The Electron-Ion Collider (EIC)

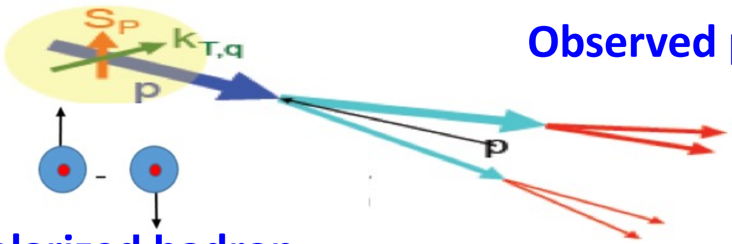
- Lec. 1: EIC & Fundamentals of QCD
- Lec. 2: Probing Structure of Hadrons
  without seeing Quark/Gluon?
  – *breaking the hadron*!
- Lec. 3: Probing Structure of Hadrons
  with polarized beam(s)
  – *Spin as another knob*
- Lec. 4: Probing Structure of Hadrons
  without breaking them?
  Dense Systems of gluons
  – *Nuclei as Femtosize Detectors*

Jianwei Qiu

Theory Center, Jefferson Lab

TMDs: Correlation between Hadron Property and Parton Flavor-Spin-Motion

- **Quantum correlation between hadron spin and parton motion:**



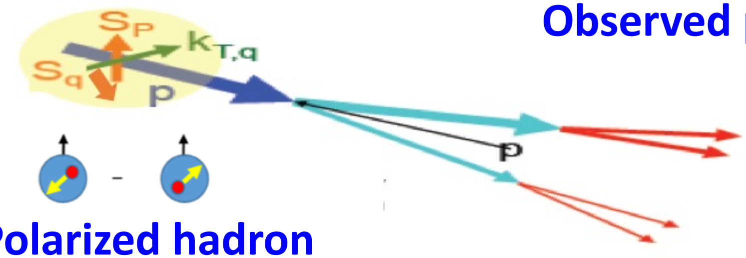
Observed particle

Polarized hadron

Sivers effect – Sivers function

Hadron spin influences
parton's transverse motion

- **Quantum correlation between hadron spin and parton spin:**



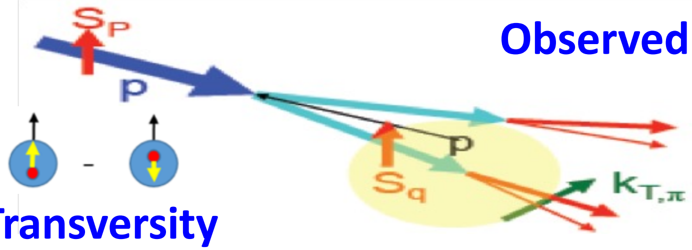
Observed particle

Polarized hadron

Pretzelosity – model OAM

Hadron spin and parton spin
influence
parton's transverse motion

- **Quantum correlation between parton's spin and its hadronization:**



Observed particle

Transversity

Polarized hadron

Collins effect – Collins function

Parton's transverse polarization
influences its hadronization

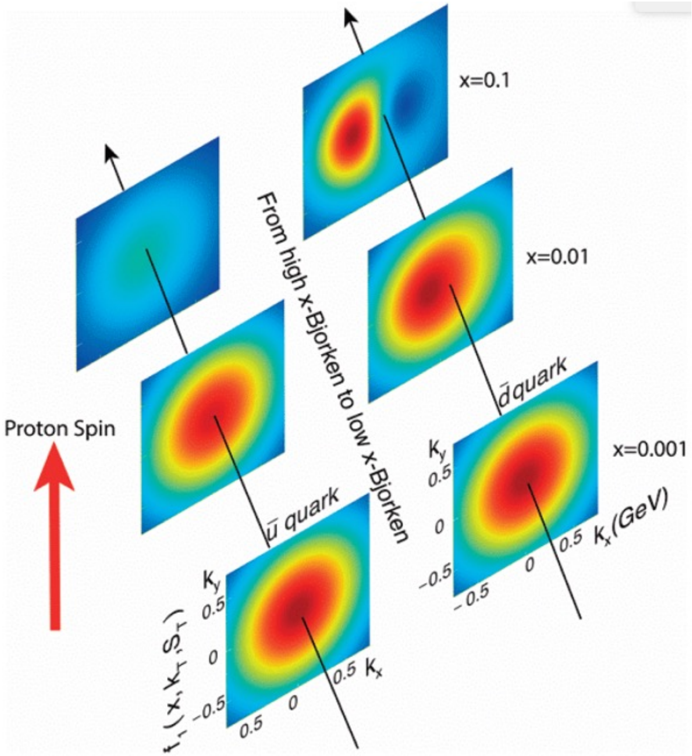


Fig. 2.7 NAS Report

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Polarization and Spin Asymmetry

Explore new QCD dynamics by varying the spin orientation

- **Cross section:**

Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

- **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ **– Positive definite**

- **Asymmetries or difference of cross sections:**

– Not necessary positive!

  - **both beams polarized** A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

  - **one beam polarized** A_L, A_N

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Dual Roles of the Proton Spin Program

- Nucleon Spin – without it, our visible world would not be the same!
- Proton is a composite particle:

  Spin is a consequence of internal dynamics of the bound state

  For example, the nucleon-nucleon interaction and shell structure determines the observed nuclear spin states

  ➡ Decomposition of proton spin in terms of quark and gluon d.o.f. helps to understand the dynamics of a fundamental QCD bound state
  – Nucleon is a building block of all hadronic matter (> 95% mass of all visible matter)

- Use the spin as a tool – asymmetries:

  Cross section is a probability – classically measured

  Spin asymmetry – the difference of two cross sections involving two different spin states

Asymmetry could be a pure quantum effect!

Spin of a Composite Particle

- **Spin:**
  - Pauli (1924): "two-valued quantum degree of freedom" of electron – 1st formulation of spin
  - Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant $\hbar$)
  - Composite particle = Total angular momentum when it is at rest
- **Spin of a nucleus:**
  - Nuclear binding: 8 MeV/nucleon << mass of nucleon
  - Nucleon number is fixed inside a given nucleus
  - Spin of a nucleus = sum of the valence nucleon's spin
- **Spin of a nucleon – Naïve Quark Model:**
  - If the probing energy << mass of constituent quark
  - Nucleon is made of three constituent (valence) quarks
  - Spin of a nucleon = sum of the constituent quark's spin

Pauli and Bohr observing spin, 1954

State: $|p\uparrow\rangle = \sqrt{\frac{1}{18}}\left[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{perm.}\right]$

Spin: $S_p \equiv \langle p\uparrow|S|p\uparrow\rangle = \frac{1}{2}, \quad S = \sum_i S_i$ *Carried by valence quarks*

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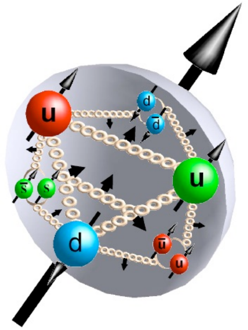
Spin of a Composite Particle



- **Spin of a nucleon – QCD:**
  - ✧ **Current quark mass << energy exchange of the collision**
  - ✧ **Number of quarks and gluons depends on the probing energy**
- **Angular momentum of a proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 | \, \hat{J}_f^z \, | P, S_z = 1/2 \rangle = \frac{1}{2}$$

- **QCD Angular momentum operator:**

$$J^i_{\rm QCD} = \frac{1}{2} \epsilon^{ijk} \int d^3x \; M^{0jk}_{\rm QCD}$$

Energy-momentum tensor

$$M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD} \, x^\mu - T^{\alpha\mu}_{\rm QCD} \, x^\nu$$

Angular momentum density

- ✧ **Quark angular momentum operator:**

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma} \gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D})) \, \psi_q \right]$$

- ✧ **Gluon angular momentum operator:**

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B}) \right]$$

Understanding how quark/gluon contribute to proton's spin needs to have the matrix elements of these partonic operators measured independently

Current Understanding for Proton Spin

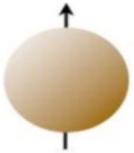
- **The sum rule:**

$$S(\mu) = \sum_f \langle P,S|\hat{J}^z_f(\mu)|P,S\rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$

  - **Infinite possibilities of decompositions – connection to observables?**
  - **Intrinsic properties + dynamical motion and interactions**

- **An incomplete story:**

Proton Spin



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …

Orbital Angular Momentum of quarks and gluons
Little known

Quark helicity
Best known

$$\frac{1}{2}\int dx\left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

$$\sim 30\%$$

Gluon helicity
Start to know

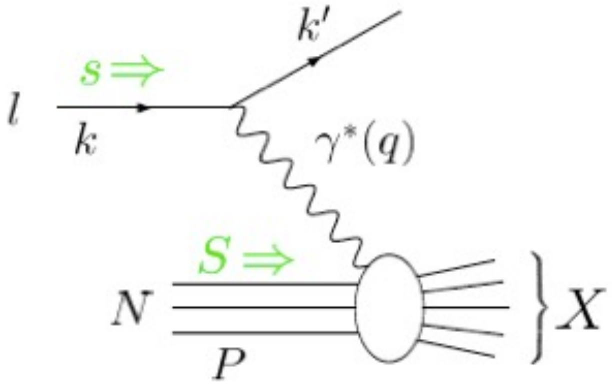
$$\Delta G = \int dx \Delta g(x)$$

$$\sim 40\%(\text{with RHIC data})$$

Net effect of partons' transverse motion?

