

# Nuclear Reactions: Application to Direct Reactions

# **Charlotte Elster**









# **Types of Direct Reactions:**

Can identify various types of DI processes that can occur in reactions of interest:

- Elastic scattering: A(a, a)A zero Q-value internal states unchanged.
- 2. Inelastic scattering: A(a,a')A Target is in excited state





# Physics extracted from direct reaction: Elastic scattering:

•The opticians among us recall that diffraction on a sharp edge results in a diffraction pattern with the first minimum at  $\theta \approx \frac{\lambda}{2R}$  and succeeding minima at roughly equal spacing, with a decreasing maxima [like the sinc( $\theta$ ) function]





# **Types of Direct Reactions cont'd:**

- 3. Breakup reactions: Usually referring to breakup of projectile *a* into two or more fragments. This may be **elastic** breakup or **inelastic** breakup depending on whether target remains in ground state.
- 4. Transfer reactions:

Stripping: Pickup:

5. Charge exchange reactions: mass numbers remain the same. Can be elastic or inelastic.





# **Elastic scattering:**



Traditionally used to extract optical potentials, rms radii, density distributions

Eur. Phys. J. A **15**, 27–33 (2002) DOI 10.1140/epja/i2001-10219-7 THE EUROPEAN Physical Journal A

# Nuclear-matter distributions of halo nuclei from elastic proton scattering in inverse kinematics

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# Matter distributions for <sup>6,8</sup>He and <sup>6,8,9,11</sup>Li measured

# **Inelastic scattering:**

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Fig. 2. Comparison of B(E1) values obtained from lifetime and Coulomb excitation measurements. The weighted average of lifetime measurements [3] (open circle) is plotted on the left along with the weighted average (solid circle) of three Coulomb excitation measurements (solid symbols). The individual Coulomb excitation measurements, GANIL (this work, square), MSU (up triangle) [6], RIKEN (down utangle) [7], and a previous GANIL experiment (diamond) [4], are plotted versus the beam energy. Traditionally used to extract electromagnetic transitions or nuclear deformations. X(<sup>11</sup>Be, <sup>11</sup>Be\*)





# Physics extracted from direct reaction: Inelastic scattering:

At a fixed angle, charged particles observed at energies below the energy expected for Elastic scattering are signatures of excited states that were populated.

#### 

- E.g. for <sup>13</sup>C(α,n)<sup>16</sup>O, the ground-state J<sup>π</sup> are known for <sup>13</sup>C, <sup>16</sup>O, α, and n.
- At low energy, only low ℓ are relevant, and here ℓ=1 is the lowest ℓ that conserves spin ...so the neutron angular distribution gives ℓ-neutron, which is determined by J of the <sup>17</sup>O state

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#### The relative intensity of each peak is related to the wavefunction overlap between the initial state (beam + target) and final state (recoil + ejectile)





Traditionally used to extract spin, parity, spectroscopic factors example: <sup>132</sup>Sn(d,p)<sup>133</sup>Sn





## **Factors influencing the direct process:**

- The general characteristics of a particular reaction type allows one to estimate whether the direct reaction mechanism is important or not
- Consider a deuteron stripping reaction, (d,p)
  - For this case, (by definition) a charged particle needs to leave the nucleus
  - It is unlikely the charged particle is going to be able to "evaporate" out of a nucleus that has absorbed energy from a projectile and shared it among the nucleons (in a statistical process), since the proton has to tunnel out of the Coulomb barrier
  - For the direct reaction, the emitted proton carries a larger portion of the reaction energy, and so tunneling out is less problematic.
  - Thus, the direct mechanism is favored for this case



#### **Angular distribution:**

- Due to the quick crossing time, there is little chance for many scattering-type events to happen for the projectile within the target
- As such, it is expected that the direct reaction products should be forward-peaked [i.e. along the beam direction], as we've seen for elastic scattering
- Consider the case where an incident projectile interacts with only the outer layer of a nucleus
   [where all deeper interactions correspond to a different reaction mechanism]
   without worrying about what the ejectile is
   [i.e. it could be the same particle as the projectile, or it could be something else]
- For a surface interaction, it's difficult to impart much momentum to the target, so generally low-lying excitations(including no excitation) will occur
- Considering a momentum triangle for the reaction (e.g. for (d,n)), it's clear that low-lying excitations imply forward-peaked reaction products



Angular distribution example: Deuteron stripping reaction <sup>90</sup>Zr(d,p) for a 5 MeV neutron

