frac{Q^2}{\bar{\mu}^2} + B \right]$$

where $A = C_F \times \alpha_s(\bar{\mu})/\pi$, $B = 3/2 \times \alpha_s(\bar{\mu})/\pi$

S_{pert} is universal

b_* prescription (CSS, 85) in S_{pert}

$$b \Rightarrow b_* = b/\sqrt{1+b^2/b_{max}^2} \; , \quad b_{max} < 1/\Lambda_{QCD}$$

Sudakov factor

- There are two parts in the Sudakov factor

$$S_{sud} \Rightarrow S_{pert}(Q;b_*) + S_{NP}(Q;b)$$

- the nonperturbative part

$$S_{NP}(Q,b) = g_2(b)\ln Q + g_1(b;z_1,z_2)$$
$$S_{NP}^T(Q,b) = g_2(b)\ln Q + g_1^T(b;z_1,z_2)$$

This term is from

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi}\ln\frac{b^2\mu^2}{c_0^2}$$

$\mu = c_0/b^*$

Q dependence always satisfies CSS equation.

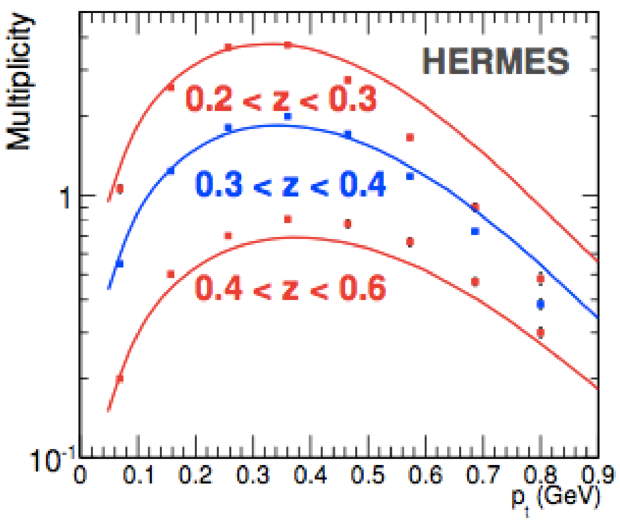
$g_2(b)$ is universal in Drell-Yan, SIDIS, and $e^+e^- \to hh$

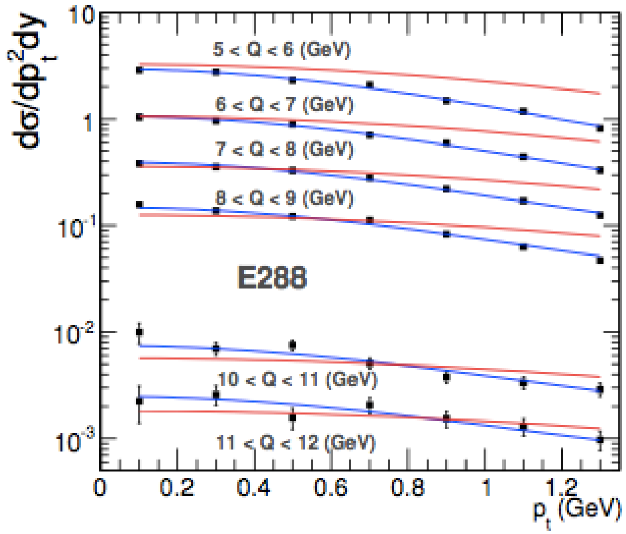
- Gaussian assumptions are usually made for $g_1(b)$ and $g_2(b)$ (BLNY 2002):

$$
\begin{aligned}
S_{NP}^{DIS} &= g_q b^2 \ln(Q/Q_0) + g_0 b^2 + g_h b^2/z_h^2 \ , \\
S_{NP}^{DY} &= g_q b^2 \ln(Q/Q_0) + 2 g_0 b^2 \ , \\
S_{NP}^{e^+e^-} &= g_q b^2 \ln(Q/Q_0) + g_h b^2 \left(1/z_{h1}^2 + 1/z_{h2}^2\right)
\end{aligned}
$$

- However, these assumptions do not work for SIDIS and Drell-Yan simultaneously in the range of Q^2~(3—100)GeV^2
  - in particular, for Q_0^2=2.4GeV^2, fitting Drell-Yan data leads to a negative g_0