Polarized Deep Inelastic Scattering

❑ DIS with polarized beam(s):



"Resolution" $Q \equiv \sqrt{-q^2}$

$$\frac{\hbar}{Q} = \frac{2\times 10^{-16}\mathrm{m}}{Q/\mathrm{GeV}} \lesssim 10^{-16}\mathrm{m} = 1/10\mathrm{fm}$$

"Inelasticity" – known as Bjorken variable

$$x_B = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2 + M_X^2 - m^2}$$

✧ **Recall – from lecture 2:**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$
$$+ iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p.q} g_1\left(x_B, Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2} g_2\left(x_B, Q^2\right)\right]$$

✧ **Polarized structure functions:** $g_1(x_B, Q^2),\ g_2(x_B, Q^2)$

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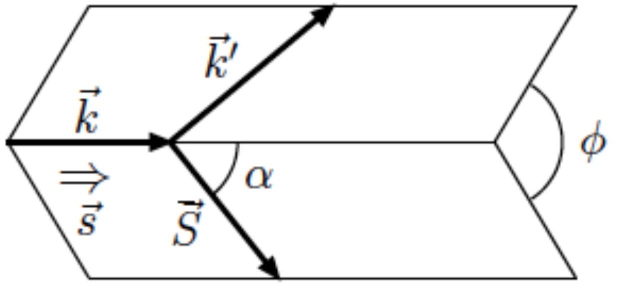
Polarized Deep Inelastic Scattering

- **Extract the polarized structure functions:**

$$\mathcal{W}^{\mu\nu}(P,q,S) - \mathcal{W}^{\mu\nu}(P,q,-S)$$



- ✧ **Define:** $\angle(\hat{k},\hat{S}) = \alpha$ **, and lepton helicity** λ

- ✧ **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times \left\{ \cos\alpha \left\{ \left[1 - \frac{y}{2} - \frac{m^2x^2y^2}{Q^2} \right] g_1(x,Q^2) - \frac{2m^2x^2y}{Q^2} g_2(x,Q^2) \right\} \right.$$

$$\left. - \sin\alpha\cos\phi \frac{2mx}{Q} \sqrt{\left(1 - y - \frac{m^2x^2y^2}{Q^2}\right)} \left(\frac{y}{2} g_1(x,Q^2) + g_2(x,Q^2) \right) \right\}$$

- ✧ **Spin orientation:**

$$\alpha = 0 : \Rightarrow \quad g_1$$

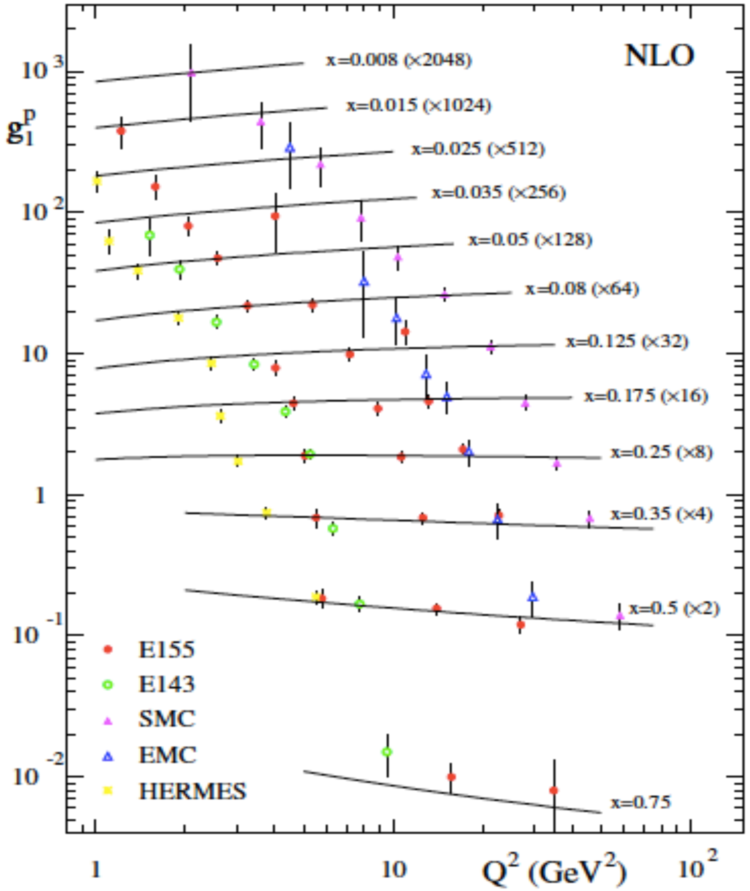
$$\alpha = \pi/2 : \Rightarrow \quad y\,g_1 + 2\,g_2$$, suppressed m/Q

Polarized Deep Inelastic Scattering

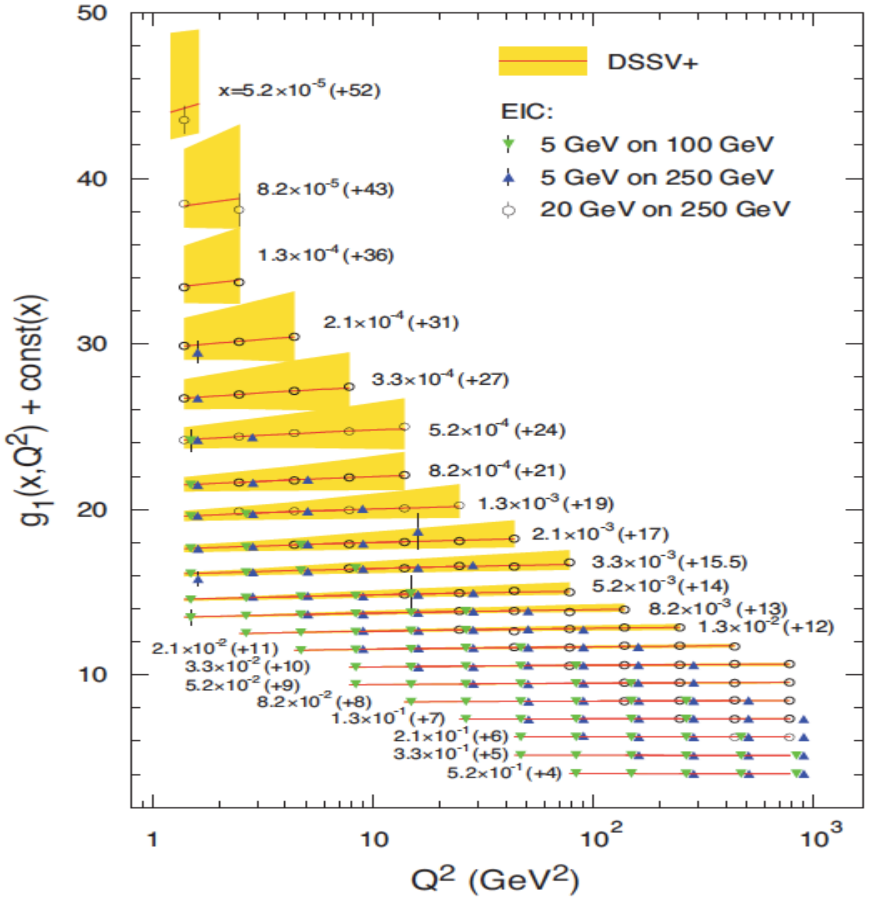
- ❑ **Spin asymmetries – measured experimentally:**
  - ✧ **Longitudinal polarization –** $\alpha = 0$

Known function

$$A_{\|} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}} = D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)} \equiv D(y)\,A_1(x,Q^2)$$

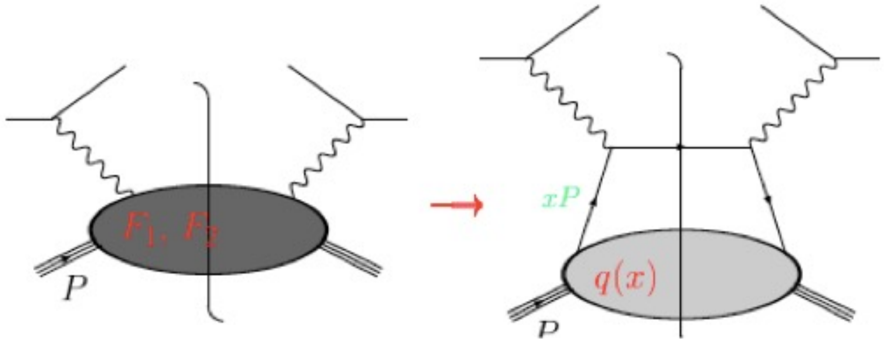


Polarized DIS at EIC



Polarized Deep Inelastic Scattering

- **Parton model results – LO QCD:**



- **Structure functions:**

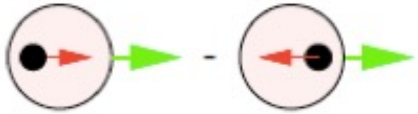
$$F_1(x) = \frac{1}{2}\sum_q e_q^2\left[q(x) + \bar{q}(x)\right]$$