- $p_d = \sqrt{2m_d E_d} \approx 140 MeV$
- The reaction Q-value and excitation energy of the recoil nucleus are much less than the incoming deuteron energy, so , so  $p_p\approx p_d\approx 140 MeV$
- Note that  $p^2 = p_a^2 + p_b^2 2p_a p_b \cos(\theta) = (p_a p_b)^2 + 2p_a p_b (1 \cos(\theta))$
- So,  $p \approx \sqrt{2p_a p_b (1 \cos(\theta))}$  and it's still true that  $p = l\hbar/R$
- Meaning,  $l \approx \frac{c}{\hbar c} R \sqrt{2p_a p_b (1 \cos(\theta))}$
- For this case  $l = \frac{c}{197 MeV fm} r_0 90^{1/3} \sqrt{2(140 MeV/c)(140 MeV/c)(1 \cos(\theta))} \approx 8 \sin(\frac{\theta}{2})$
- I.e. l=0 at  $0^\circ$  , l=1 at  $14^\circ$ , etc.
- This of course is a classical estimate, what it really tells us is the angle  $\theta_l$  at which the angular distribution for a given l transfer will peak



#### **Angular distribution example cont'd:**

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\*These are plots of  $\theta_{cm}$  and we did our calculations for  $\theta_{lab}$ , but for a light low-energy beam on a heavy target,  $\theta_{cm} \approx \theta_{lab}$ 

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### **Spectrocopy:**

- Since the angular distribution of the ejectile is directly related to the *l* transfer in the reaction, we can use direct reactions to do spectroscopy
- If J<sup>π</sup> is known for the target (which is presumably in the ground state) and l is the angular momentum brought into the nucleus by the particle stripped from the projectile, these can combine to form a state of some spin in the recoil nucleus
- For, e.g. X(d,p)Y, the allowed spin for the excited state populated is in the range:  $|(|J_X - l_n| - \frac{1}{2})| \le J_{Y^*} \le J_X + l_n + \frac{1}{2}$ with parity constrained by  $\pi_X \pi_{Y^*} = (-1)^l$
- Note there that the transferred angular momentum is  $l \pm s$ , where l and s correspond to the transferred nucleon



# **Breakup:**





Fig. 1. Doppler corrected  $\gamma$ -ray spectra measured in coincidence with an <sup>22</sup>O fragment and one neutron for Pb (symbols) and C (shaded area) targets. Arrows indicate the strongest  $\gamma$  transitions as expected from the <sup>22</sup>O level scheme of Ref. [10] (partial level scheme shown as inset; level energies are in keV).

# <sup>23</sup>O(Pb,Pb)<sup>22</sup>O+n+γ

[Nociforo et al, PLB 605 (2005) 79]





# What is or can be measured:

- Z and A of emitted particles
- Laboratory energies and angles of emitted particles
- Cross sections (probability of a reaction taking place)
  - $\sigma_{tot}$ ,  $\sigma(\theta)$  or  $d\sigma/d\Omega$ ,  $\sigma(E)$  or  $d\sigma/dE$ ,  $d^2\sigma/dEd\Omega$  etc.
  - Shapes of angular distributions can inform about reaction mechanisms and properties of the residual nuclei, e.g. sizes, shapes, spins and parities of levels
  - Energy dependence can be used to identify resonances
- Orientation of spins of projectiles and/or emitted particles
  - Spin observables like analyzing powers A<sub>y</sub> or spin rotation functions Q may inform about spin dependent processes

# Theory needs to predict or postdict the same quantities.



# **Exotic nuclei are usually short lived:**

Thus one can only study them through reactions: Have to be studied with reactions in inverse kinematics





Many-body problem



Quantum mechanical scattering problem

"idealist" approach: Just do it!





## "Idealist Approach": Ab initio Theory

### This should contain:

- Degrees of freedom for all nucleons in the problem
- "Realistic" nucleon-nucleon (NN) and three-nucleon forces (3NFs)
- Pauli principle treated exactly

extremely difficult multi-channel scattering problem

Exactly solvable for A=3,4,5 at low energies Work by groups at TRIUMF and LLNL and others

Exactly solvable for A=3,4 via Faddeev methods (no energy restriction) Work by A. Deltuva, R. Lazauskas, H. Witala, R. Skibinski and others

Exactly solvable for A=3,4 via Hyperspherical Harmonics methods Work by A. Kievsky, M. Viviani, S. Bacca and others



# **Realistic today ≡** Chiral Expansion of nuclear forces:



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pnusl



#### Ab initio no-core shell model with continuum (NCSMC)

Seeks many-body solutions in the form of a generalized cluster expansion



- Ab initio no-core shell model (NCSM):
  - Clusters' structure, short range
- Resonating-group method (RGM):
  - Dynamics between clusters, long range

Slide from S. Quaglioni

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Deuterium-Tritium fusion

 $N = N_{\text{max}} + T$ 

N =

N =

Big Bang nucleosysthesis of light nuclei \_

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 $= N_{max}\hbar\Omega$ 

Fusion research and plasma physics





#### G. Hupin, S. Quaglioni, and P. Navratil, Phys. Rev. C 90, 061601(R) (2014)





G. Hupin, S. Quaglioni, and P. Navrátil, in progress

#### **Chart of light nuclei**



"Idealists" current domain

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#### For heavier nuclei:

#### A more "pragmatic" approach:



#### **Reduce the Many-Body to a Few-Body Problem**

#### Isolate relevant degrees of freedom

# Solve the few-body problem with effective interactions



# Intuitive ideas leading to effective interaction theory

Basic ideas for dealing with the many-body, stong (non-perturbative) nuclear interaction problem began with scattering.