Sun,Yuan,1308.5003





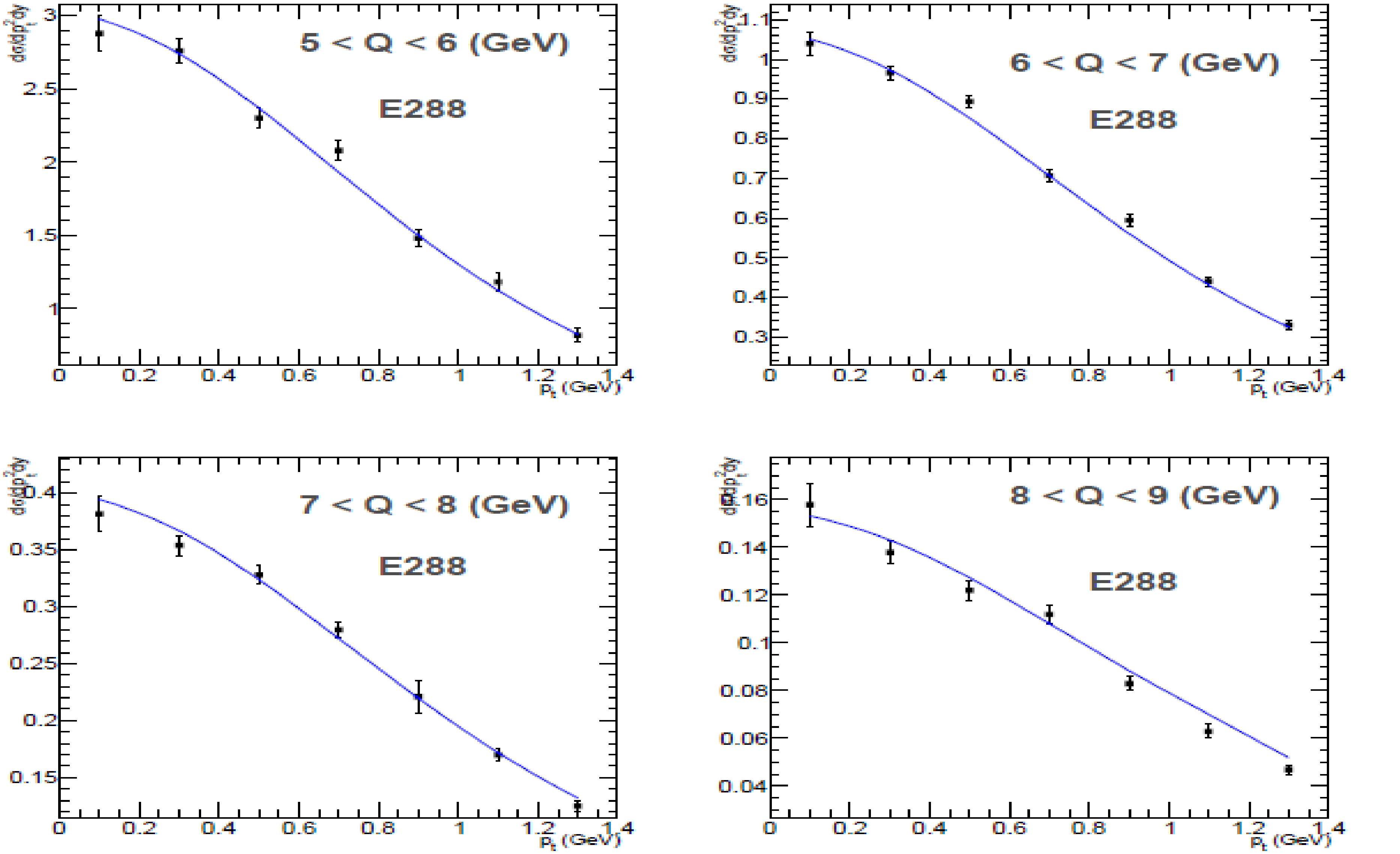
CT10 and DSS are used here, so are our other fittings.

Sun,Yuan, 1304.5037, 1308.5003

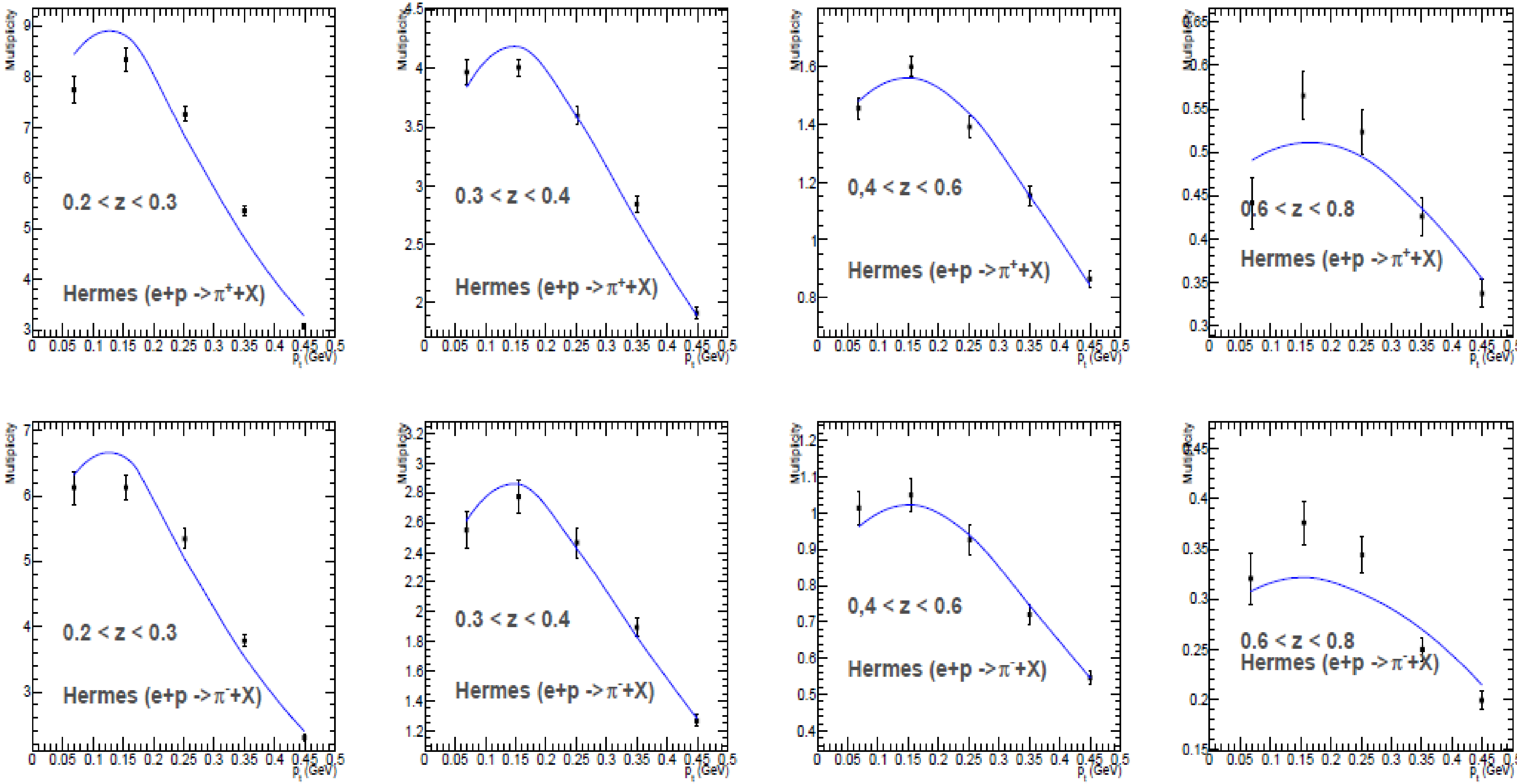
- Sun-Yuan (1308.5003) has shown that direct integration of the evolution kernel from low to high Q can describe both Drell-Yan and SIDIS data
  - This suggests that Log(b) maybe a good choice for $g_2(b)$.