$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right]$$

$$g_1 = \frac{1}{2}\left[\frac{4}{9}(\Delta u + \Delta\bar{u}) + \frac{1}{9}(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s})\right]$$

- **Polarized quark distribution:**

$$\boxed{\Delta f(\xi) \equiv f^{+}(\xi) - f^{-}(\xi)}$$



Information on nucleon's spin structure

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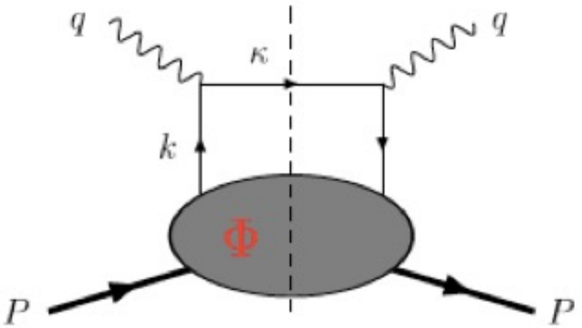
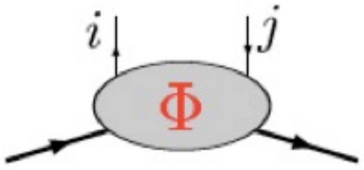
Polarized Deep Inelastic Scattering

❑ Systematics polarized PDFs – LO QCD:





✧ **Two-quark correlator:**

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4\, \delta^4(P-k-P_X)\, \langle PS|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|PS\rangle$$

$$= \int d^4z\, e^{ik\cdot z} \langle PS\,|\,\bar{\psi}_j(0)\,\psi_i(z)\,|\,PS\rangle$$

✧ **Hadronic tensor (one –flavor):**

$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4} \delta\left((k+q)^2\right)\ \mathrm{Tr}\left[\Phi\, \gamma^\mu (\not{k}+\not{q})\gamma^\nu\right]$$

Polarized Deep Inelastic Scattering

✧ **General expansion of $\phi(x)$:**

must have general expansion in terms of $\not{P}$, $\not{n}$, $\not{s}$ etc.

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{||}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_\perp\right]$$

✧ **3-leading power quark parton distribution:**

$$q(x) = \frac{1}{4\pi}\int dz^- \,\mathrm{e}^{iz^- xP^+}\langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\,(0,z^-,\mathbf{0}_\perp)\,|P,S\rangle$$

$$\Delta q(x) = \frac{1}{4\pi}\int dz^- \,\mathrm{e}^{iz^- xP^+}\langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\,(0,z^-,\mathbf{0}_\perp)\,|P,S\rangle$$

$$\delta q(x) = \frac{1}{4\pi}\int dz^- \,\mathrm{e}^{iz^- xP^+}\langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_\perp\gamma_5\,\psi\,(0,z^-,\mathbf{0}_\perp)\,|P,S\rangle$$

"unpolarized" – "longitudinally polarized" – "transversity"

Polarized Deep Inelastic Scattering

❑ Physical interpretation:

$$q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right) \times \left[\left|\langle X|\, \mathcal{P}^+ \psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2 + \left|\langle X|\, \mathcal{P}^- \psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2\right]$$

$$\Delta q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right) \times \left[\left|\langle X|\, \mathcal{P}^+ \psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2 - \left|\langle X|\, \mathcal{P}^- \psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2\right]$$

$$\delta q(x) = \frac{1}{2}\sum_X \delta(P_X^+ - (1-x)P^+) \times \left[\left|\langle X|\, \mathcal{P}^\uparrow \psi_+(0)\,|P,S_\perp=\frac{1}{2}\rangle\right|^2 - \left|\langle X|\, \mathcal{P}^\downarrow \psi_+(0)\,|P,S_\perp=\frac{1}{2}\rangle\right|^2\right]$$

Spin projection:

$$\mathcal{P}^\pm \equiv \frac{1\pm\gamma_5}{2}$$

and

$$\mathcal{P}^{\uparrow\downarrow} \equiv \frac{1\pm\gamma_\perp\gamma_5}{2}$$

Basics for Spin Observables

- **Factorized cross section:**

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$e.g.\ \mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-)$

with $\hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$

- **Parity and Time-reversal invariance:**

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

- **IF:** $\langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

or $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

Operators lead to the "+" sign ➡ **spin-averaged cross sections**

Operators lead to the "-" sign ➡ **spin asymmetries**

- **Example:**

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \;\Rightarrow\; q(x)$$

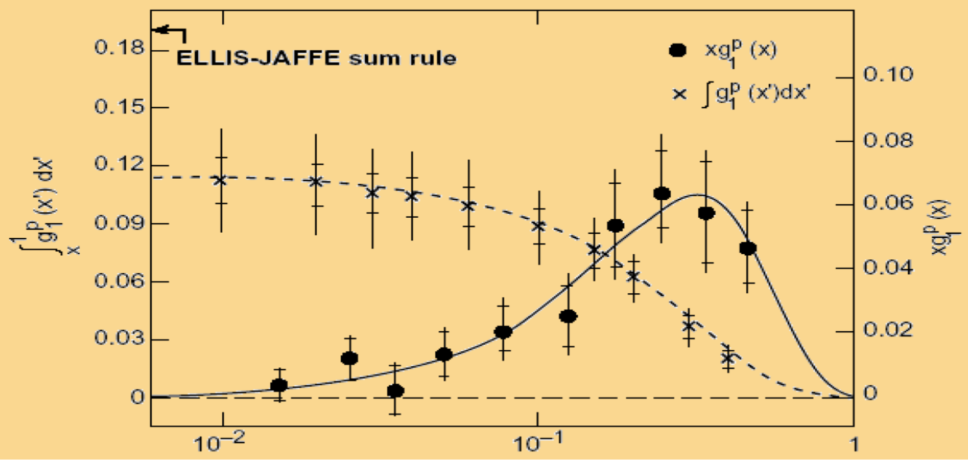
$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \;\Rightarrow\; \Delta q(x)$$

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \;\Rightarrow\; \delta q(x) \rightarrow h(x)$$

$$\mathcal{O}(\psi,A^\mu) = \frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow \Delta g(x)$$

Proton "Spin Crisis" – Excited the Field

- **EMC (European Muon Collaboration '87) – "the Plot":**



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

✧ **Combined with earlier SLAC data:** $$\int_0^1 g_1^p(x)dx = 0.126 \pm 0.018$$

✧ **Combined with:** $g_A^3 = \Delta u - \Delta d$ **and** $g_A^8 = \Delta u + \Delta d - 2\Delta s$

from low energy neutron & hyperon β decay

$$\Rightarrow \quad \Delta\Sigma = \sum_q\left[\Delta q + \Delta\bar{q}\right] = 0.12 \pm 0.17$$

- **"Spin crisis" or puzzle:**

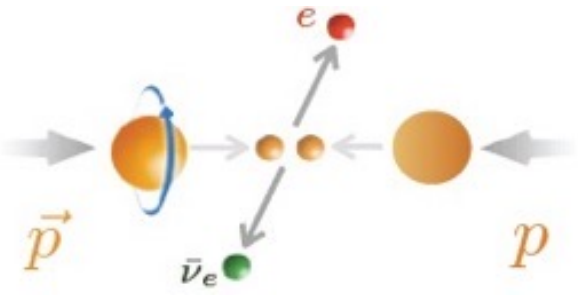
✧ **Strange sea polarization is sizable & negative**

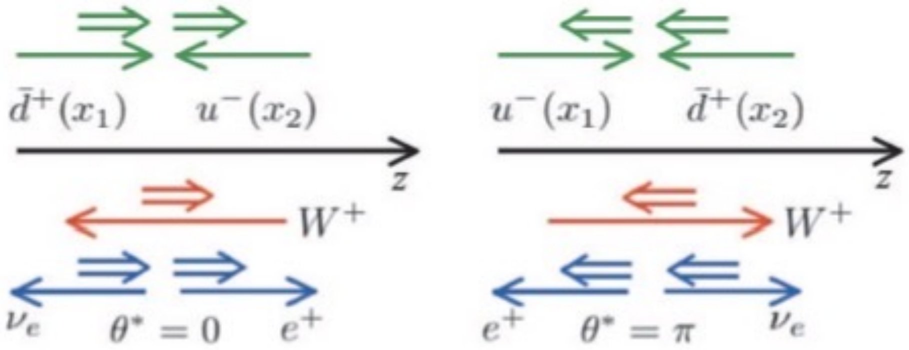
✧ **Very little of the proton spin is carried by quarks**

→ ***New era of spin physics***

Determination of Δq and Δq

- **W's are left-handed:**





- **Flavor separation:**
  - **Lowest order:**

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}}e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}}e^{-y_W}$$