A seminal idea was due to Leslie L. Foldy working on sonar during WWII.

Foldy described projectile scattering from a nucleus as a wave propagating through many, dense scattering sources with a complex (absorptive) index of refraction. His essential idea was to express the total scattered wave in terms of individual N+N scattered waves, rather than in terms of the very strong N+N interaction which can not be expanded in a perturbation theory.



L.L. Foldy, Phys. Rev. 67, 107 (1945)

#### check out:

The Journal of the Acoustical Society of America, Vol. **132**, 1960 (2012) Mulltiple Scattering in the Spirit of Leslie Foldy



# Intuitive ideas leading to effective interaction theory

In 1950 Geoffrey Chew introduced the **"impulse approximation"** as a suitable way to simplify the intractable A+1 – body problem (e.g. p+A, n+A) to an effective two-body scattering problem.



#### Seminal papers that introduced multiple scattering theory:

Chew, Phys. Rev. **80**, 196 (1950). Chew and Wick, Phys. Rev. **85**, 636 (1952). Chew and Goldberger, Phys. Rev. **87**, 778 (1952).

#### Basic ideas:

- 1) the full A+1 scattering can be accurately represented as a coherent sum of individual hadron + nucleon scatterings and re-scatterings from nucleons in the target nucleus
- 2) at high energies the free-space hadron + nucleon scattering amplitude is unaffected by the nuclear medium
- 3) the hadron + nucleus scattering amplitude should be expanded in terms of the two-body scattering amplitudes, rather than directly in terms of the N+N potential.

# Intuitive ideas leading to effective interaction theory

In 1951 **Melvin Lax** extended these approaches to obtain an effective interaction potential, later called the "optical potential" to represent the effective p+A interaction.

Rev. Mod. Phys. 23, 287 (1951), Phys. Rev. 85, 621 (1952)



First representation of such an effective potential. Introduced the socalled "tp" form, where  $\rho$  is the nuclear density and t represents an effective N+N interaction

# **Global Phenomenological Optical Potential**





Same importance as NN phase shift analysis

Fig. 2. Comparison of predicted neutron total cross sections and experimental data, for nuclides in the Mg-Ca mass region, for the energy range 10 keV-250 MeV.

# Phenomenological effective interaction for describing elastic scattering as single-channel problem

Contributions to an optical potential U(r)

- Coulomb part:  $V_c(r) = Z_1 Z_2 e^2/r$
- Real Nuclear (short range) part: V(r)
  - Should describe nuclear attraction
- Imaginary Nuclear Part: W(r)
  - Other things can happen so that flux is lost  $\rightarrow W(r)$  negative
- Spin-Orbit (L·S) part:  $V_{SO}(r)$ 
  - Spin-orbit force in the NN interaction, needed to describe polarization data

General form:

$$U(r) = V_C(r) + V(r) + iW(r) + V_{SO}(r)$$



# Phenomenological effective interaction for describing elastic scattering as single-channel problem

The form follows our rough understanding of the density profile of nuclei:

"Woods-Saxon" or "Saxon-Woods" parameterization

$$V(r) = \frac{-V_0}{1 + e^{(r - R_R)/a_R}}$$

$$W(r) = \frac{-W_0}{1 + e^{(r - R_I)/a_I}}$$

 $R=r_0(A_1^{1/3}+A_2^{1/3})$  is the radius where the potential is  $\frac{1}{2}$  its maximum

*"a"* is the "diffuseness" parameter. Describes the "spread" of the potential about *R* 

"Volume" terms: parameters are  $V_0$ ,  $R_R$ ,  $a_R$ ,  $W_0$ ,  $R_I$ ,  $a_I$ 



# Phenomenological potentials fitted to stable nuclei







# Landscape of global optical potentials



energy **†** 





# Phenomenological effective interaction for describing elastic scattering as single-channel problem

Best fit of elastic scattering data for a wide range of nuclei and energies Cross sections, angular distributions, polarizations