With $g_2(b) = g_q \mathrm{Log}(b/b_*)$ and $g_1(b)=g_0\, b^2$, we refit the Drell-Yan data

$g_q = 1.3$, $g_0=0.08$, $Q_0^2=2.4\mathrm{GeV}^2$



SIDIS from HERMES

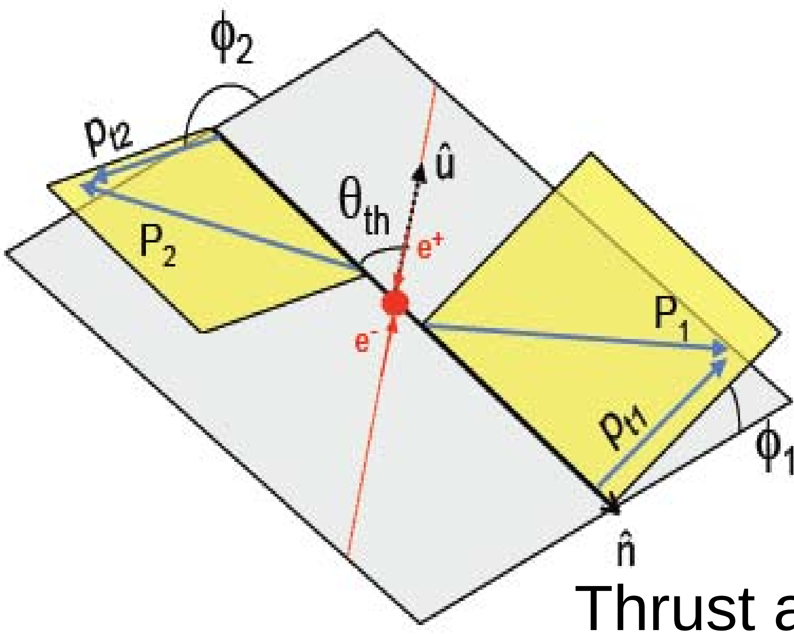


g_h is fitted to the HERMES data: $g_h = 0.038$

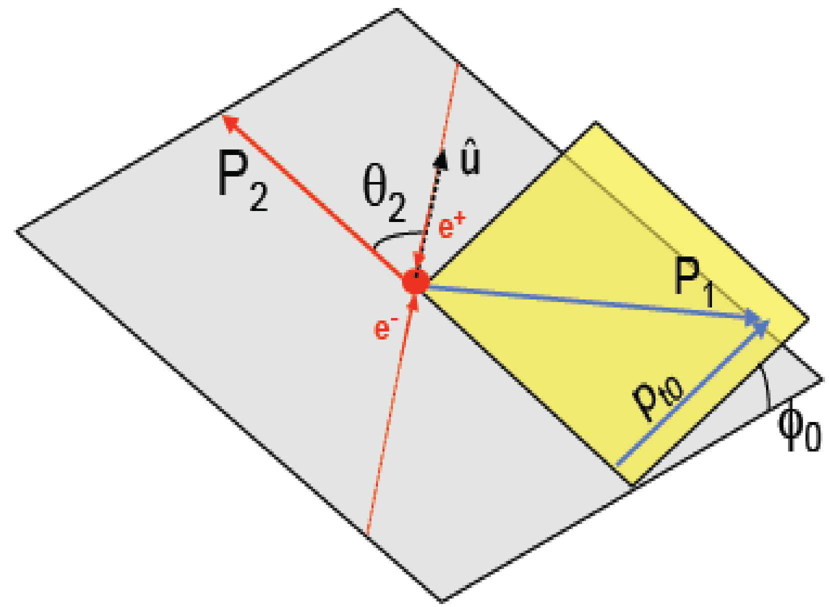
- So we have found a good NP form factor which can describe the data for the Q^2 from 3 to100GeV^2

- Now we can study the TMD evolution for the Collins asymmetries in e^+e^- annihilation and SIDIS

Collins asymmetries in $e^+e^- \rightarrow hh+X$ from BELLE and BABAR



Thrust axis



A_{12} A_0

The Collins asymmetries is proportional to $\cos(\phi_1+\phi_2)$ or $\cos(2\phi_0)$

- Besides Collins effect, the gluon radiation effect can also contribute $\cos(\phi_1+\phi_2)$ or $\cos(2\phi_0)$ terms

$$R_{12} = \frac{N(\phi_1+\phi_2)}{\langle N_{12}\rangle}$$

$$\propto \left[(1+\cos^2\theta)\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2) + \sin^2\theta\cos(\phi_1+\phi_2)\left[\sum_q e_q^2 f(H_1^\perp(z_1)\overline{H}_1^\perp(z_2)) + C\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)\right]\right] \cdot \left[(1+\cos^2\theta)\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)\right]^{-1}$$

$$= 1 + \frac{\sin^2}{1+\cos^2\theta}\cos(\phi_1+\phi_2)\left[\frac{\sum_q e_q^2 f(H_1^\perp(z_1)\overline{H}_1^\perp(z_2))}{\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)} + C\right] .$$

$$e^+e^- \to q\bar{q}g \to h_1h_2X$$

$$\frac{dN}{d\Omega} \propto \frac{Q_t^2}{Q^2+Q_t^2}$$

At BELLE and BABAR, they choose $Q_t < 3.5$GeV to suppress this background.