  - **Forward W⁺ (backward e⁺):**

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

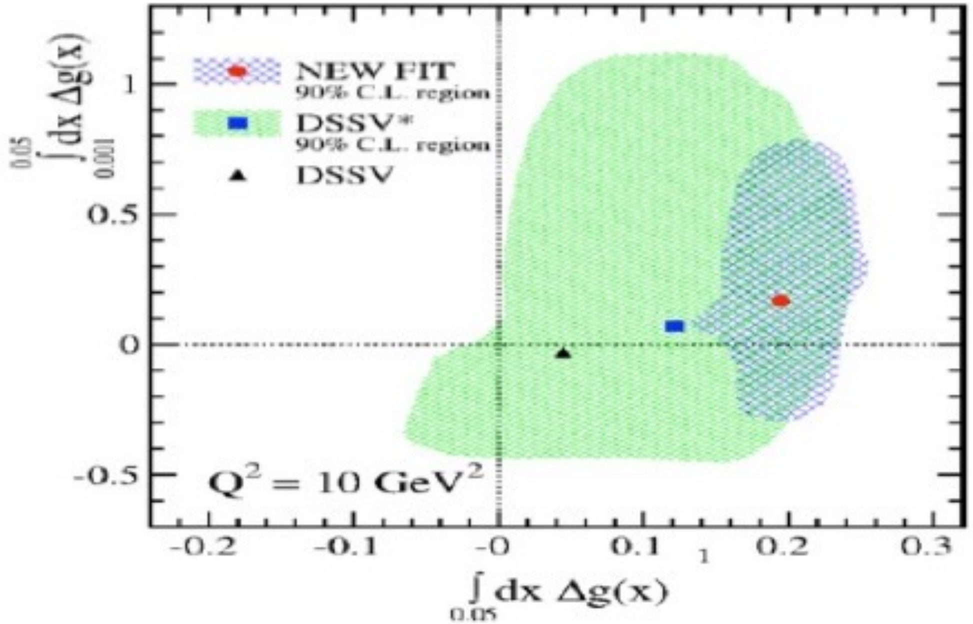
  - **Backward W⁺ (forward e⁺):**

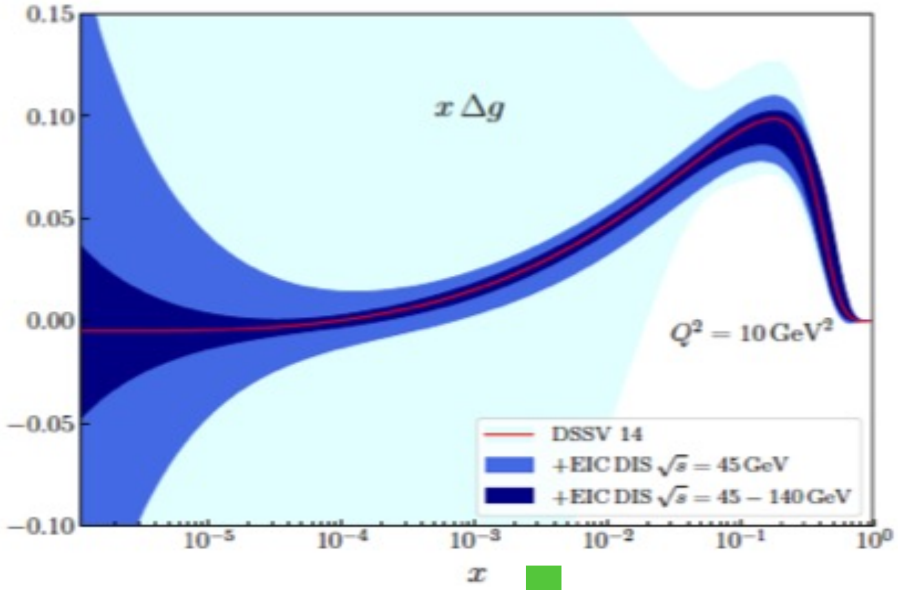
$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

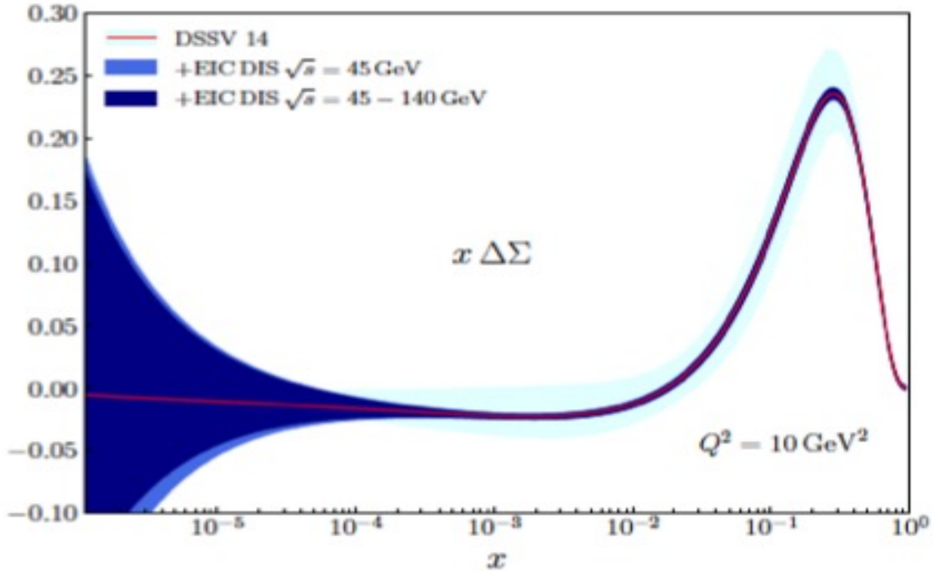
- **Complications:**

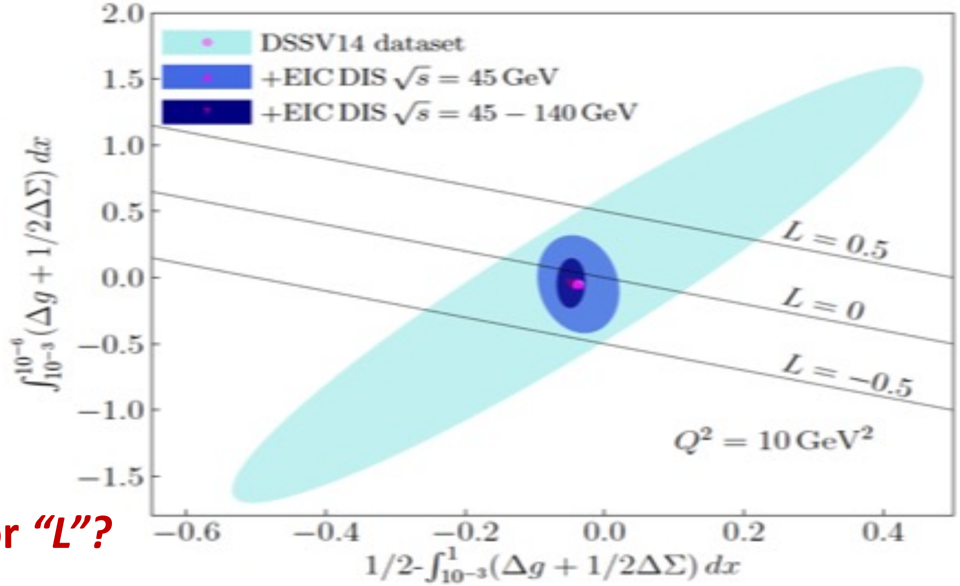
High order, W's p_T-distribution at low p_T

What the EIC can do – EIC Yellow Report?







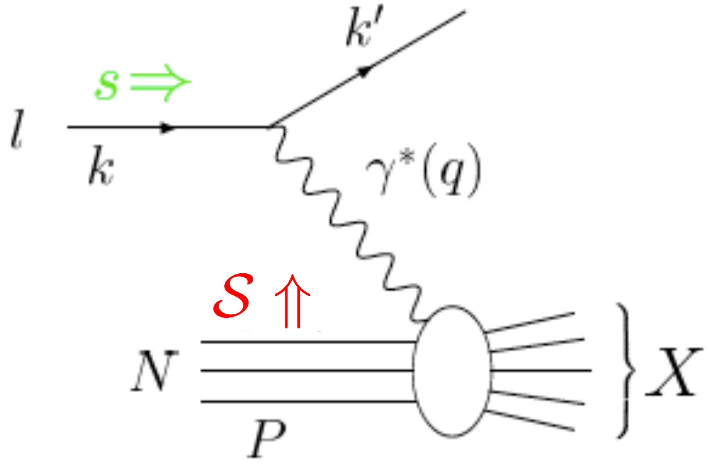


Room for *“L”*?

