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Nuclear Reactions:
Introduction, Motivation, and Definitions

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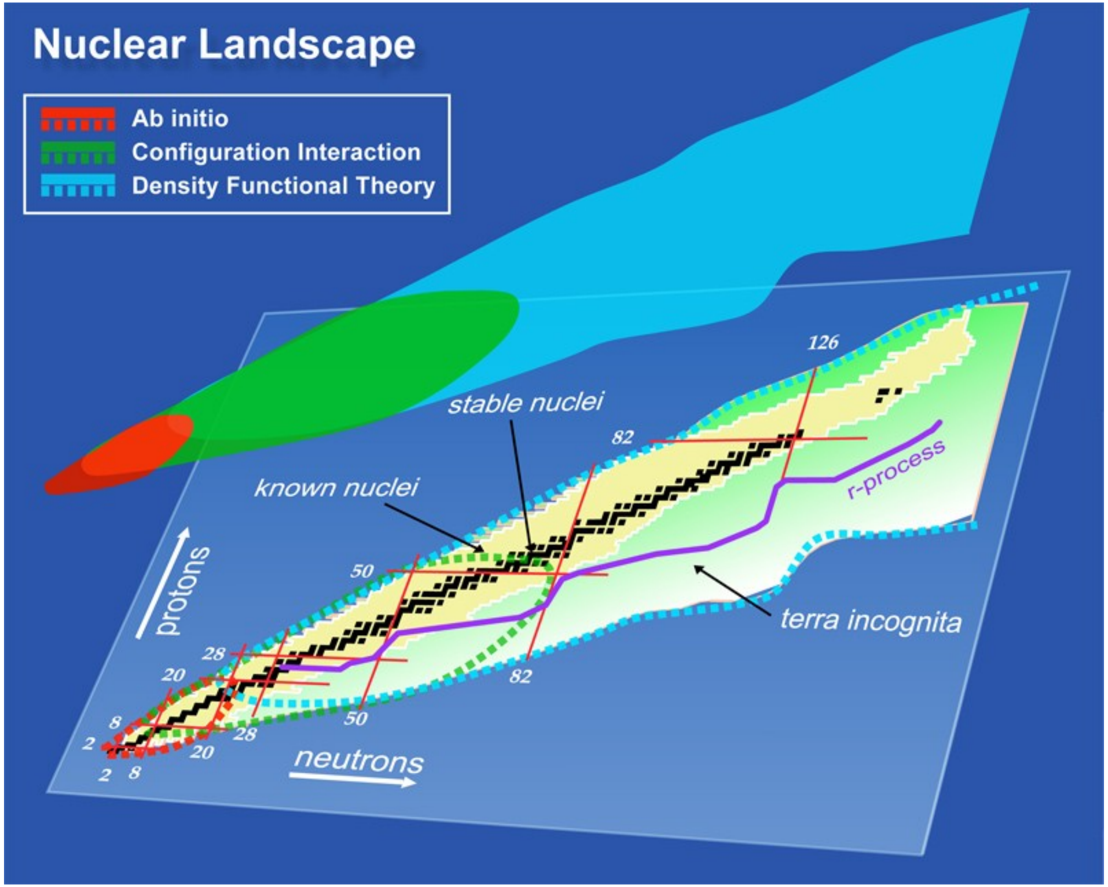
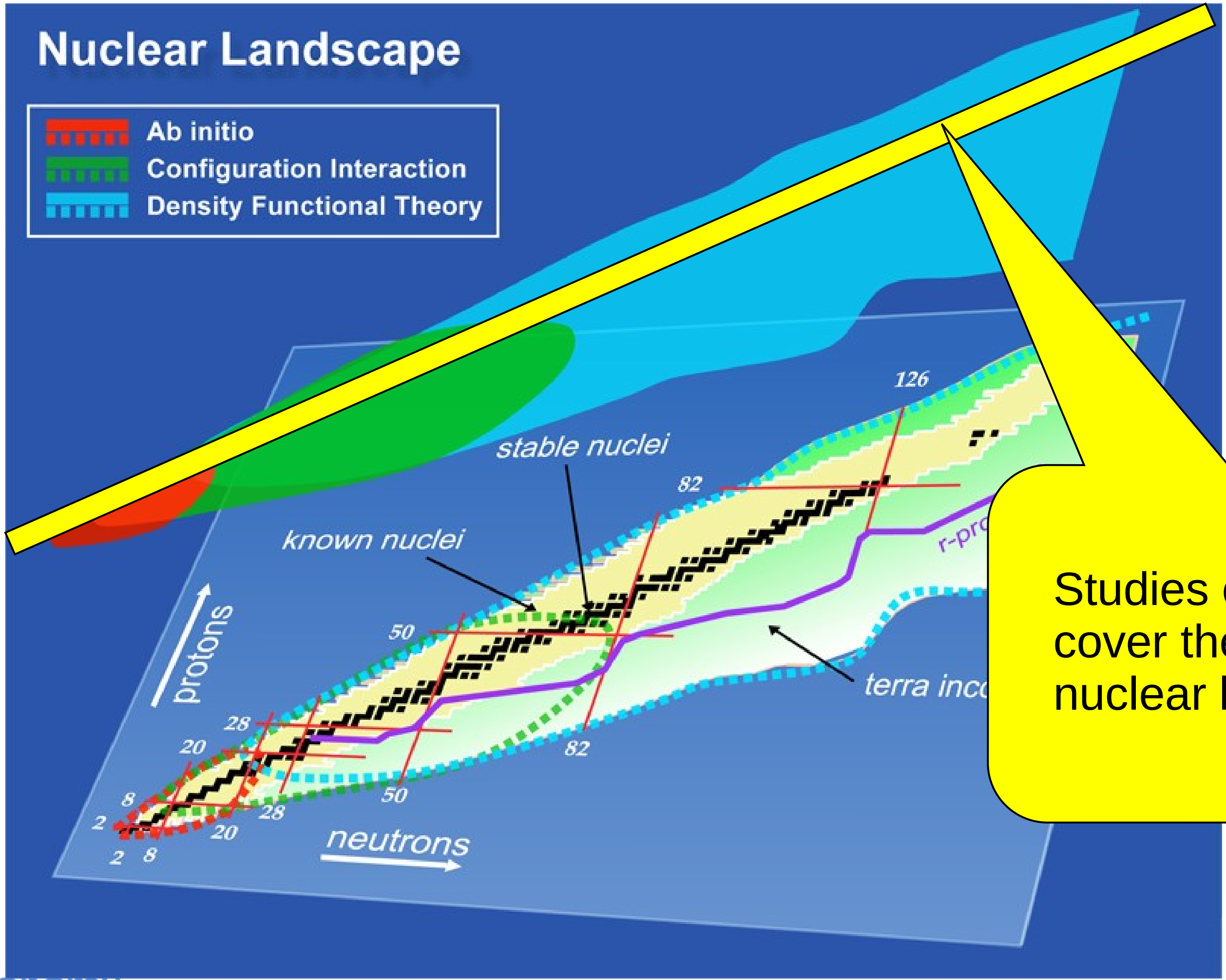
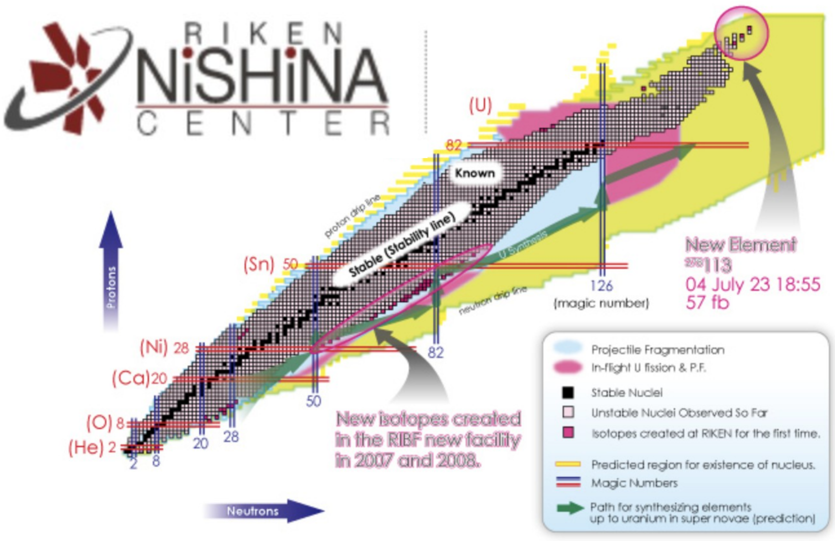