**Examples:** 

- Becchetti Greenlees, Phys. Rev. 182, 1190 (1969)
- E.D. Cooper et al, PRC47, 297 (1993)
- Koning Delaroche, NP A713, 231 (2003)
- Weppner-Penney, PRC80, 034608 (2009)
- W. Dickhoff et al. (review Prog.Part.Nucl.Phys 105, 252 (2019) dispersive opt. Model [DOM]

Woods-Saxon parameterized phenomenological potentials widely used in Scattering codes like FRESCO, TALYS, ECIS

#### In the era of *ab initio* nuclear theory we want to do better

# Ideas leading to effective interaction theory

In 1953 **Kenneth M. Watson** gathered up emerging ideas and published the first *formal* scattering solution for the p+A problem



K. M. Watson

K. M. Watson, Phys. Rev. 89, 575 (1953).

Later more explicit description

## Ideas leading to effective interaction theory

In 1959 Arthur Kerman, Hugh McManus and Roy Thaler modified the Watson theory by re-organizing the expansion and paved the way an accurate (numerical) application for Chew's impulse approximation:

Ann. Phys. 8, 551 (1959)



### Ideas leading to effective interaction theory



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At the same time **Herman Feshbach** and collaborators developed a powerful projection operator formalism, which they used to generate a perturbation expansion of the optical potential and which could be applied to reactions other than elastic scattering.

Ann. Rev. Nucl. Sci 8, 49 (1958)

Ann. Phys. (NY) 5, 357 (1958)

Ann. Phys. (NY) 19, 287 (1962)

### Start with theory:

#### Isolate relevant degrees of freedom



Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on **P** space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly (Feshbach, Annals Phys. 5 (1958) 357-390)

#### **Effective Interactions: non-local and energy dependent**





#### Isolate relevant degrees of freedom



Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on **P** space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly (Feshbach, Annals Phys. 5 (1958) 357-390)







### Goal: effective interaction from ab initio methods

Start from many-body Hamiltonian with 2 and 3 body forces

#### Theoretical foundations laid by Feshbach and Watson in the 1950s

#### Feshbach:

effective nA interaction via Green's functions from a solution of the many body problem using basis function expansion, e.g. CCGF, SCGF (current truncation to singles and doubles)



<sup>40</sup>Ca(n,n)<sup>40</sup>Ca

Rotureau, Danielewicz, Hagen, Jansen, Nunes PRC 98, 044625 (2018)



Idini, Barbieri, Navratil J.Phys.Conf. 981. 012005 (2018) Acta Phys. Polon. B48, 273 (2017)



# Goal: effective interaction from ab initio methods

Start from many-body Hamiltonian with 2 (and 3) body forces

### Theoretical foundations laid by Feshbach and Watson in the 1950s

#### Feshbach:

effective nA interaction via Green's function from solution of many body problem using basis function expansion,
 e.g. SCGF, CCGF (current truncation to singles and doubles)

energy ~ 10 MeV

### Watson:

#### Multiple scattering expansion, e.g. spectator expansion (current truncation to two active particles)

#### **Spectator Expansion:**

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Chinn, Elster, Thaler, Weppner (1995)

### **Expansion in:**

- particles active in the reaction
- antisymmetrized in active partic

Intended for "fast reaction", i.e. ≥ 80 MeV





# **Framework for ab initio Elastic Scattering**



Structure theory: no-core shell model





# Reaction theory: **spectator expansion**





## Framework for ab initio Elastic Scattering (spectator expansion)





physics + astronomy Target nucleus has 0<sup>+</sup> ground state

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#### **Beyond NCSM: SA-NCSM One-Body Densities**

NNLO<sub>opt</sub> chiral potential



### Spin-orbit local part $U_{so}(\zeta)$



#### Central local part $U_{c}(\zeta)$







# **Connect back to phenomenological Optical Potential**

- The optical model is a way to do this, where the potential is something like U(r) = V(r) + iW(r)
- As we saw much earlier in class, the Woods-Saxon form is the best bet for V(r)
- Since absorption is mostly going to happen on the surface, typically  $W(r) \propto dV/dr$
- Solving for the optical model parameters for one case means reaction cross sections can be solved for many more cases with the same projectile and similar A for the target

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Ab initio calculations do not support dV/dr terms Potential well is given very well. Spin-orbit potential is also a Woods-Saxon function multiplied with r Imaginary part of the spin-orbit potential is very small.





#### TOWARDS A CONSISTENT APPROACH FOR NUCLEAR STRUCTURE AND REACTIONS: MICROSCOPIC OPTICAL POTENTIALS



17 June 2024 — 21 June 2024

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Work on a consistent approach to structure and reactions is very timely. Microscopic optical potentials are needed in many aspects in calculations of nuclear reactions.

#### White paper: Optical Potentials for the rareisotope beam era

Published in: .Phys.G 50 (2023) 6, 060501

e-Print: 2210.07293 [nucl-th]