Transverse spin phenomena in QCD

- **40 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance**

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



Single Transverse-Spin Asymmetry (SSA)

$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions

A_N for inclusive DIS

- **DIS cross section:**

$$\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$$

- **Leptionic tensor is symmetric:**

$$L^{\mu\nu} = L^{\nu\mu}$$

- **Hadronic tensor:**

$$W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$$

- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right]$$

- **Vanishing single spin asymmetry:**

$$\boxed{\begin{aligned} A_N = 0 \quad \Leftrightarrow \quad & \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle \\ & \overset{?}{=} \langle P, -\vec{s}_\perp |\, j^\dagger_\nu(0)\, j_\mu(y)\, | P, -\vec{s}_\perp \rangle \end{aligned}}$$

A_N for inclusive DIS

- **Define two quantum states:**

$$\langle\beta| \equiv \langle P, \vec{s}_\perp | \, j^\dagger_\mu(0) \, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P, \vec{s}_\perp\rangle$$

- **Time-reversed states:**

$$|\alpha_T\rangle = V_T \, |P, \vec{s}_\perp\rangle = |-P, -\vec{s}_\perp\rangle$$

$$|\beta_T\rangle = V_T \left[j^\dagger_\mu(0) \, j_\nu(y) \right]^\dagger |P, \vec{s}_\perp\rangle$$
$$= \left(V_T j^\dagger_\nu(y) V_T^{-1} \right) \left(V_T j_\mu(0) V_T^{-1} \right) |-P, -\vec{s}_\perp\rangle$$

- **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha| \, V_T^\dagger \, V_T \, |\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\boxed{\begin{aligned}&\langle -P, -\vec{s}_\perp| \, \left(V_T j^\dagger_\nu(y) V_T^{-1} \right) \left(V_T j_\mu(0) V_T^{-1} \right) |-P, -\vec{s}_\perp\rangle \\ &= \langle P, \vec{s}_\perp | \, j^\dagger_\mu(0) \, j_\nu(y) \, |P, \vec{s}_\perp\rangle\end{aligned}}$$

A_N for inclusive DIS

- **Parity invariance:**

$$1 = U_P^{-1} U_P = U_P^{\dagger} U_P$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^\dagger(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) | -P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^\dagger(y) V_T^{-1} U_P^{-1}\right) \left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}\right) | P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | j_\nu^\dagger(-y)\, j_\mu(0) | P, -\vec{s}_\perp \rangle$$

Translation invariance:

$$\Downarrow$$

$$\boxed{\begin{aligned} &\langle P, -\vec{s}_\perp | j_\nu^\dagger(0)\, j_\mu(y) | P, -\vec{s}_\perp \rangle \\ &= \langle P, \vec{s}_\perp | j_\mu^\dagger(0)\, j_\nu(y) | P, \vec{s}_\perp \rangle \end{aligned}}$$

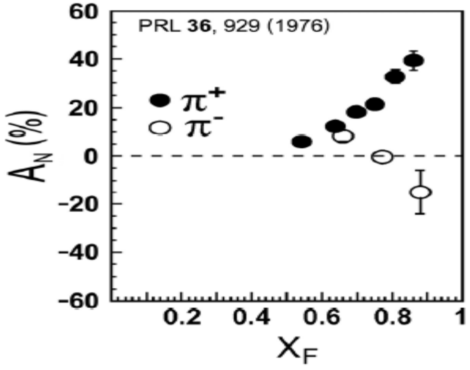
- **Polarized cross section:**

$$\boxed{\begin{aligned} \Delta\sigma(\vec{s}_\perp) &\propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right] \\ &= L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp)\right] = 0 \end{aligned}}$$

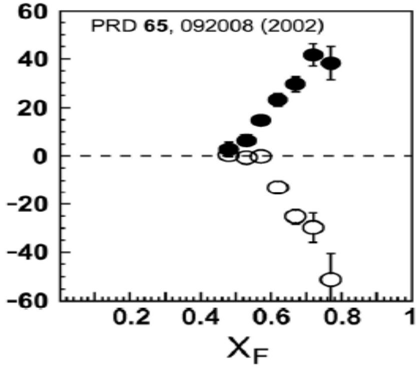
A_N in Hadronic Collisions

- A_N - consistently observed for over 40 years!

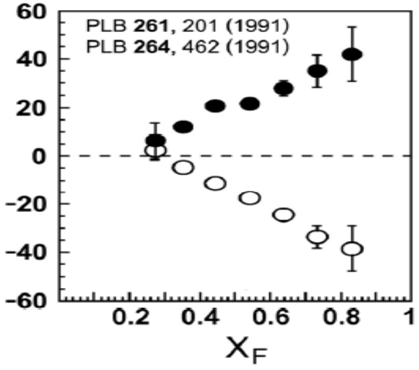
ANL – 4.9 GeV



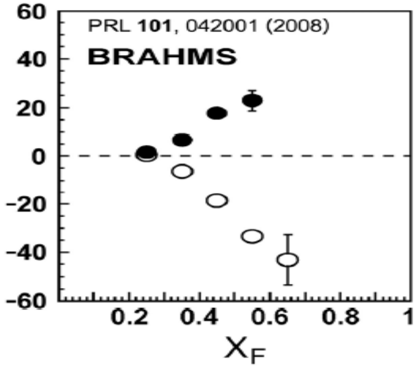
BNL – 6.6 GeV



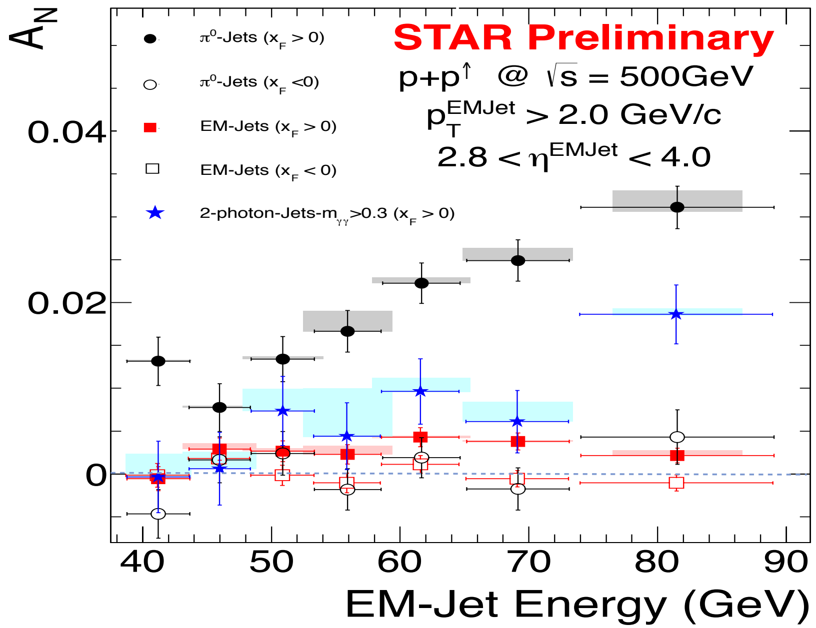
FNAL – 20 GeV

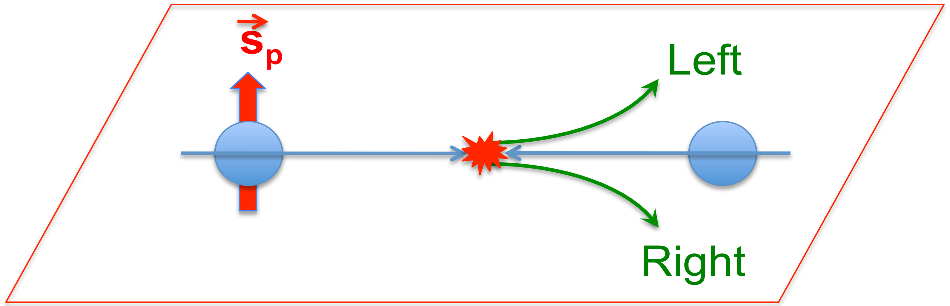


BNL – 62.4 GeV



- Survived the highest RHIC energy:





$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

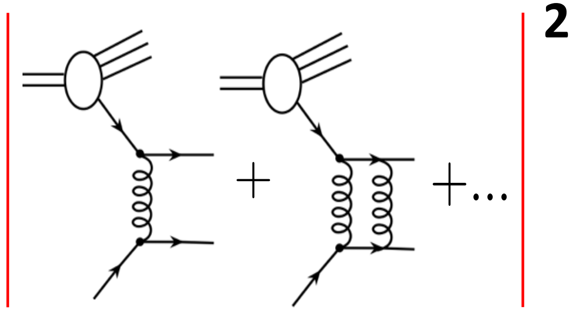
Do we understand this?

A_N in Hadronic Collisions

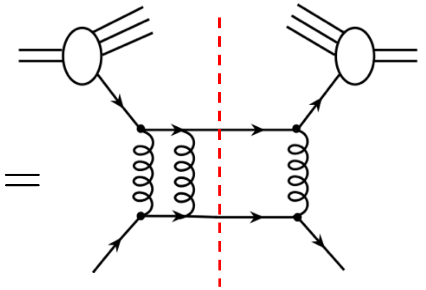
- **Early attempt:**

Kane, Pumplin, Repko, PRL, 1978

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) =$



$\propto \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

- **What do we need?**

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

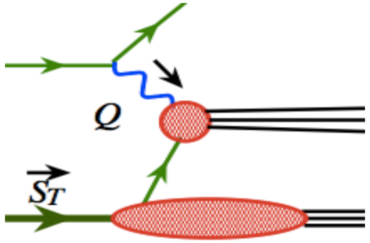
- **Vanish without parton's transverse motion:**

A direct probe for parton's transverse motion,

Spin-orbital correlation, QCD quantum interference

Current Understanding of A_N

- Symmetry plays important role:

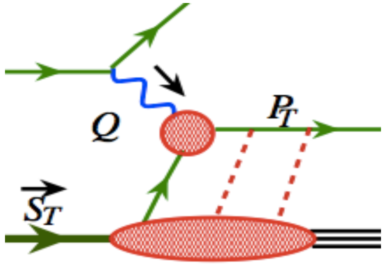


Inclusive DIS
Single scale
Q

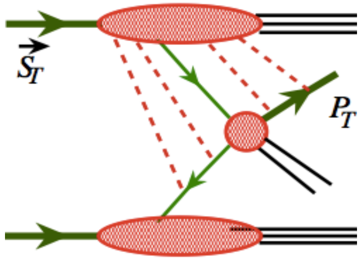
Parity
Time-reversal

→ $A_N = 0$

- One scale observables – $Q \gg \Lambda_{QCD}$:



SIDIS: $Q \sim P_T$

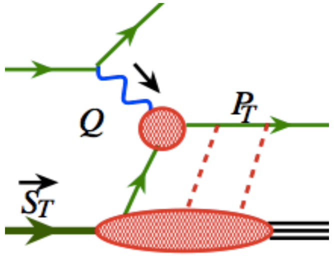


DY: $Q \sim P_T$; Jet, Particle: P_T

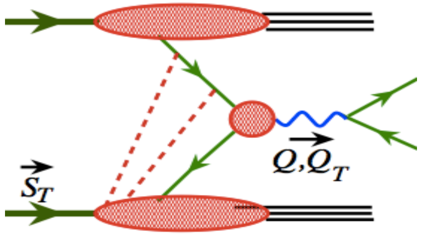
→

Collinear factorization
Twist-3 distributions

- Two scales observables – $Q_1 \gg Q_2 \sim \Lambda_{QCD}$:



SIDIS: $Q \gg P_T$



DY: $Q \gg P_T$ or $Q \ll P_T$

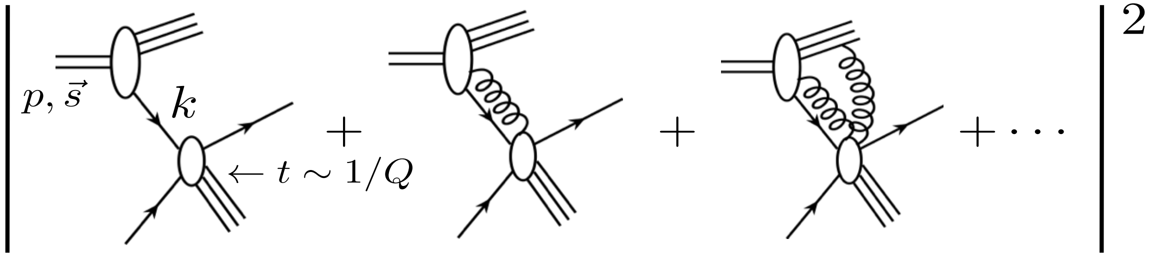
→

TMD factorization
TMD distributions

How Collinear Factorization Generates A_N?

❑ Collinear factorization beyond leading power:

$\sigma(Q, \vec{s}) \propto$



$$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$$ – **Expansion**

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

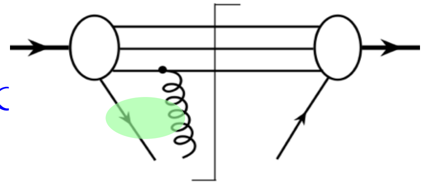
Too large to compete! Three-parton correlation

❑ Single transverse spin asymmetry:

$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

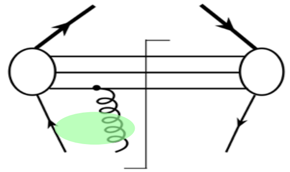
Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$T^{(3)}(x,x) \propto$



Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$



Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Needed Phase: **Integration of "dx" using unpinched poles**

Twist-3 Distributions Relevant to A_N

- **Twist-2 distributions:**
  - **Unpolarized PDFs:**
  - **Polarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$
$$\Delta q(x) \propto \langle P,S_{\|}|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_{\|}\rangle$$
$$\Delta G(x) \propto \langle P,S_{\|}|F^{+\mu}(0)F^{+\nu}(y)|P,S_{\|}\rangle(i\epsilon_{\perp\mu\nu})$$

- **Two-sets Twist-3 correlation functions:** ***No probability interpretation!***

Kang, Qiu, 2009

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

- **Twist-3 fragmentation functions:**

See Kang, Yuan, Zhou, 2010, Kang 2010

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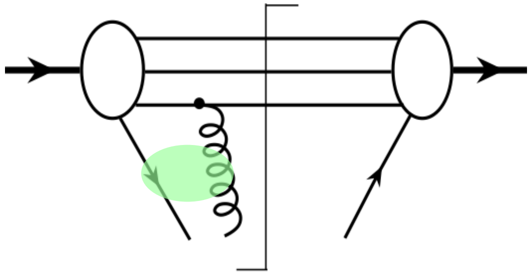
"Interpretation" of twist-3 correlation functions

- **Measurement of direct QCD quantum interference:**

Qiu, Sterman, 1991, …

$$T^{(3)}(x,x,S_\perp) \;\propto$$



Interference between a single active parton state and an active two-parton composite state

- **"Expectation value" of QCD operators:**

$$\langle P,s|\overline{\psi}(0)\gamma^+\psi(y^-)|P,s\rangle \;\longrightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[\epsilon_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta^{\;+}(y_2^-)\right]\psi(y^-)|P,s\rangle$$

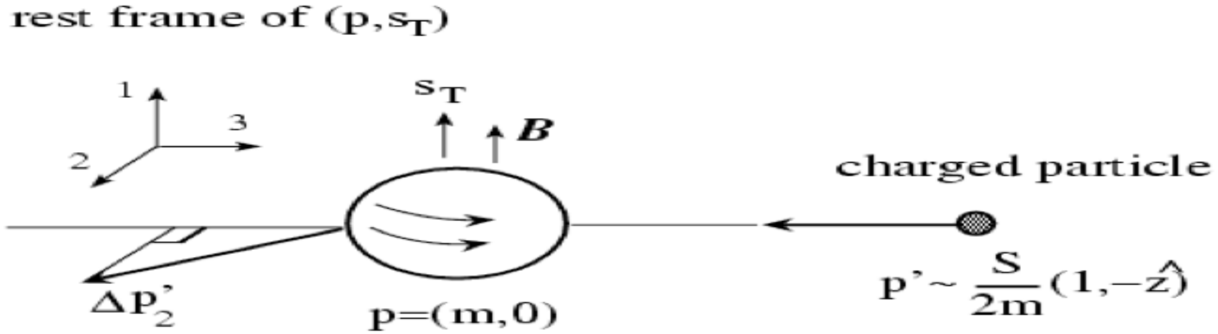
$$\langle P,s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P,s\rangle \;\longrightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[i\,g_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta^{\;+}(y_2^-)\right]\psi(y^-)|P,s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

- The operator in Red – a classical Abelian case:

Qiu, Sterman, 1998



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\, \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

- The total change:

$$\boxed{\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)}$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

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Test QCD at Twist-3 Level

❑ Scaling violation – "DGLAP" evolution:

Kang, Qiu, 2009

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \underbrace{\begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}}_{(x,\, x+x_2,\, \mu,\, s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix}}_{(\xi,\, \xi+\xi_2;\, x,\, x+x_2,\, \alpha_s)} \underbrace{\otimes}_{\int d\xi \int d\xi_2} \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}$$

❑ Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3$

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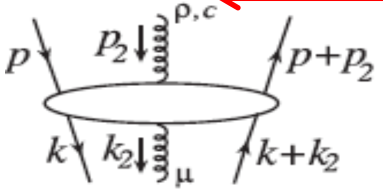
Evolution Kernels – an Example

❑ Quark to quark:

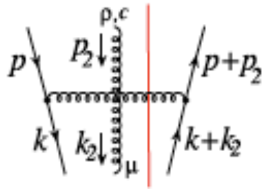
Kang, Qiu, 2009



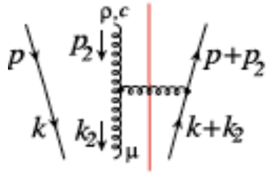
$$\mathcal{P}^{(\mathrm{LC})}_{q,F} = \frac{1}{2}\gamma\cdot P\left(\frac{-1}{\xi_2}\right)(i\epsilon^{s_T\rho n\bar{n}})\tilde{C}_q$$

$$\mathcal{V}^{\mathrm{LC}}_{q,F} = \frac{\gamma^+}{2P^+}\delta\left(x-\frac{k^+}{P^+}\right)x_2\delta\left(x_2-\frac{k_2^+}{P^+}\right)(i\epsilon^{s_T\sigma n\bar{n}}[-g_{\sigma\mu}]C_q$$

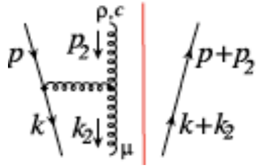
❑ Feynman diagram calculation:



$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi_2)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[C_F-\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1+z^2}{1-z}\right)$$



$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi-x)\frac{1}{\xi_2}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\frac{2x+\xi_2}{x+\xi_2}\right)$$
$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\mathcal{T}_{q,F}(x,x)$$

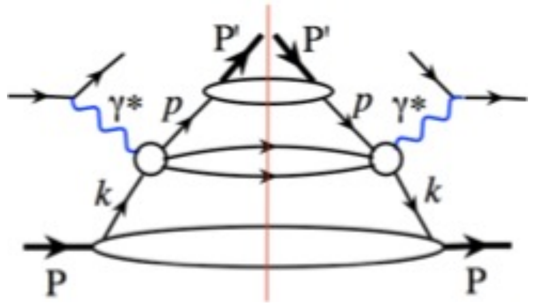


$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi+\xi_2-x)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\frac{1+z}{1-z}\right)$$
$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\mathcal{T}_{q,F}(x,x)$$