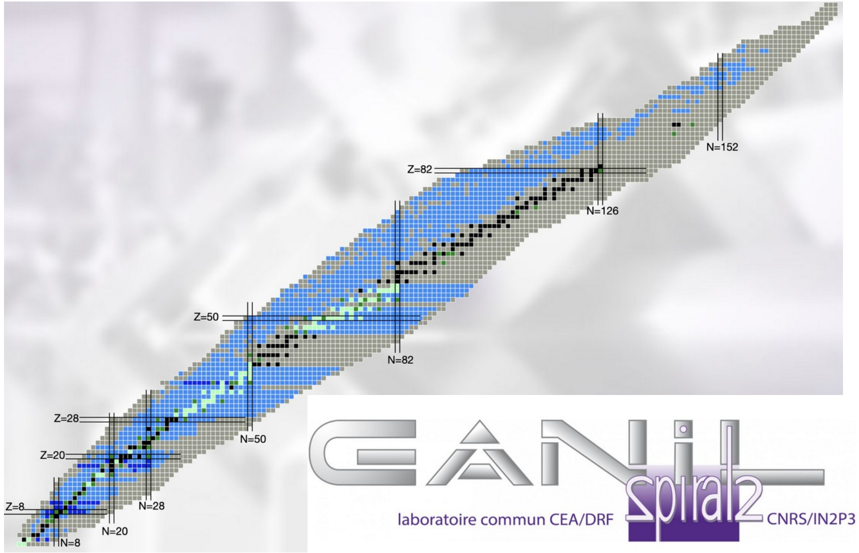
Goal:

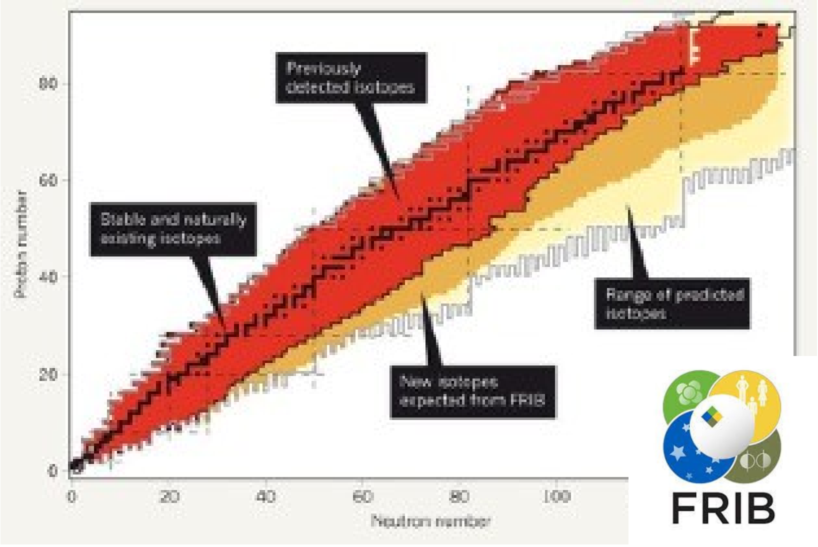
- Understand the richness of the nuclear landscape
- Most nuclei in this landscape are unstable
- Properties are experimentally mostly discovered/determined by nuclear reactions.
- Specifically, unstable nuclei can not be target material and must be studied in inverse kinematics.



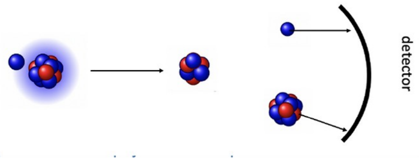
Studies of reactions cover the entire nuclear landscape





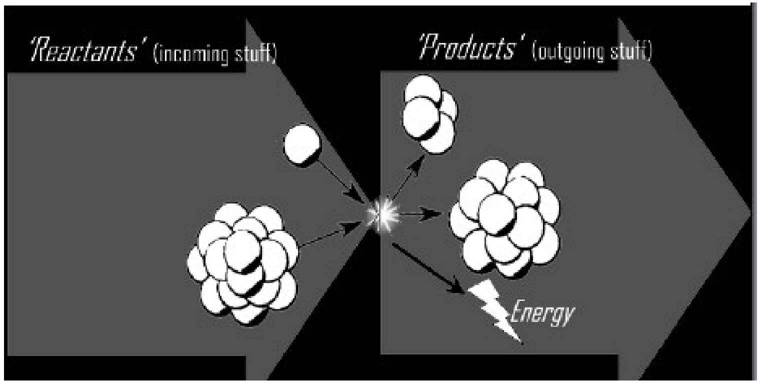


Unstable nuclei are synthetized and often studied through **<u>nuclear reactions</u>**



Defining a reaction:

- A nuclear reaction consists of the **interaction of two or more nuclei or nucleons** that result in some **final product**
- Nuclear reactions are governed by the strong force and thus they **conserve baryon number, energy, nuclear charge, linear and angular momentum, parity.**



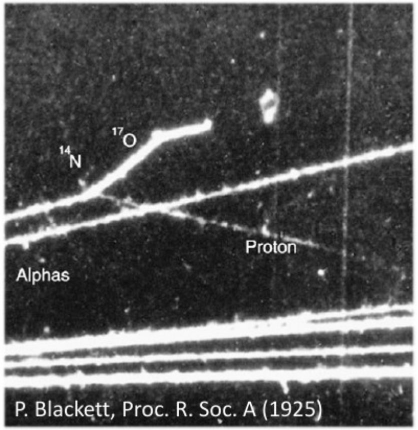
The modern notation for a reaction is always to put the lighter of the reactants and products on the inside of a pair of brackets, like A(b,c)D, where MA>Mb and MD>Mc

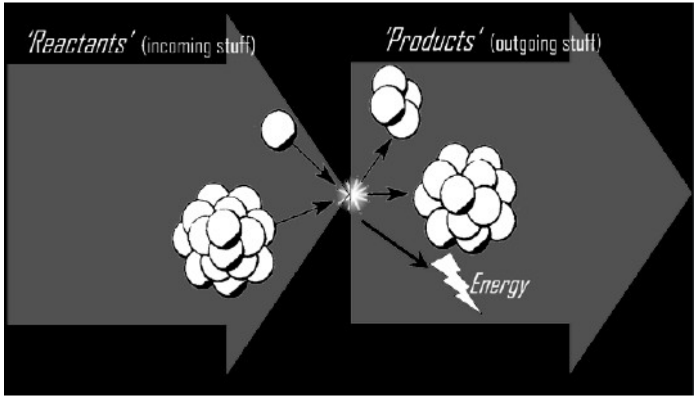
Defining a reaction cont'd:

- The initial stuff is known as reactants [projectile and target, in the lab]
- The final stuff is known as the products [recoil and ejectile, in the lab]

- Several sets of products are often possible for a pair of reactants colliding at a given energy, including simple scattering.
- The ways of "decaying" from the nucleus briefly formed by the reaction are usually called channels

1st nuclear fusion reaction observed in the lab





Reminder of the basics: Single channel scattering

General Setup: 2 particles with masses m_1 and m_2 move with momenta $\mathbf{k}_1$ and $\mathbf{k}_2$

Relative momentum: $\mathbf{p} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$

Total momentum: $\mathbf{P} = (\mathbf{k}_1 + \mathbf{k}_2)$

Total mass: $M = m_1 + m_2$

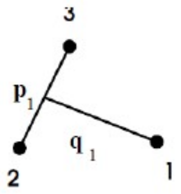
Reduced mass: $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

Hamiltonian for free motion: $H_0 = \frac{k_1^2}{2m_1} + \frac{k_2^2}{2m_2} = \frac{P^2}{2M} + \frac{p^2}{2\mu}$

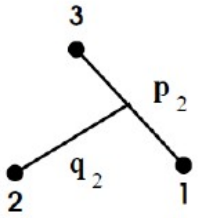
Two body wavefunction: $\boldsymbol{\Psi}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \boldsymbol{\Psi}(\boldsymbol{P}, \boldsymbol{p})$