First, we define:

Valence quarks go to pion

$$N^U(\phi)=\frac{d\sigma(e^+e^-\to\pi^\pm\pi^\mp X)}{d\Omega dz_1 dz_2}\propto\frac{5}{9}D^{\mathrm{fav}}(z_1)\overline{D}^{\mathrm{fav}}(z_2)+\frac{7}{9}D^{\mathrm{dis}}(z_1)\overline{D}^{\mathrm{dis}}(z_2)$$

Sea quarks go to pion

$$N^L(\phi)=\frac{d\sigma(e^+e^-\to\pi^\pm\pi^\pm X)}{d\Omega dz_1 dz_2}\propto\frac{5}{9}D^{\mathrm{fav}}(z_1)\overline{D}^{\mathrm{dis}}(z_2)+\frac{5}{9}D^{\mathrm{dis}}(z_1)\overline{D}^{\mathrm{fav}}(z_2)+\frac{2}{9}D^{\mathrm{dis}}(z_1)\overline{D}^{\mathrm{dis}}(z_2)$$

$$N^C(\phi)=\frac{d\sigma(e^+e^-\to\pi\pi X)}{d\Omega dz_1 dz_2}=N^U(\phi)+N^L(\phi)\propto\frac{5}{9}[D^{\mathrm{fav}}(z_1)+D^{\mathrm{dis}}(z_1)][\overline{D}^{\mathrm{fav}}(z_2)+\overline{D}^{\mathrm{dis}}(z_2)]+\frac{4}{9}D^{\mathrm{dis}}(z_1)\overline{D}^{\mathrm{dis}}(z_2)$$

By a double ratio:

$$\frac{R^U_\alpha}{R^L_\alpha}:=\frac{N^U_\alpha(\beta_\alpha)/\langle N^U_\alpha\rangle}{N^L_\alpha(\beta_\alpha)/\langle N^L_\alpha\rangle},(\alpha=0,12)$$

$$\frac{R^U_{12}}{R^L_{12}}=1+\cos(\phi_1+\phi_2)\frac{\sin^2\theta}{1+\cos^2\theta}\left\{\frac{f\left(H_1^{\perp,fav}\overline{H}_2^{\perp,fav}+H_1^{\perp,dis}\overline{H}_2^{\perp,dis}\right)}{\left(D_1^{fav}\overline{D}_2^{fav}+D_1^{dis}\overline{D}_2^{dis}\right)}-\frac{f\left(H_1^{\perp,fav}\overline{H}_2^{\perp,dis}\right)}{\left(D_1^{fav}\overline{D}_2^{dis}\right)}\right\}$$

⇓

A^{UL}

Similarly, we can also get A^{UC} from the ratio R^U/R^C

$$A^{UL} \sim \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \frac{\pi \langle k_{tC}^2 \rangle}{4M^2} \left[\frac{H_1^{fav}\overline{H}_2^{fav} + H_1^{dis}\overline{H}_2^{dis}}{D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}} - \frac{H_1^{fav}\overline{H}_2^{dis} + H_1^{dis}\overline{H}_2^{fav}}{D_1^{fav}\overline{D}_2^{dis} + D_1^{dis}\overline{D}_2^{fav}} \right]$$

$$A^{UC} \sim \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \frac{\pi \langle k_{tC}^2 \rangle}{4M^2} \left[\frac{H_1^{fav}\overline{H}_2^{fav} + H_1^{dis}\overline{H}_2^{dis}}{D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}} - \frac{\left(H_1^{fav} + H_1^{dis}\right)\left(\overline{H}_2^{fav} + \overline{H}_2^{dis}\right)}{\left(D_1^{fav} + D_1^{dis}\right)\left(\overline{D}_2^{fav} + \overline{D}_2^{dis}\right)} \right]$$

Collins functions from BELLE and BABAR data

$$\hat{H}_{\pi^+/u}(z) = N_u z^{a_u}(1-z)^{b_u} D_{\pi^+/u}(z)$$
$$\hat{H}_{\pi^+/d}(z) = N_d z^{a_d}(1-z)^{b_d} D_{\pi^+/d}(z)$$