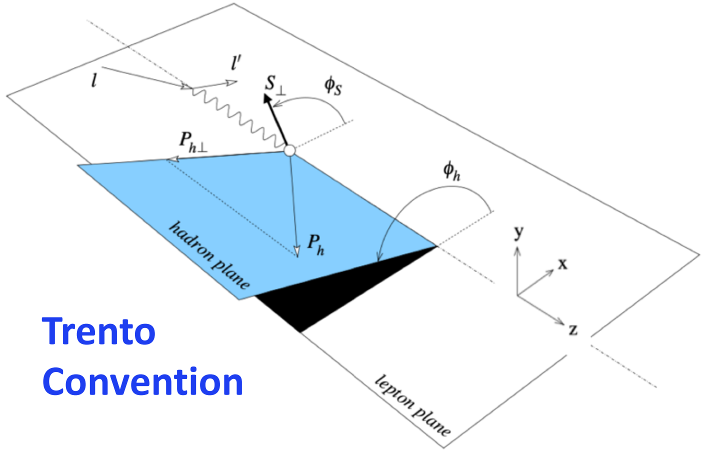
+ Virtual loop diagrams

How TMD Factorization Generates A_N?

- **SIDIS – "one-photon approximation":**
  - **18 Structure functions**
  - **A_N = at least one of 6 F_{UT} structure functions needs to be finite!**



$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$
$$+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$
$$+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$
$$+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$
$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$
$$+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\}$$



Trento Convention

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}$$

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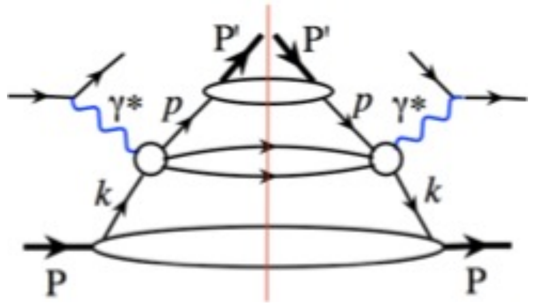
How TMD Factorization Generates A_N?

❑ TMD factorization for SIDIS:



In the photon-hadron frame, 8 of **18 structure functions can be factorized in terms of convolution of TMDs at leading power**

- **Unpolarized:**

$$F_{UU,T} = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) f^a(x, p_T^2) \, D^a(z, k_T^2)$$

- **Transverse Single-Spin Asymmetry – Sivers:**

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right] \qquad \hat{\boldsymbol{h}} = \frac{\boldsymbol{P}_{h\perp}}{|\boldsymbol{P}_{h\perp}|}$$

- **Transverse Single-Spin Asymmetry – Collins:**

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

With:

$$\mathcal{C}[w f D] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T) \, f^a(x, p_T^2) \, D^a(z, k_T^2)$$

Orbital Angular Momentum

OAM: Correlation between parton's position and its motion
– in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**
  - ✧ **generated by a "torque" of color Lorentz force**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2 y_T}{(2\pi)^3} \langle P' | \overline{\psi}_q(0) \frac{\gamma^+}{2} \int_{y^-}^{\infty} dz^- \Phi(0, z^-)$$
$$\times \underbrace{\sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{"Chromodynamic torque"}} \Phi(z^-, y)\psi(y)|P\rangle_{y^+=0}$$

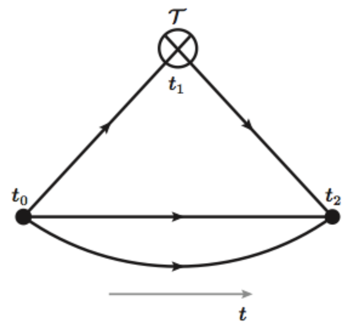
Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

Nucleon Spin and OAM From Lattice QCD

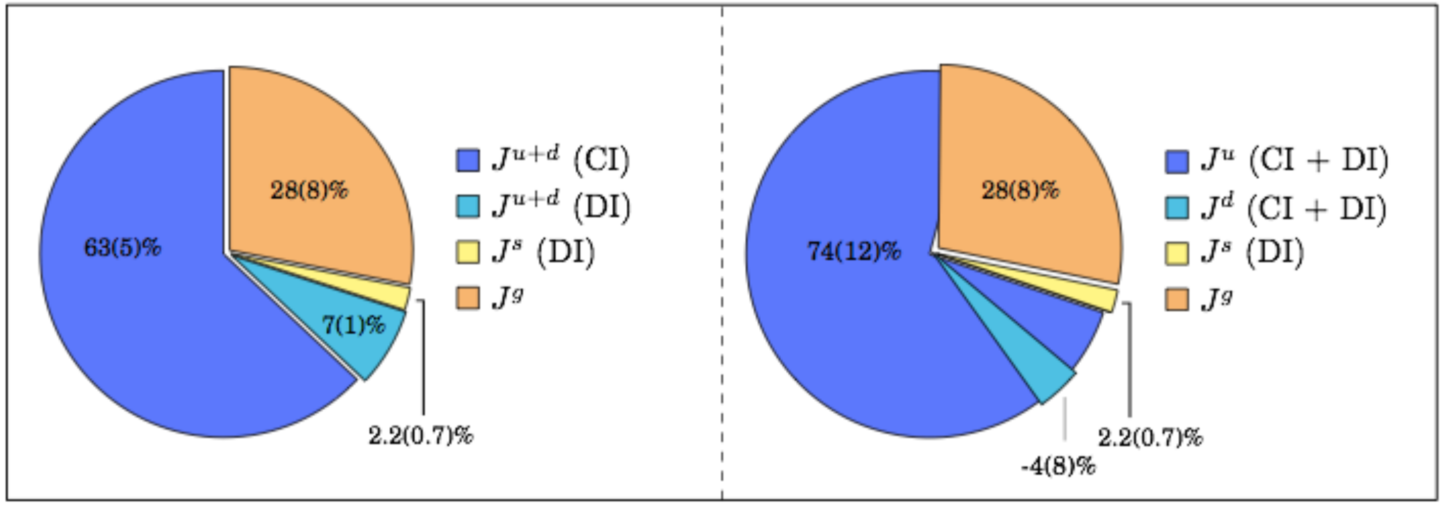
See lectures by Huey-Wen Lin

- **QCD Collaboration:**

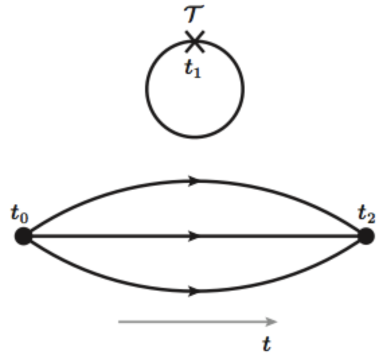
Deka *et al.* Phys.Rev.D91 (2015) 014505



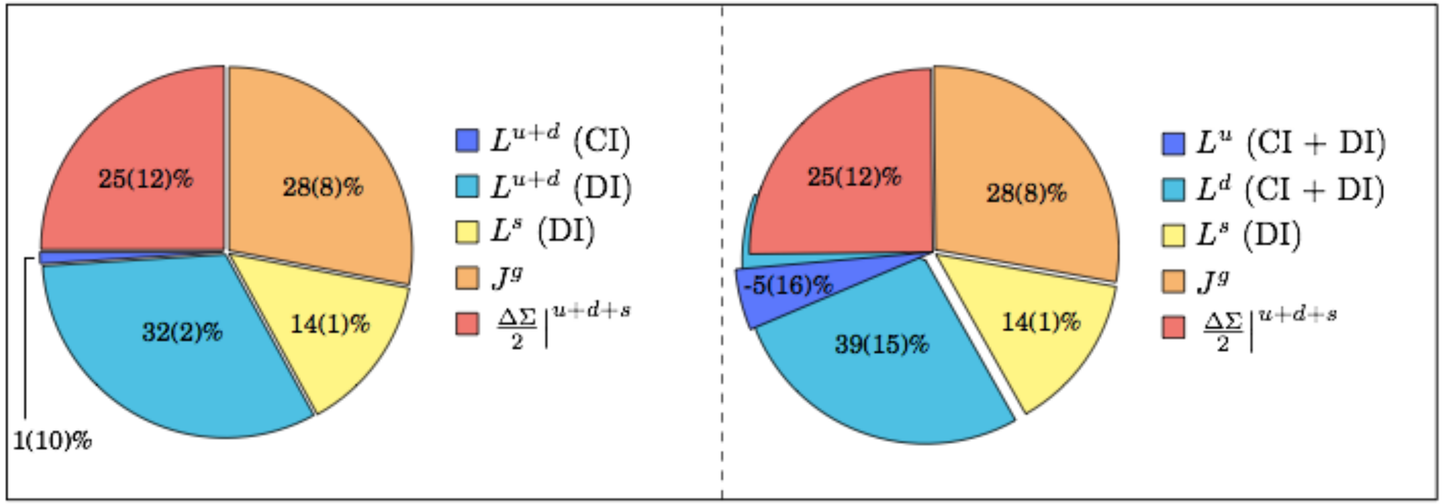
Connected Interaction (CI)