In case of 3 particles: Jacobi coordinates

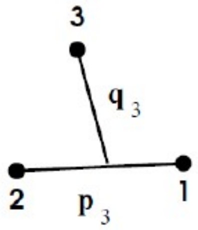
General Setup For 3 equal mass particles (easier case):



$$\vec{p}_1 = \frac{1}{2}(\vec{k}_2 - \vec{k}_3)$$
$$\vec{q}_1 = \frac{2}{3}(\vec{k}_1 - \frac{1}{2}(\vec{k}_2 + \vec{k}_3)).$$



$$\vec{p}_2 = \frac{1}{2}(\vec{k}_3 - \vec{k}_1)$$
$$\vec{q}_2 = \frac{2}{3}(\vec{k}_2 - \frac{1}{2}(\vec{k}_3 + \vec{k}_1))$$



$$\vec{p}_3 = \frac{1}{2}(\vec{k}_1 - \vec{k}_2)$$
$$\vec{q}_3 = \frac{2}{3}(\vec{k}_3 - \frac{1}{2}(\vec{k}_1 + \vec{k}_2))$$

In case of 3 particles with different masses: Jacobi coordinates

$$\mathbf{p_i} = \mu_{jk}\left(\frac{\mathbf{k_j}}{m_j} - \frac{\mathbf{k_k}}{m_k}\right)$$

$$\mathbf{q_i} = M_i\left(\frac{\mathbf{k_i}}{m_i} - \frac{\mathbf{k_j}+\mathbf{k_k}}{m_j+m_k}\right),$$

pair $(\mathbf{q_i}, \mathbf{p_i})$, where $(i = 1, 2, 3)$

where $(\mu_{jk} = \mu_i)$ is the two-body reduced mass of the pair(jk)

Reduced masses:

$$\frac{1}{\mu_{jk}} = \frac{1}{m_j} + \frac{1}{m_k}$$

$$\mu_{jk} = \frac{m_j m_k}{m_j + m_k},$$

$$\frac{1}{M_i} = \frac{1}{m_i} + \frac{1}{m_j + m_k}$$

$$M_i = \frac{m_i(m_j + m_k)}{M},$$

Total mass: $M = m_i + m_j + m_k$

Total momentum: $\mathcal{K} = \sum_i \mathbf{k}_i.$

Free Hamiltonian:

$$H_0 = \sum_{i=3} \frac{k_i^2}{2m_i} = \frac{\mathcal{K}^2}{2M} + \frac{p_i^2}{2\mu_i} + \frac{q_i^2}{2M_i},$$

c.m. motion – Galilean invariance: can be separated off

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IF forces depend only on relative momenta (coordinates): $\boldsymbol{\Psi}(\boldsymbol{P},\boldsymbol{p}) = \Psi(\boldsymbol{P})\psi(\boldsymbol{p})$

And we can consider the problem only in relative variables.

Introduce momentum eigenstates: $|\vec{p}\rangle$ normalized as $\langle\vec{p}\mid\vec{p}\,'\rangle = \delta^3(\vec{p}-\vec{p}\,')$

Simplify for the moment to equal mass situation: $m_1 = m_2 = m \rightarrow 2\mu = m$

Assume potential to be energy independent, then

Schrödinger equation for a scattering state $\Psi_{\vec{p}}^{(+)}$

$$(H_0+V)\,\Psi_{\vec{p}}^{(+)} = E\,\Psi_{\vec{p}}^{(+)}$$

or

$$(H_0-E)\,\Psi_{\vec{p}}^{(+)} = -V\Psi_{\vec{p}}^{(+)}$$

J. Kupsch, W. Sandhas, Commun. Math. Phys. 2, 147 (1966)

Assumption: $V(r\rightarrow\infty)\approx\frac{1}{r^{1+\varepsilon}}$ [Kupsch-Sandhas Theorem]

Schrödinger equation can be cast into integral form,
the Lippmann-Schwinger (LS) equation:

$$| \Psi_{\vec{p}}^{(+)} \rangle = | \vec{p} \rangle + \frac{1}{E + i\epsilon - H_0} V | \Psi_{\vec{p}}^{(+)} \rangle$$

configuration space representation

conjugate variable to $\vec{p}$ is $\quad \vec{x} = \vec{r}_1 - \vec{r}_2$

choose $| \vec{x} \rangle$ to be normalized as

$$\langle \vec{x} \mid \vec{x}' \rangle = \delta^3(\vec{x} - \vec{x}')$$

Then the Fourier transform is given by

$$\langle \vec{x} \mid \vec{p} \rangle = \frac{1}{(2\pi)^{3/2}} \, e^{i\vec{p}\cdot\vec{x}}$$

The configuration space representation of the free propagator

$$G_0 \equiv \frac{1}{E + i\epsilon - H_0}$$

is given as

$$\begin{aligned}
\langle \vec{x} \mid \frac{1}{E + i\epsilon - H_0} \mid \vec{x}\,' \rangle &= \int d\vec{p}\, \langle \vec{x} \mid \vec{p} \rangle \frac{1}{E + i\epsilon - p^2/m} \langle \vec{p} \mid \vec{x}\,' \rangle \\
&= \frac{1}{(2\pi)^{3/}} \int d^3p\, e^{i\vec{p}\cdot\vec{x}} \frac{1}{E + i\epsilon - p^2/m} e^{-i\vec{p}\cdot\vec{x}\,'} \\
&= \frac{1}{(2\pi)^3} \int d^3p\, e^{i\vec{p}\cdot(\vec{x}-\vec{x}')} \frac{1}{E + i\epsilon - p^2/m} \\
\text{with } \rho \equiv \mid \vec{x} - \vec{x}' \mid \qquad &= \frac{1}{(2\pi)^3} 4\pi \int_0^\infty dp\, p^2 j_0(p\rho) \frac{1}{E + i\epsilon - p^2/m}
\end{aligned}$$

Standard residue techniques lead to

$$\langle \vec{x} \mid G_0 \mid \vec{x}\,' \rangle = -\frac{m}{4\pi} \frac{e^{i\sqrt{mE}|\vec{x}-\vec{x}'|}}{\mid \vec{x} - \vec{x}' \mid}$$

exhibits the outgoing wave behavior from the source point $\vec{x}'$ to $\vec{x}$.

configuration space representation of the LSE

$$\langle \vec{x} \mid \Psi_{\vec{p}}^{(+)} \rangle \equiv \Psi_{\vec{p}}^{(+)}(\vec{x})$$

$$= \frac{1}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{x}} - \frac{m}{4\pi} \int d^3x' \frac{e^{i\sqrt{mE}|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} V(x') \Psi_{\vec{p}}^{(+)}(\vec{x}')$$

Thereby we assumed a local potential

$$\langle \vec{x} \mid V \mid \vec{x}' \rangle = \delta^3(\vec{x}-\vec{x}\,') V(x')$$

asymptotic form for $|\vec{x}| \to \infty$:

$$\Psi_{\vec{p}}^{(+)}(\vec{x}) \to \frac{1}{(2\pi)^{\frac{3}{2}}} \left(e^{i\vec{p}\vec{x}} + \frac{e^{ipx}}{x} f(\hat{x}) \right)$$

also known as Sommerfeld radiaton condition

with the scattering amplitude $f(\hat{x})$ depending on the direction $\hat{x}$ of observation

$$f(\hat{x}) = -m\sqrt{\frac{\pi}{2}} \int d^3x' \, e^{-ip\hat{x}\cdot\vec{x}'} V(x') \Psi_{\vec{p}}^{(+)}(\vec{x}') \, .$$

Interpret in terms of scattered momentum $\vec{p}' \equiv \hat{x} p$

Introduce transition amplitude

$$\begin{aligned} \langle \vec{p}' \mid t \mid \vec{p} \rangle &\equiv \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3x' \, e^{-i\vec{p}'\vec{x}'} \, V(x') \, \Psi^{(+)}_{\vec{p}}(\vec{x}') \\ &= \langle \vec{p}' \mid V \mid \Psi^{(+)}_{\vec{p}} \rangle \, . \end{aligned}$$

where *t* is the result of the scattering process and determines all scattering observables