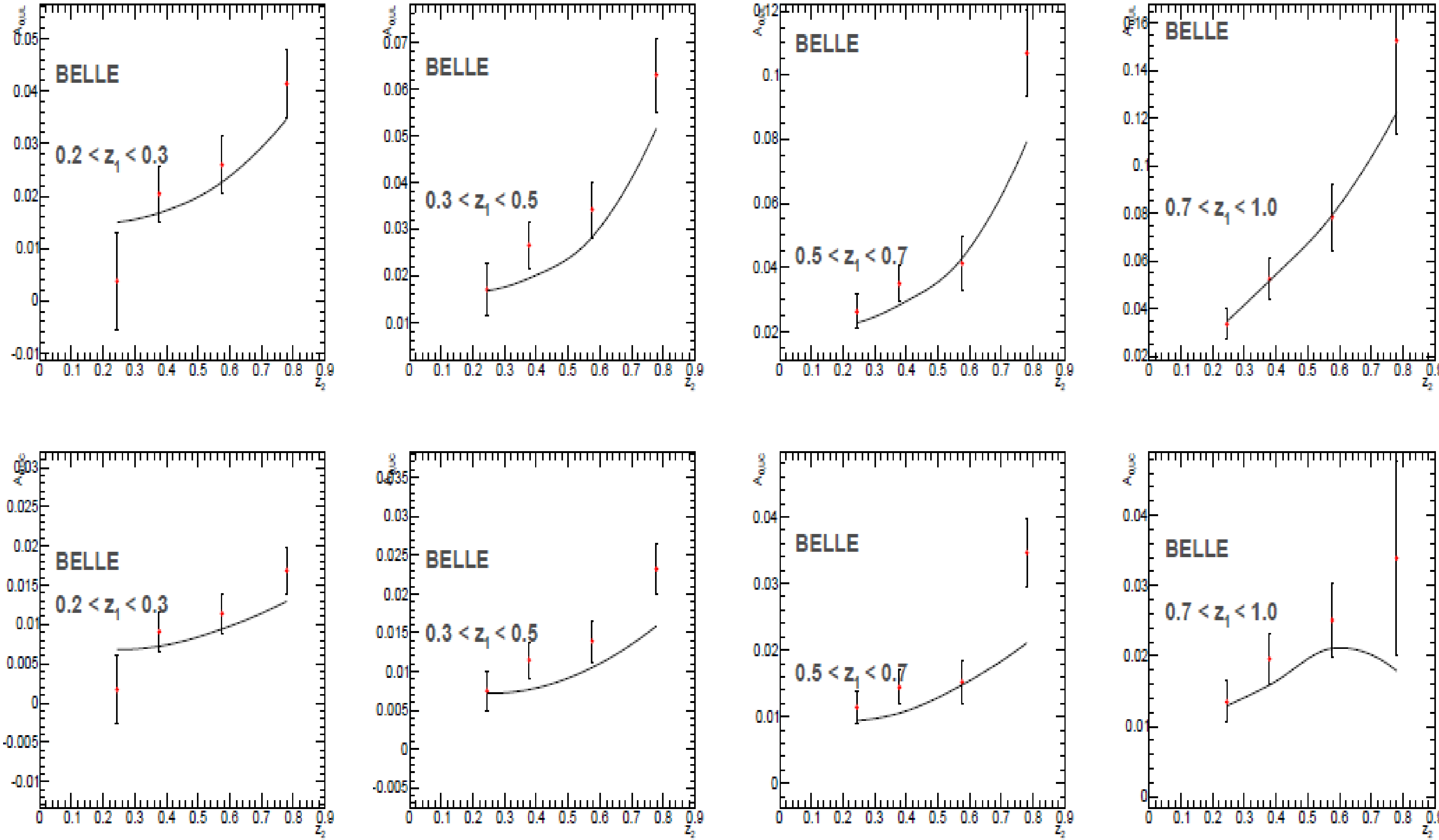
nonperturbative factor:

$$S^{e^+e^-}_{\text{collins}}(Q,b) = g_q \ln(Q/Q_0)\ln(b/b^*) + (g_h - g_c)b^2\left(1/z^2_{h1} + 1/z^2_{h2}\right)$$