We read off: $t \mid \vec{p} \rangle \equiv V \mid \Psi^{(+)}_{\vec{p}} \rangle$

with $\mid \Psi^{(+)}_{\vec{p}} \rangle = \mid \vec{p} \rangle + \frac{1}{E + i\epsilon - H_0} V \mid \Psi^{(+)}_{\vec{p}} \rangle$

follows

$$\begin{aligned} t \mid \vec{p} \rangle &= V \mid \vec{p} \rangle + V \, G_0 \, V \mid \Psi^{(+)}_{\vec{p}} \rangle \\ &= V \mid \vec{p} \rangle + V \, G_0 \, t \mid \vec{p} \rangle \end{aligned}$$

free propagator $G_0 = \frac{1}{E + i\epsilon - H_0}$

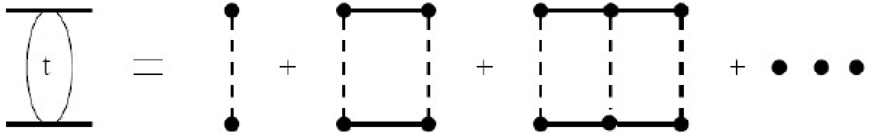
strip off the initial state $|\vec{p}\rangle$ and get the operator relation

$$t = V + V\, G_0\, t$$

LS operator equation for the transition amplitude

Simple physical interpretation by iterating

$$\begin{aligned} t &= V + VG_0\,(\,V + VG_0 t\,) \\ &= V + VG_0V + VG_0VG_0V + VG_0VG_0VG_0V + \ldots \end{aligned}$$



LS equation contains V iterated to all orders as does the Schrödinger equation.

Scattering amplitude and cross section:

Cross section

$$\sigma(\theta,\phi) = \frac{v_f}{v_i}|f(\theta,\phi)|^2$$

$$k = \sqrt{2\mu E/\hbar^2}$$
$$\mathbf{v} = \mathbf{p}/\mu = \hbar\mathbf{k}/\mu$$

Initial velocity v_i = final velocity v_f $\equiv$ elastic scattering

Renormalized scattering amplitude

$$\tilde{f}(\theta,\phi) = \sqrt{\frac{v_f}{v_i}} f(\theta,\phi)$$

$$\sigma(\theta,\phi) = |\tilde{f}(\theta,\phi)|^2$$

How do we solve the scattering problem in practice?

Choose basis

Option 1: **momentum space eigenstates** $|\,\boldsymbol{p}\rangle$

Free propagator $G_0 \equiv \dfrac{1}{E + i\epsilon - H_0}$ simple

LS equation $t = V + V\ G_0\ t$ becomes integral equation of Fredholm type

Solve numerically with standard methods

Option 2: **coordinate space eigenstates** $|\boldsymbol{r}\rangle$

For local potentials $\langle \vec{x} \mid V \mid \vec{x}'\rangle = \ \delta^3(\vec{x} - \vec{x}\,')\ V(x')$

2nd order differential equation

Solve numerically with standard methods

Assume rotationally invariant potential: $V(\mathbf{r}_1-\mathbf{r}_2) = V(|\mathbf{r}_1-\mathbf{r}_2|) \equiv V(r)$

$$\rightarrow [H_0,L^2] = [H_0,L_3] = [L^2,L_3] = 0$$

Assume potential short ranged (i.e. falling off faster than $1/r^{1+\varepsilon}$

Scattering: initial beam is symmetric around beam axis (m=0)
→ scattered wave is also → $f(\theta,\phi) = f(\theta)$

Solve $(H_0 + V - E)\,\psi\,(R,\theta) = 0$

$$H_0 \equiv \quad \hat{T} = -\frac{\hbar^2}{2\mu}\nabla_R^2$$

$$= \frac{\hbar^2}{2\mu}\left[-\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}\right) + \frac{\hat{L}^2}{R^2}\right]$$

Partial wave expansion

Solution of Schrödinger equation: $$\psi(R,\theta) = \sum_{L=0}^{\infty}(2L+1)\mathrm{i}^L P_L(\cos\theta)\frac{1}{kR}\chi_L(R)$$

Plane wave: $$\mathrm{e}^{\mathrm{i}kz} = \sum_{L=0}^{\infty}(2L+1)\mathrm{i}^L P_L(\cos\theta)\frac{1}{kR}F_L(0,kR)$$

$$\mathrm{e}^{\mathrm{i}kz} = \sum_{L=0}^{\infty}(2L+1)\mathrm{i}^L P_L(\cos\theta)\frac{1}{kR}\frac{\mathrm{i}}{2}[H_L^-(0,kR) - H_L^+(0,kR)]$$

incoming outgoing

- at large distances the radial wavefunction should behave as

$$\chi_L^{\mathrm{ext}}(R) = A_L\left[H_L^-(0,kR) - \mathbf{S}_L H_L^+(0,kR)\right]$$

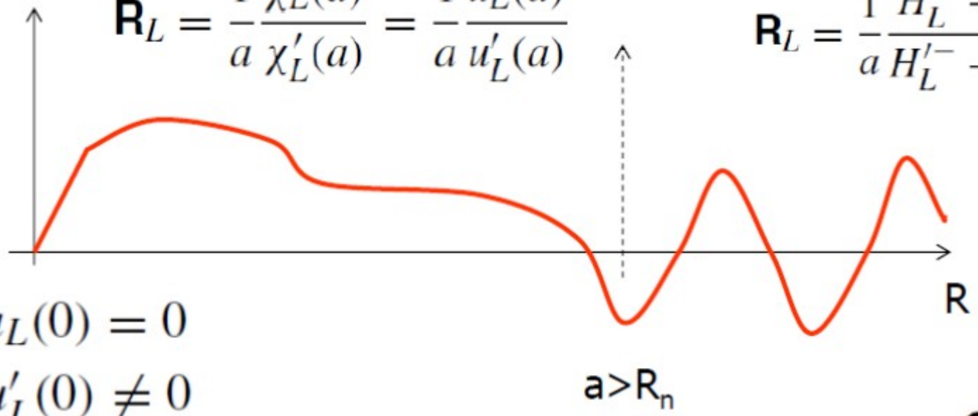
partial wave S-matrix element

Match logarithmic derivative → don't worry about normalization

- The matching can be done with the inverse log derivative R_L
- any potential will produce R_L which relates to S_L

$$\mathbf{R}_L = \frac{1}{a}\frac{\chi_L(a)}{\chi'_L(a)} = \frac{1}{a}\frac{u_L(a)}{u'_L(a)}$$

$$\mathbf{R}_L = \frac{1}{a}\frac{H_L^- - \mathbf{S}_L H_L^+}{H_L'^- - \mathbf{S}_L H_L'^+}$$



$$u_L(0) = 0$$

$$u'_L(0) \neq 0$$

$$\mathbf{S}_L = \frac{H_L^- - a\mathbf{R}_L H_L'^-}{H_L^+ - a\mathbf{R}_L H_L'^+}$$

S-matrix and scattering amplitude

- to obtain the scattering amplitude need to sum the partial waves

$$\psi(R,\theta) \overset{R>R_n}{\longrightarrow} \frac{1}{kR}\sum_{L=0}^{\infty}(2L+1)\mathrm{i}^L P_L(\cos\theta)A_L[H_L^-(0,kR)-\mathbf{S}_L H_L^+(0,kR)]$$

$$\psi^{\mathrm{asym}}(R,\theta) = \mathrm{e}^{\mathrm{i}kz} + f(\theta)\frac{\mathrm{e}^{\mathrm{i}kR}}{R}$$

Relations:

$$f(\theta) = \frac{1}{2\mathrm{i}k}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)(\mathbf{S}_L - 1)$$

$$\sigma(\theta) \equiv \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{1}{2\mathrm{i}k}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)(\mathbf{S}_L - 1)\right|^2$$

Phase shifts:

- Each partial wave S-matrix can be equivalently described with a phase shift $\mathbf{S}_L = e^{2i\delta_L}$

$$\delta_L(E) = \frac{1}{2i} \ln \mathbf{S}_L + n_B(E)\,\pi$$

Levinson's Theorem $\delta(k=0) - \delta(k \equiv \infty) = n_B \pi$

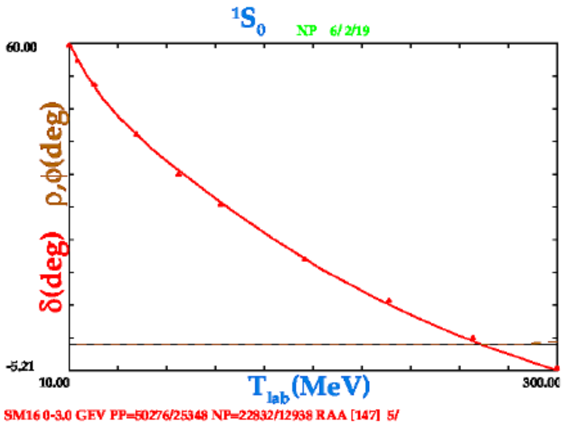
- scattering amplitude in terms of phase shifts

$$f(\theta) = \frac{1}{k} \sum_{L=0}^{\infty} (2L+1) P_L(\cos\theta) e^{i\delta_L} \sin\delta_L$$

- asymptotic form in terms of phase shift

$$\chi_L^{\mathrm{ext}}(R) \to e^{i\delta_L}[\cos\delta_L \sin(kR - L\pi/2) + \sin\delta_L \cos(kR - L\pi/2)]$$
$$= e^{i\delta_L} \sin(kR + \delta_L - L\pi/2).$$

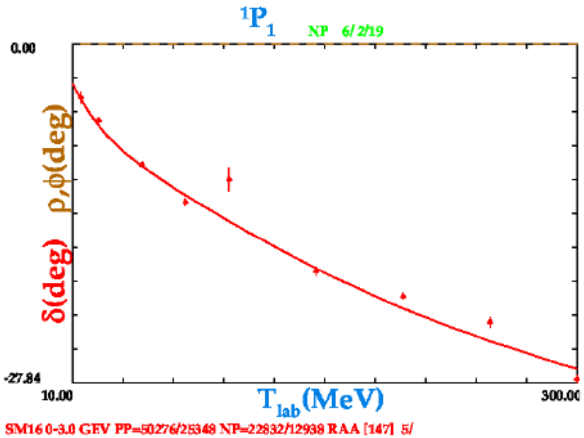
Phase shifts as function of energy:

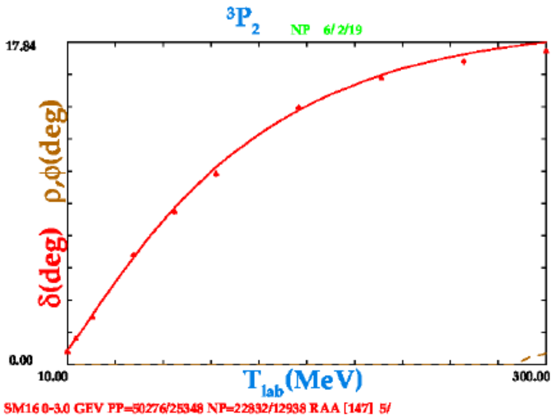


- attractive potentials: delta>0
- repulsive potentials: delta<0

NN phase shifts from

http://gwdac.phys.gwu.edu/





Relation to t-matrix

- the partial wave T-matrix is defined as the amplitude of the outgoing wave

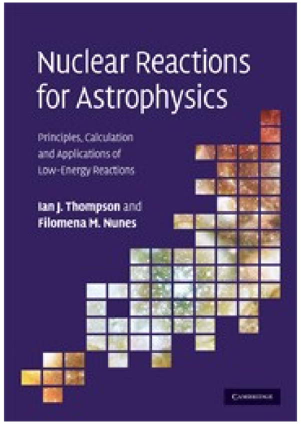
$$\chi_L^{\mathrm{ext}}(R) = F_L(0,kR) + \mathbf{T}_L H_L^+(0,kR) \qquad \mathbf{S}_L = 1 + 2\mathrm{i}\mathbf{T}_L$$

- simple relation with the scattering amplitude

$$f(\theta) = \frac{1}{k}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)\mathbf{T}_L$$

Useful relations:

Using:	δ	$\mathbf{K}$	$\mathbf{T}$	$\mathbf{S}$
$\chi(R) =$	$e^{i\delta}[F\cos\delta + G\sin\delta]$	$\frac{1}{1-i\mathbf{K}}[F+\mathbf{K}G]$	$F + \mathbf{T}H^+$	$\frac{i}{2}[H^- - \mathbf{S}H^+]$
$\delta =$	δ	$\arctan \mathbf{K}$	$\arctan \frac{\mathbf{T}}{1+i\mathbf{T}}$	$\frac{1}{2i}\ln \mathbf{S}$
$\mathbf{K} =$	$\tan\delta$	$\mathbf{K}$	$\frac{\mathbf{T}}{1+i\mathbf{T}}$	$i\frac{1-\mathbf{S}}{1+\mathbf{S}}$
$\mathbf{T} =$	$e^{i\delta}\sin\delta$	$\frac{\mathbf{K}}{1-i\mathbf{K}}$	$\mathbf{T}$	$\frac{i}{2}(1-\mathbf{S})$
$\mathbf{S} =$	$e^{2i\delta}$	$\frac{1+i\mathbf{K}}{1-i\mathbf{K}}$	$1+2i\mathbf{T}$	$\mathbf{S}$
$V = 0$	$\delta = 0$	$\mathbf{K} = 0$	$\mathbf{T} = 0$	$\mathbf{S} = 1$
V real	δ real	$\mathbf{K}$ real	$\lvert 1+2i\mathbf{T}\rvert = 1$	$\lvert\mathbf{S}\rvert = 1$



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Integrated cross section and optical theorem

$$f(\theta) = \frac{1}{2ik}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)(\mathbf{S}_L - 1)$$

○ use properties of legendre polynomials

$$\begin{aligned}\sigma_{\text{el}} &= \int_0^{2\pi} d\phi \int_0^{\pi} \mathrm{d}\theta \sin\theta\ \sigma(\theta) \\ &= 2\pi \int_0^{\pi} \mathrm{d}\theta \sin\theta |f(\theta)|^2 \\ &= \frac{\pi}{k^2}\sum_{L=0}^{\infty}(2L+1)|1-\mathbf{S}_L|^2 \\ &= \frac{4\pi}{k^2}\sum_{L=0}^{\infty}(2L+1)\sin^2\delta_L,\end{aligned}$$

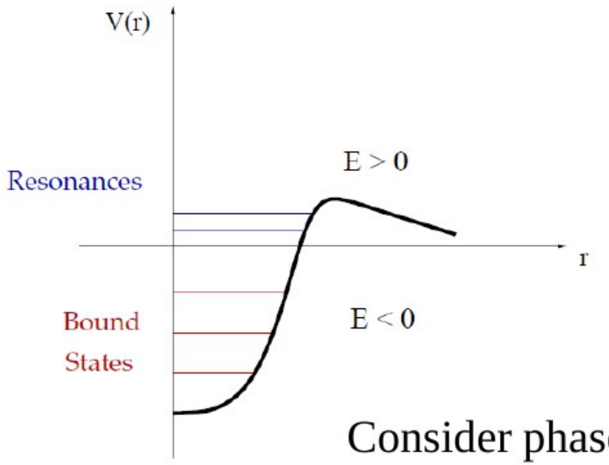
○Optical theorem:
total elastic cross section related to zero-angle scattering amplitude

$$f(0) = \frac{1}{2ik}\sum_{L=0}^{\infty}(2L+1)(\mathrm{e}^{2\mathrm{i}\delta_L} - 1),$$

$$\begin{aligned}\operatorname{Im} f(0) &= \frac{1}{k}\sum_{L=0}^{\infty}(2L+1)\sin^2\delta_L \\ &= \frac{k}{4\pi}\sigma_{\text{el}}.\end{aligned}$$

Resonances and phase shifts

Consider:



Resonance may be characterized by rapid change in

- cross section
- phase shift

$$\Delta t \sim \hbar \mathrm{d}\delta(E)/\mathrm{d}E$$

Consider phase shift for small momenta

$$\tan\delta_\ell \xrightarrow{p\to 0} \frac{(\ell+1) - R\,\gamma_\ell(R)}{\ell + R\,\gamma_\ell(R)} \frac{(pR)^{2\ell+1}}{[1\cdot 3\cdot 5\cdots(2\ell-1)]^2\,(2\ell+1)}$$

$$\frac{1}{R_{\ell,p}(r)}\frac{d}{dr}R_{\ell,p}(r)$$

Pole at $R\,\gamma_\ell(R) = -\ell$

⇨ $\tan\delta_\ell \to \infty$, i.e. $\delta_\ell = \frac{\pi}{2} + \mathrm{n}\pi$

condition occurs for a specific momentum p_R at a specific energy $E_R = p_R/\tilde{2}\mu$