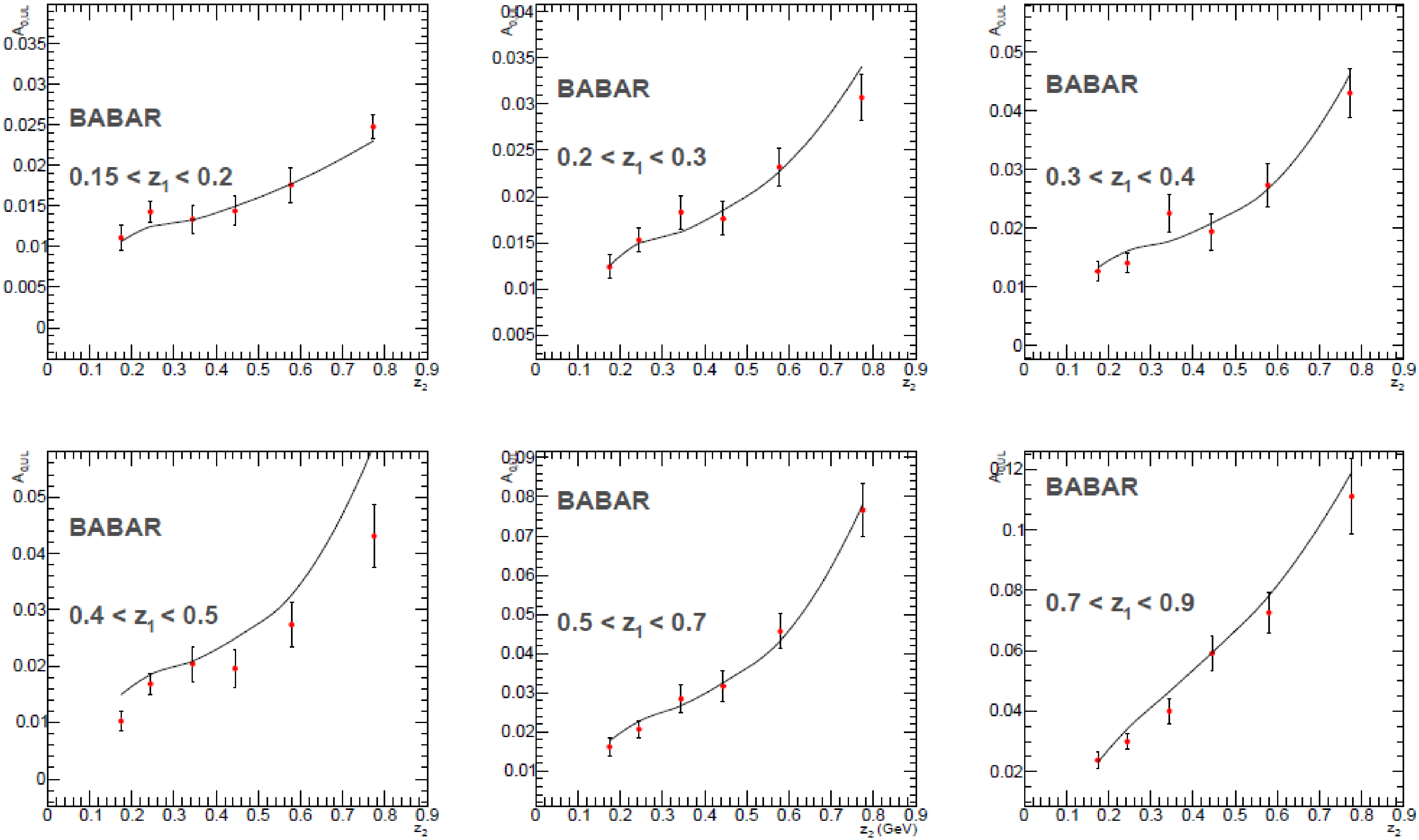
Minuit package is uesd

$\mathcal{X}^2 \approx 120$ vs 122 data points

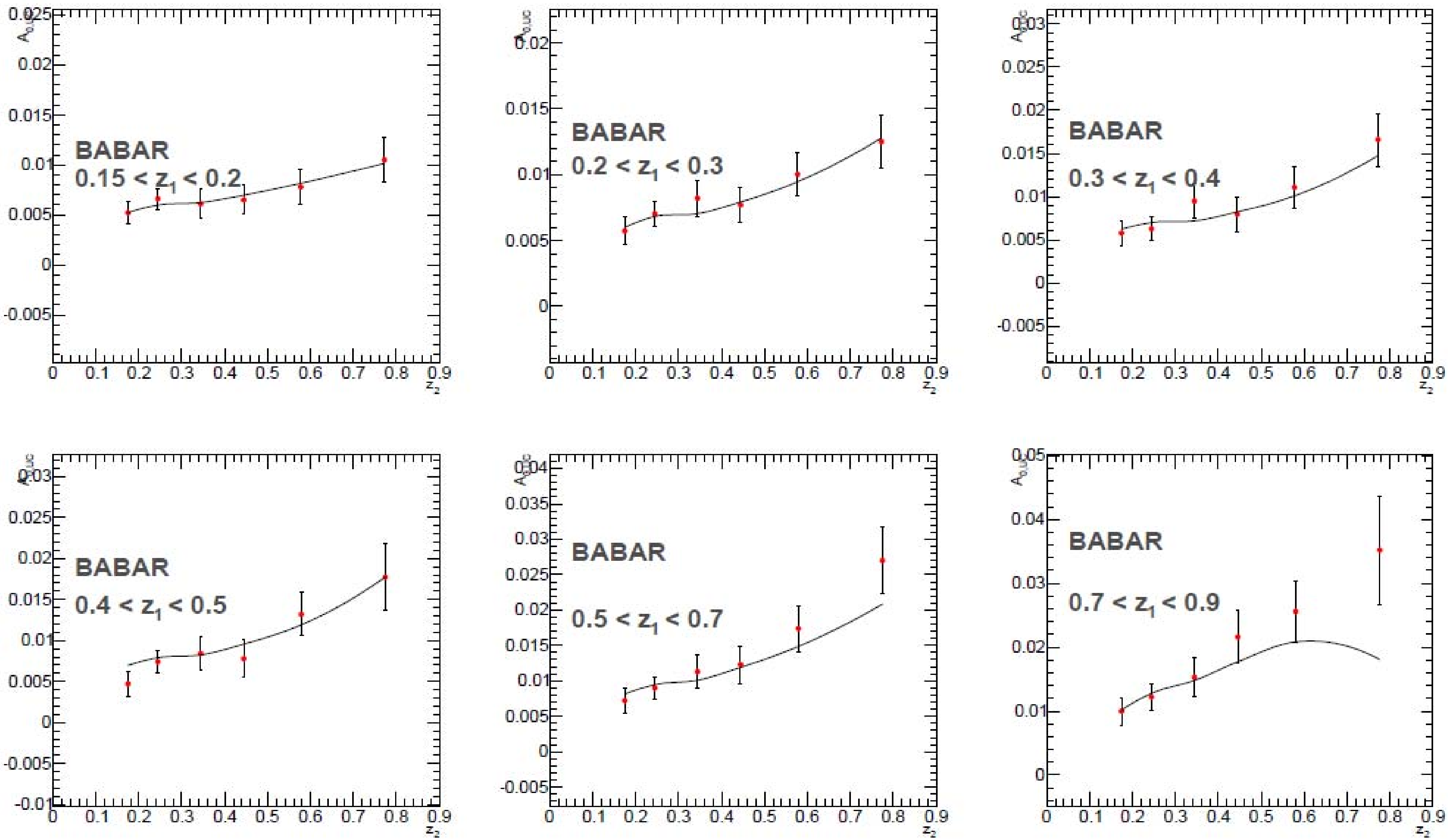
NO.	NAME	VALUE	ERROR
1	g_c	1.97919e-02	1.79702e-03
2	Nu	1.00000e+01	1.68748e+01
3	Nd	-1.53931e+00	1.22775e-01
4	a_u	7.96969e+00	4.45691e-01
5	a_d	1.43211e+00	7.83941e-02
6	b_u	1.18983e+00	6.43297e-02
7	b_d	7.20196e-09	1.94833e-01