Expanding $\gamma(R)$ around E_R gives

$$\gamma_\ell(R)R \approx -l + (E - E_R)\frac{d(\gamma_\ell R)}{dE}\,|_{E=E_R}$$

Inserting and neglecting terms *(E-E$_R$)* in numerator

$$\tan\delta_\ell \approx \frac{1}{E - E_R}\frac{1}{2}\frac{2(pR)^{2\ell+1}}{[(2\ell-1)!!]^2\frac{d(\gamma R)}{dE}}$$

$$= \frac{1}{E_R - E}\frac{\Gamma_l}{2}$$

with

$$\Gamma_l = \frac{-2(pR)^{2\ell+1}}{[(2\ell-1)!!]^2\frac{d(\gamma R)}{dE}}.$$

Breit-Wigner resonance form of the amplitude

$$f_\ell = e^{i\delta_\ell}\sin\delta_\ell = \frac{\Gamma_l/2}{E_R - E - i\Gamma_l/2}$$

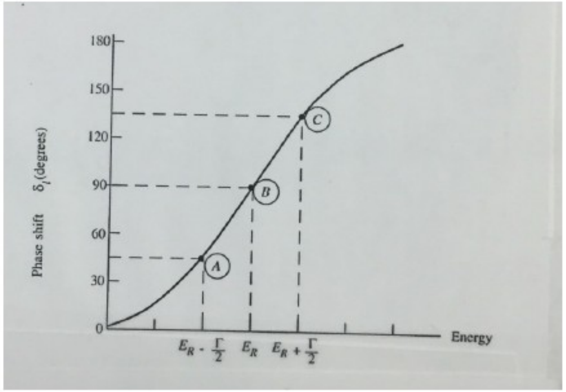
Breit-Wigner cross section in a specific partial wave partial wave ℓ.

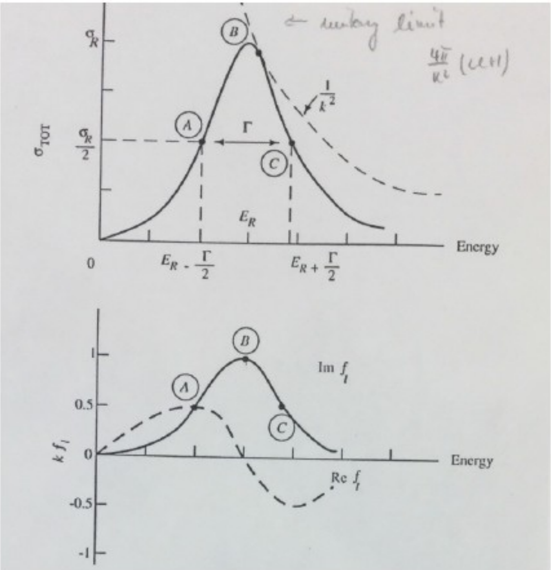
$$\sigma_\ell^{tot} = \frac{4\pi(2\ell+1)}{p^2}\frac{\Gamma_l^2/4}{(E-E_R)^2 + \Gamma_l^2/4} \qquad (11.126)$$

$$\sigma_\ell^{tot} = \frac{4\pi(2\ell+1)}{p^2} \frac{\Gamma_l^2/4}{(E-E_R)^2 + \Gamma_l^2/4}$$

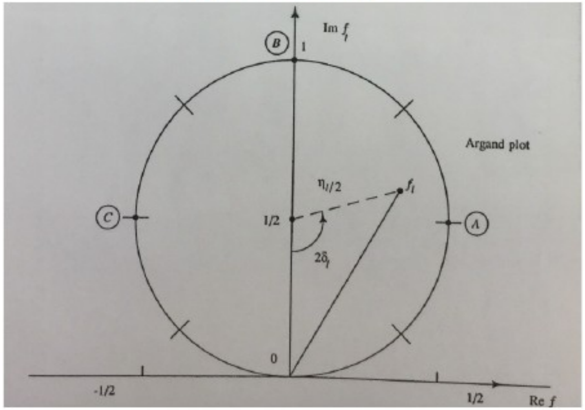
$$f_\ell = e^{i\delta_\ell} \sin \delta_\ell = \frac{\Gamma_l/2}{E_R - E - i\Gamma_l/2}$$

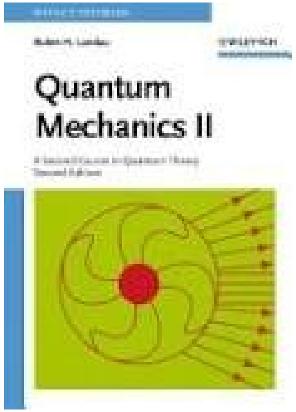
$$\tan \delta_\ell \to \infty, \text{ i.e. } \delta_\ell = \frac{\pi}{2} + n\pi$$





Argand diagram





Quantum Mechanics II

A Second Course in Quantum Theory

By Rubin H. Landau · 2008

OHIO UNIVERSITY

Physics realization of an "ideal" resonance: Δ (1232)

Pion nucleon scattering

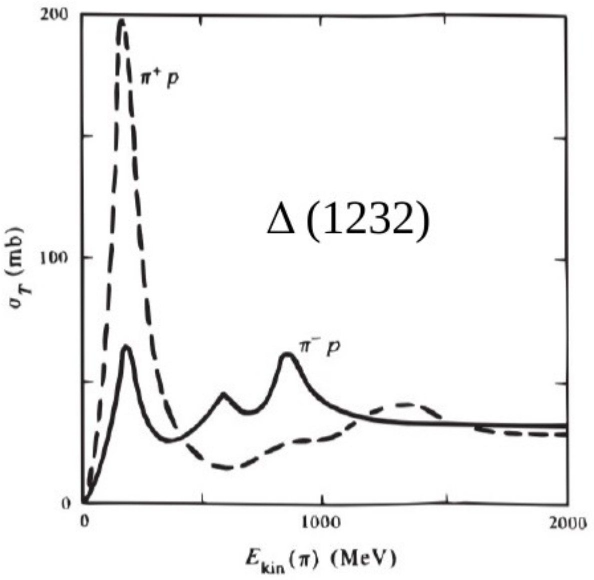
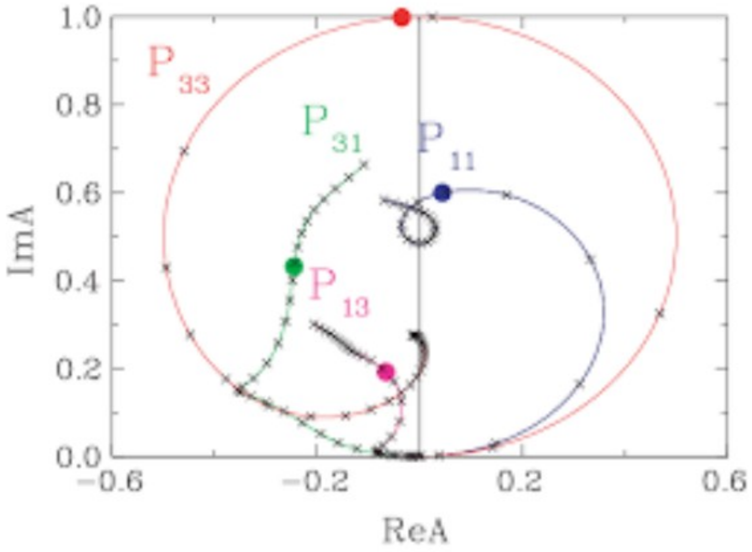


Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm^2.)



Usually there is background in addition to the resonant part: $\delta(E) = \delta_{bg}(E) + \delta_{res}(E)$

n + α scattering

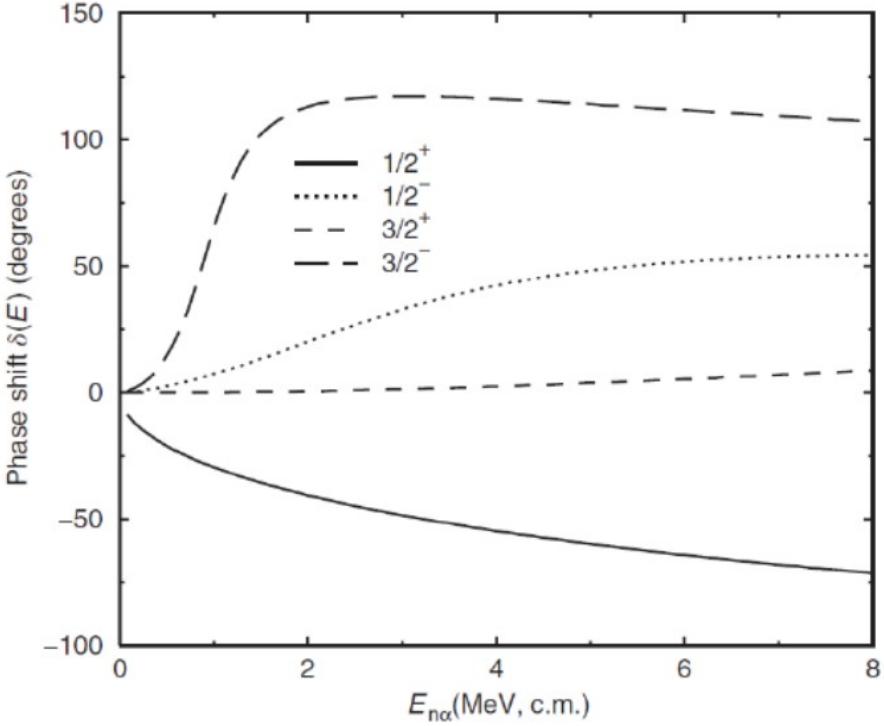


Fig. 3.2. Examples of resonant phase shifts for the $J^\pi = 3/2^-$ channel in low-energy n–α scattering, with a pole at $E = 0.96 - i0.92/2$ MeV. There is only a hint of a resonance in the phase shifts for the $J^\pi = 1/2^-$ channel, but it does have a wide resonant pole at $1.9 - i6.1/2$ MeV.

Back to reactions: Types of reactions

Reactions are categorized by the participants and the reaction mechanism

Participants:

- The projectile and target are not changed and exit in their ground state: ***elastic scattering***
- The projectile and target are not changed and one (or both) is excited: ***inelastic scattering***
- A multi-nucleon beam transfers a nucleon (or nucleons) to/from the target: ***transfer***
  - A projectile exits as an ejectile, taking one or more nucleons with it: ***pick-up***
  - A projectile exits as an ejectile, loosing one or more nucleons in the process: ***knock-out***
- A beam comes in and only one γ comes out : ***radiative capture***
- A γ comes in and one or few particles come out: ***photodisintegration***

Mechanism:

- Few nucleons take part in the reaction (e.g. transfer): ***direct***
- Projectile and target briefly fuse, forming a loosely bound state: ***resonant***
- Projectile and target briefly fuse, sharing energy among all nucleons: ***statistical***

Classification by Outcome:

1. **Elastic scattering:**
   projectile and target stay in their g.s.
2. **Inelastic scattering:**
   projectile or target left in excited state
3. **Transfer reaction:**
   1 or more nucleons moved to the other nucleus
4. **Fragmentation/Breakup/Knockout:**
   3 or more nuclei/nucleons in the final state
5. **Charge Exchange:**
   A is constant but Z (charge) varies, e.g. by pion exchange
6. **Multistep Processes:**
   intermediate steps can be any of the above
   ('virtual' rather than 'real')

Classification by Outcome cont'd

7. **Deep inelastic collisions:**
   Highly excited states produced
8. **Fusion:**
   Nuclei stick together
9. **Fusion-evaporation:**
   fusion followed by particle-evaporation and/or gamma emission
10. **Fusion-fission:**
    fusion followed by fission

The first 6 processes are *Direct Reactions* (DI)
The last 3 processes give a *Compound Nucleus* (CN).

Compound and Direct Reactions

When two nuclei collide there are **two** types of reactions:

1. Nuclei can coalesce to form highly excited **Compound nucleus (CN)** that lives for relatively long time.
   Long lifetime sufficient for excitation energy to be shared by all nucleons. If sufficient energy localised on one or more nucleons (usually neutrons) they can escape and CN decays.
   Independence hypothesis: CN lives long enough that it loses its memory of how it was formed. So probability of various decay modes independent of entrance channel.
2. Nuclei make 'glancing' contact and separate immediately, said to undergo **Direct reactions(DI)**.
   Projectile may lose some energy, or have one or more nucleons transferred to or from it.

"Location" of Reactions:

CN reactions at small impact parameter,

DI reactions at surface & large impact parameter.

CN reaction involves the whole nucleus.

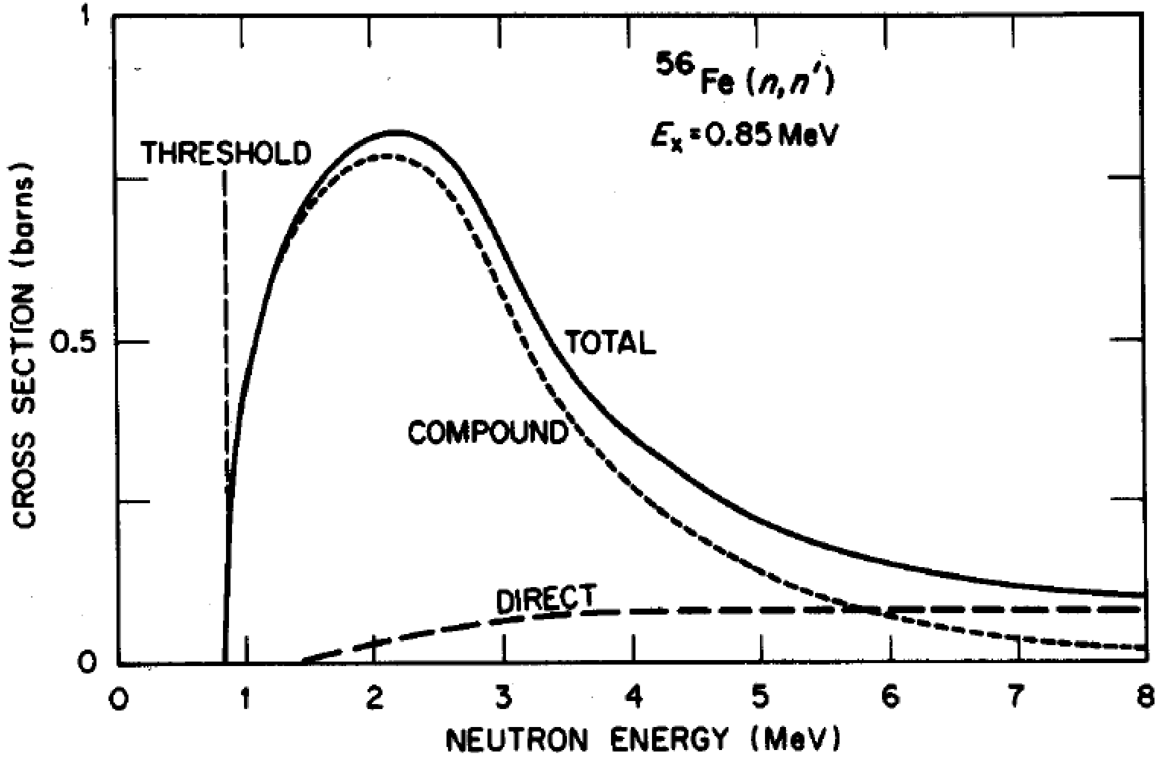
DI reaction usually occurs on the surface of the nucleus. This leads to diffraction effects.

Duration of reactions:

A typical nucleon orbits within a nucleus with a period of $\sim 10^{-22}$ sec. [as K.E. $\sim$ 20 MeV].

If reaction complete within this time scale or less then no time for distribution of projectile energy around target $\Rightarrow$ DI reaction occurred. CN reactions require $\gg 10^{-22}$ sec.