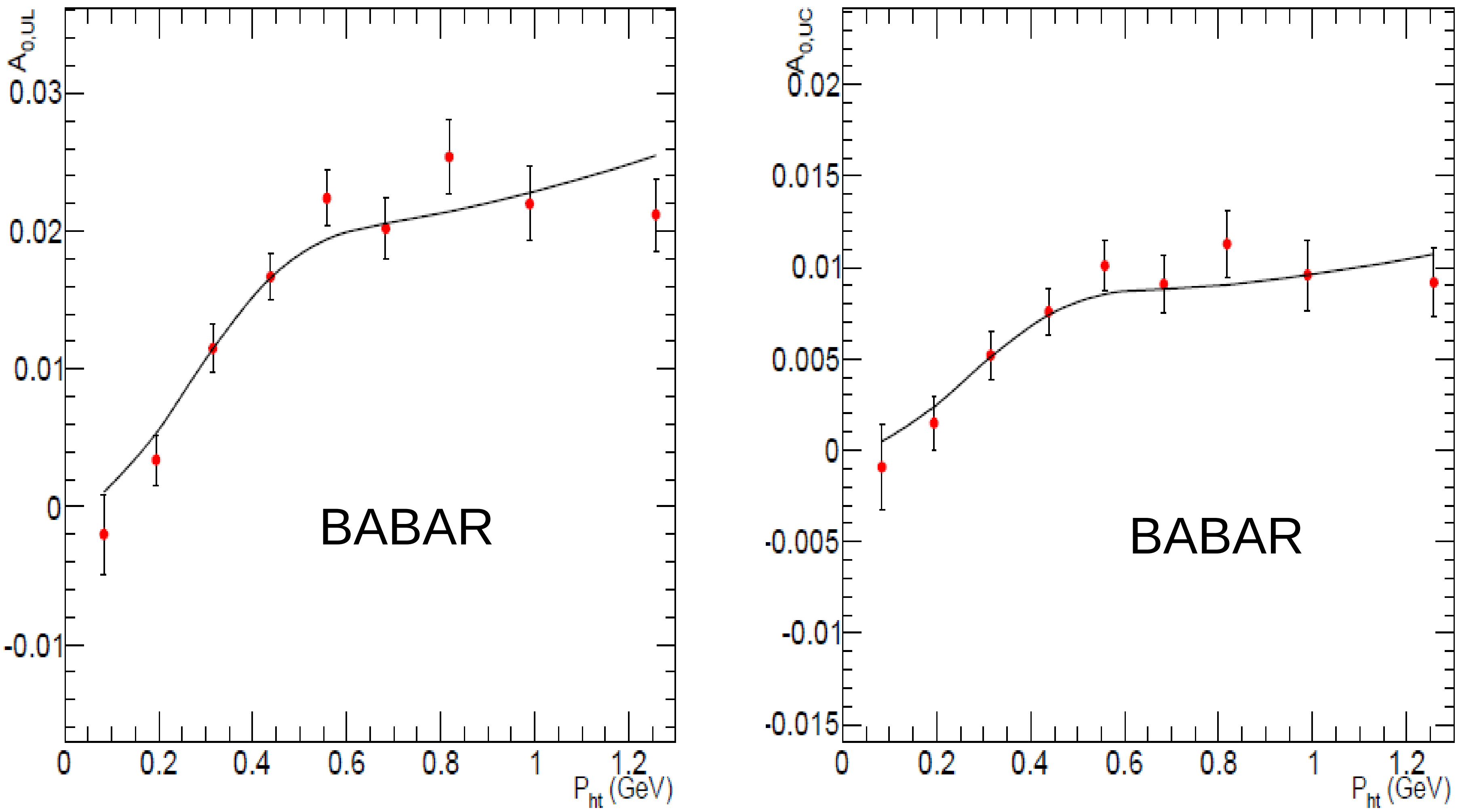
Our fitting result for BELLE.



Our fitting result for BABAR.

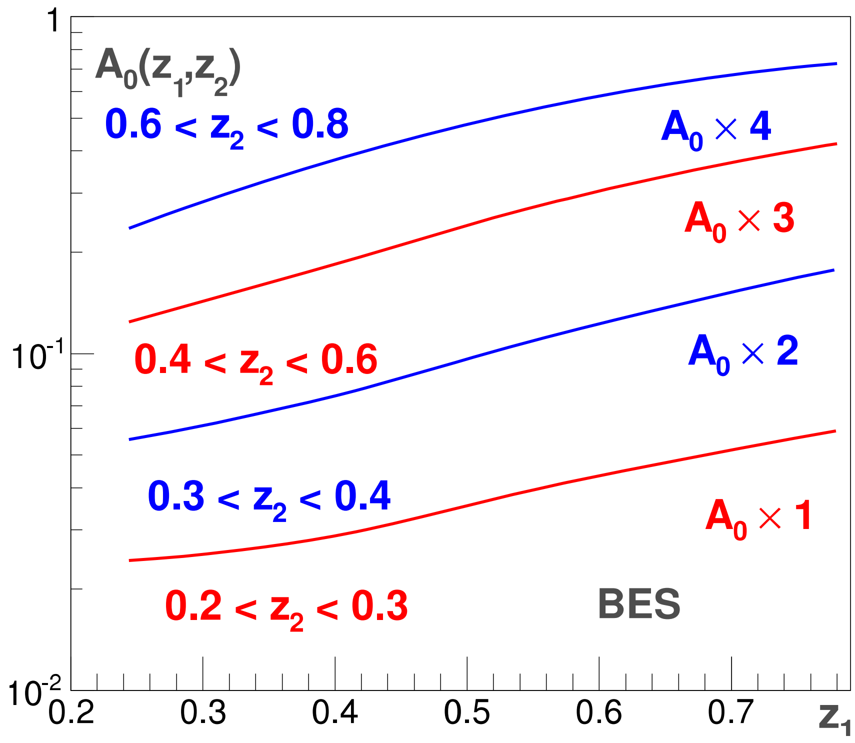


BABAR
0.15 < z1 < 0.2
BABAR
0.2 < z1 < 0.3
BABAR
0.3 < z1 < 0.4
BABAR
0.4 < z1 < 0.5
BABAR
0.5 < z1 < 0.7
BABAR
0.7 < z1 < 0.9
A0,UC
z2



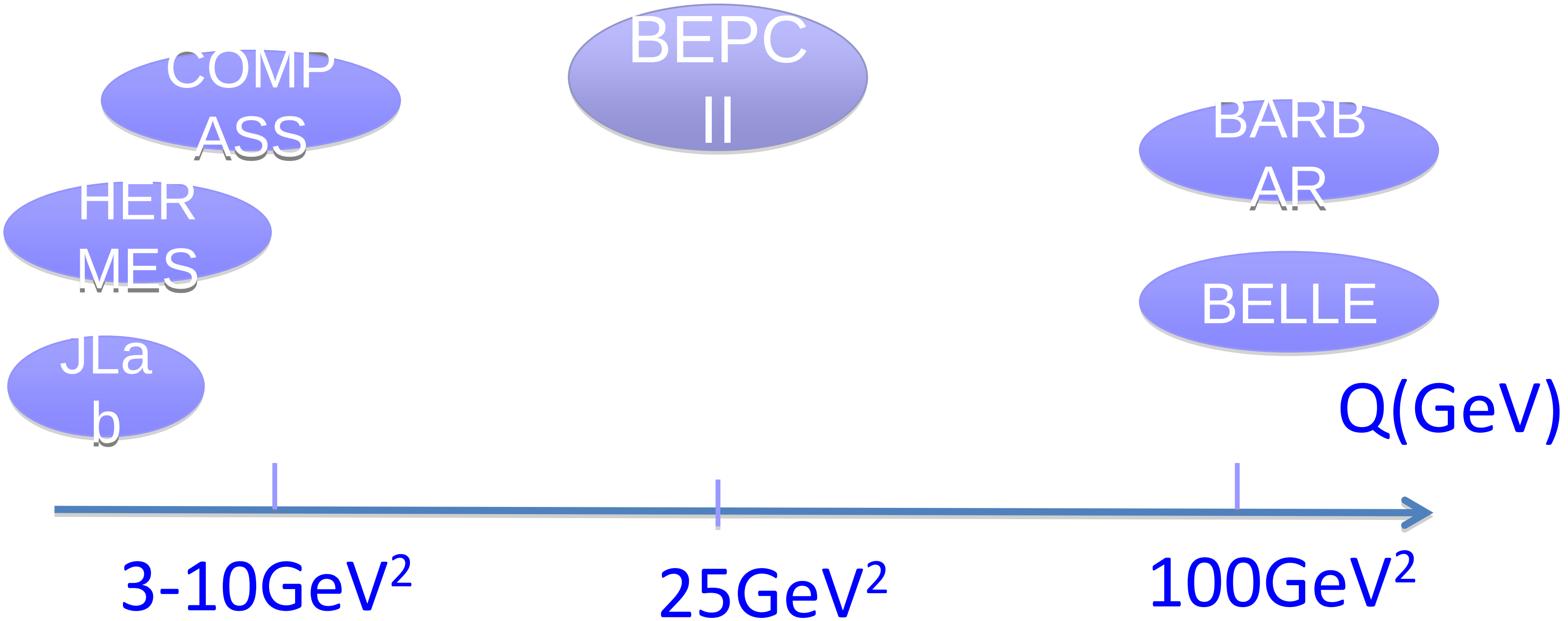
Pt distribution from BABAR

Test the evolution at BEPC



- $E_{c.m.}$=4.6GeV, di-pion in e^+e^- annihilation
- Because of energy evolution effect, It will be larger than that at BELLE and BABAR by a factor 2

Q^2 for all these experiments



Importance

- Reliable determination of the Collins functions.

- Study the QCD evolution effects
  - By theory
  - By experiments

Summary

- TMD evolution is studied for the Collins effects in e^+e^- annihilation
- Collins functions fitted from the existing data at BELLE and BABAR with the CSS resummation form
- Predictions at BEPCII have been made following the TMD evolution
- The experimental result at BEPC will provide an important test for TMD factorization

Thank you very much!

- Fit the Collins function from BELLE and BARBAR data
- Energy dependence follows the CSS resummation formulism

$$D_q(z_1, C_0/b)$$

$$\widetilde{Z}_{uu}(Q;b) = e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}(Q,b)} \Sigma_{i,j} \hat{C}_{qi}^{(e^+e^-)} \otimes D_{i/A}(z_1) \hat{C}_{qj}^{(e^+e^-)} \otimes D_{j/B}(z_2') \,,$$

$$\widetilde{Z}^{\alpha\beta}_{\rm collins}(Q;b) = \left(\frac{-ib_\perp^\alpha}{2}\right)\left(\frac{-ib_\perp^\beta}{2}\right) e^{-\mathcal{S}_{pert}(Q^2,b_*)-S^T_{NP}(Q,b)}$$

$$\times \Sigma_{i,j} \Delta\hat{C}_{qi}^{\rm collins(e^+e^-)} \otimes D^{(3)}_{i/A} \hat{C}_{qj}^{\rm collins(e^+e^-)} \otimes D^{(3)}_{j/B} \,,$$

$$\hat{H}_{1q}(z_{h1}, C_0/b)$$

- then, we can predict the Collins effect at BEPC