Classification of Reactions:



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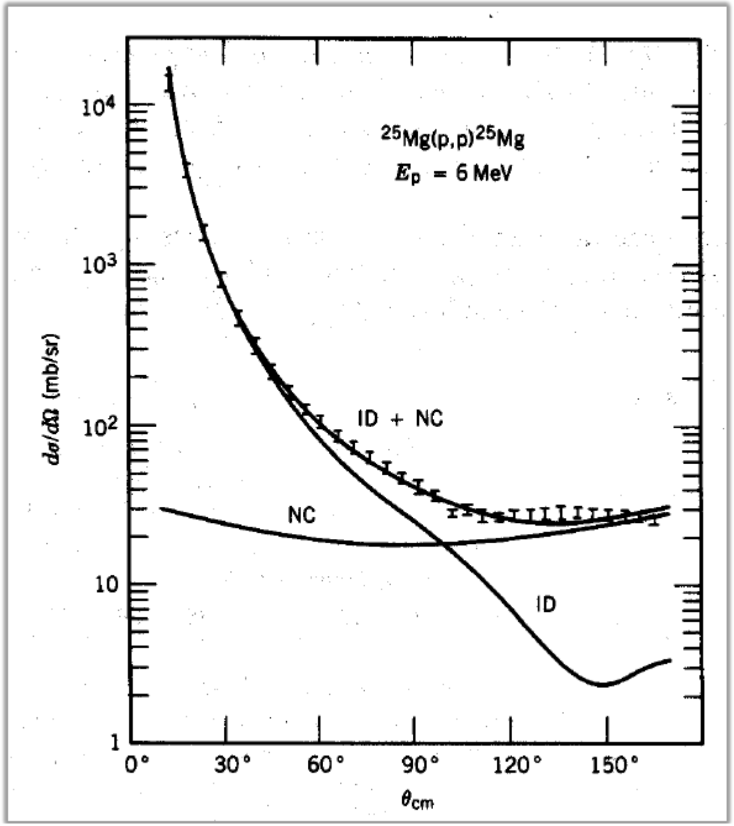
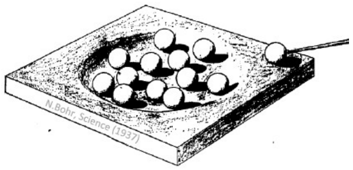
Angular Distributions:

Direct reactions (ID):
Forward peaked (large b)

Compound reactions (NC):
Distribution is generally isotropic (except for heavy ion collision where L large)



Statistical Reactions: Semiclassical Picture



- Consider the case where a projectile fuses with the target, sharing its energy amonst many nuleons in the nucleus, like a billard ball entering a well and causing several others to rattle around.

- The nucleon energies will be distributed statistically and they will scatter with each other until one nucleon happens to pick up sufficient energy to escape the nucleus.

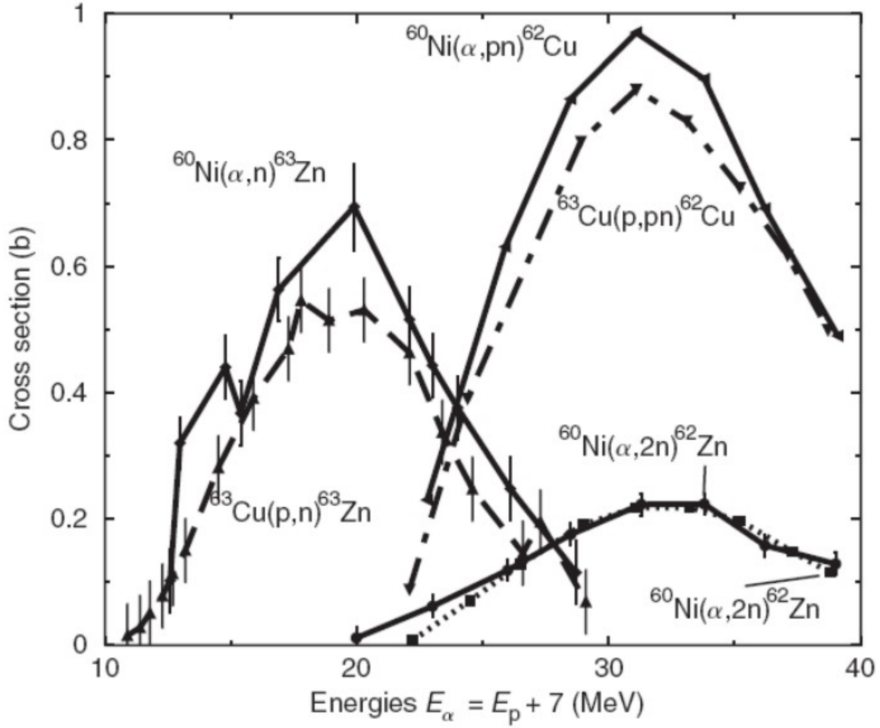
- Adopting this qualitative picture, we expect

  - The de-excitation of the compound nucleus is akin to evaporation, meaning the ejectile energy distribution should have a Maxwell-Boltzmann character
  - The multiple collisions occuring with the nucleus erases any signature left by the initial reaction, so

    - The ejectiles should be isotropic (in the c.m. frame, since momentum still has to be conserved)
    - The de-excitation characteristics for a given compound nucleus excited state energy should not depend on how the compound nucleus was created

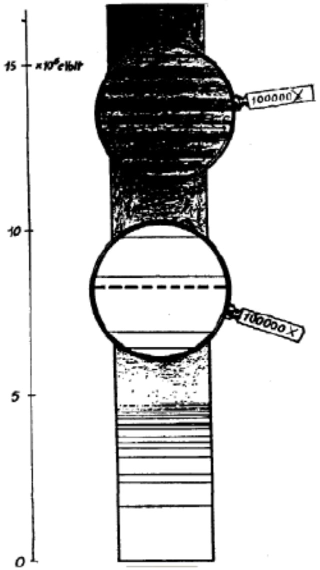
Independence Hypothesis:

- This statistical reaction picture implies that the compound nucleus forgets how it was formed, so the decay properties depend only on the compound nucleus itself
- This is confirmed by decay spectra for nuclei populated by various channels
- The key implication here is that compound nucleus formation and decay probabilities are separated
- One key result which follows is that other probes, e.g. β-decay, can be used to determine key properties needed to understand much harder to measure reactions, e.g. (p,γ)



Hauser-Feshbach Formalism:

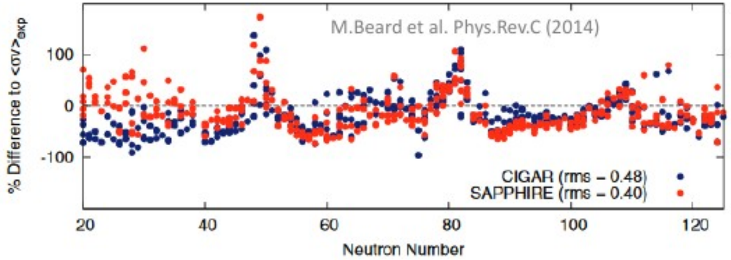
N.Bohr, Science (1937)

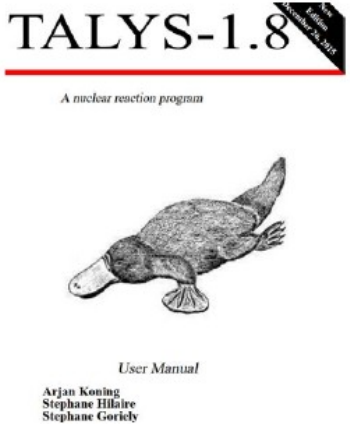


- The semi-classical picture of nucleons rattling around in the nucleus sampling many configurations until one configuration occurs in which the compound nucleus can de-excite by evaporation corresponds to the case of many nearby resonances
- Anyhow, if you don't buy this, it's true that the characteristics of statistical nuclear reactions [isotropic ejectile emission with a Maxwell-Boltzmann energy distribution] are seen for reactions where high level-density regions are populated in the compound nucleus
- In this picture, many resonances are closely spaced and the projectile will experience an interaction that's the statistical average of said resonances
- Note that Hauser-Feshbach (HF) assumes the level spacing $D \gg \Gamma$.
  It is frequently misstated that HF assumes overlapping resonances ($\Gamma \gg D$), but this is not the case.

This HF business sounds like a lot of busy work, how should I actually do these calculations?

- Generally speaking, your best bet is to use open-source tools
- Many options are available, each with their own strengths, weaknesses, and assumptions.
- Unfortunately many under-the-hood assumptions lead to disagreements up to a factor of a few for what looks like the same inputs chosen by the user.
- At present, the most popular, best documented, easiest to use, and likely most tested HF code on the market is Talys, though another front-runner is EMPIRE
- If you use these, remember GI-GO (Garbage In, Garbage Out)





Talys: https://nds.iaea.org/talys/

Empire-II: https://www-nds.iaea.org/empire218/