(b)



Disconnected Interaction (DI)



(c)

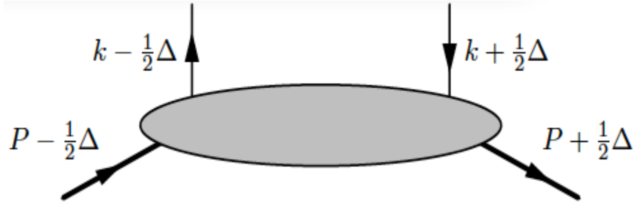
Partonic Motion Seen by a Hard Probe – GTMD

Meissner, Metz, Schiegel, 2009

- **Fully unintegrated distribution:**

$$W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta)=\frac{1}{2}\int\frac{d^4z}{(2\pi)^4}\,e^{ik\cdot z}\,\langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\Gamma\,\mathcal{W}(-\tfrac{1}{2}z,\tfrac{1}{2}z\,|\,n)\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle$$

– in general, not factorizable from the rest of the scattering



- **Generalized TMDs – hard probe:**

$$\mathcal{W}(x,k_T,\Delta)_\Gamma=\int dk^2\,W(P,k,\Delta)_\Gamma$$

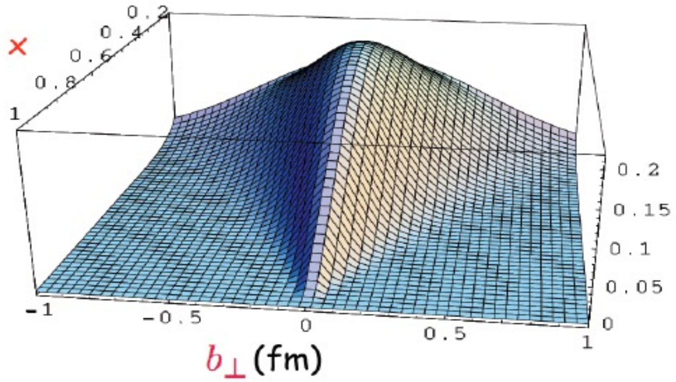
– Could be factorized assuming on-shell parton for the hard probe

- **Wigner function:**

Belitsky, Ji, Yuan

$$W(x,k_T,b)\propto\int d^3\Delta\,e^{i\vec{b}\cdot\vec{\Delta}}\,\mathcal{W}(x,k_T,\Delta)_{\Gamma=\gamma^+}$$

- **Connection to all other known distributions:**



$$W(x,k_T,b)\Longrightarrow$$ Tomographic image of nucleon

$$q(x,b_\perp)=\int d^2k_T db^-\,W(x,k_T,b)_{\gamma^+}$$

Burkardt, 2002

$\mathcal{W}(x,k_T,\Delta)_\Gamma\Rightarrow$ **TMDs** $(\Delta=0)$, **GPDs** $(\int d^2k_T)$, **PDFs** $(\Delta=0,\int d^2k_T)$

The Electron-Ion Collider (EIC)

- Lec. 1: EIC & Fundamentals of QCD
- Lec. 2: Probing Structure of Hadrons without seeing Quark/Gluon?
  – *breaking the hadron*!
- Lec. 3: Probing Structure of Hadrons with polarized beam(s)
  – *Spin as another knob*
- Lec. 4: Probing Structure of Hadrons without breaking them?
  Dense Systems of gluons
  – *Nuclei as Femtosize Detectors*

Jianwei Qiu
Theory Center, Jefferson Lab

BACKUP SLIDES

Lattice QCD Calculation of Hadron Structure

- **Hadron structure measured with a hard probe:**

  Probing operators living on the "light-Cone",

$$q(x) \propto \mathrm{F.T.}\langle P|\overline{\psi}_q(-y^-)\Gamma\Phi\psi_q(y^-)|P\rangle|_{y^+=0,y_\perp=0_\perp}$$

  PDFs are boost invariant with twist-2 operators

  Such matrix elements are non-perturbative, and Cannot be calculated by lattice QCD directly, because of its Euclidean space formulation

- **Quasi-PDFs approach:**

$$\tilde{q}(\tilde{x}, P_z) \propto \mathrm{F.T.}\langle P|\overline{\psi}_q(-z)\Gamma\Phi\psi_q(z)|P\rangle|_{y^0=0,y_\perp=0_\perp}$$

$$\to q(x) \quad \text{as } P_z \to \infty$$

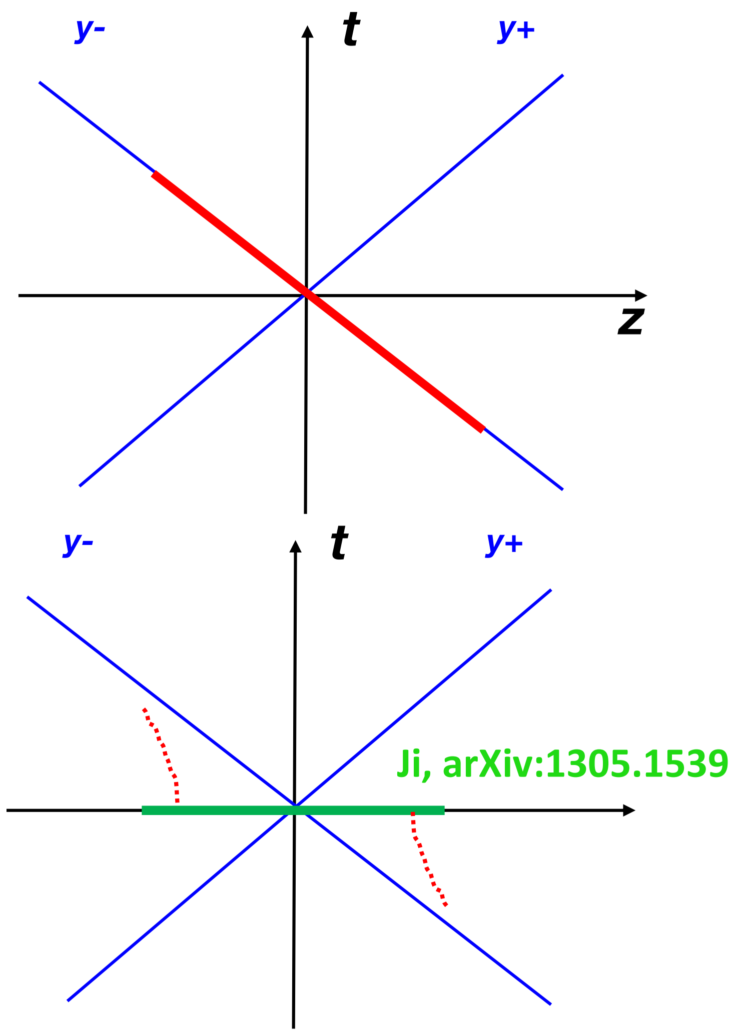
Calculable in LQCD

  Quasi-PDFs are NOT boost invariant, not by twist-2 operators

  In Lattice QCD calculation, difficult to take $P_z \to \infty$

  Matching - Formulated in LaMET:

$$\tilde{q}(\tilde{x}, P_z) = \int_x^1 \frac{dx}{x} Z\left(\frac{\tilde{x}}{x}, \frac{\mu}{P_z}\right) q(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\tilde{x}^2(1-\tilde{x})P_z^2}, \frac{M^2}{P_z^2}\right)$$



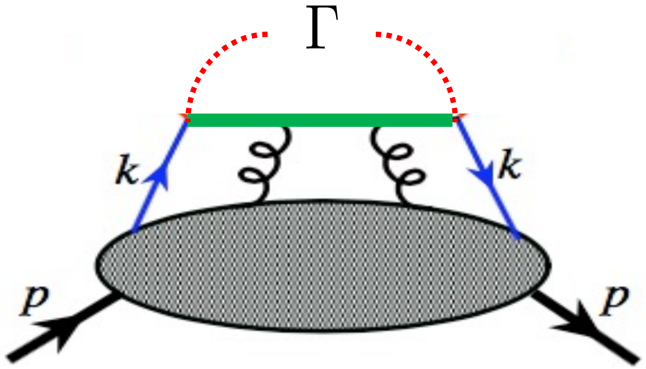
Approach to light-cone under a **boost** $P_z \to \infty$

Extracting PDFs requires solving the inverse problem

Lattice QCD Calculation of Hadron Structure

- **Space-like parton correlation functions (PCFs):**

$$\langle P|\overline{\psi}_q(-z)\Gamma\Phi\psi_q(z)|P\rangle|_{y^0=0,y_\perp=0_\perp} \qquad \langle P|F^{\alpha\beta}(-z)\Phi F^{\mu\nu}(z)|P\rangle|_{y^0=0,y_\perp=0_\perp}$$



Unlike measured cross section,

- **They are not physically measured observable**
- **Their value depend on UV renormalization**
- **They have UV power divergence**
- **They are multiplicatively renormalizable**

- **Renormalization of space-like PCFs:**

UV divergence is a property of the operator, not the state

$$\langle P|\mathcal{O}(z)|P\rangle_{\rm Ren} \equiv \frac{\langle P|\mathcal{O}(z)|P\rangle}{\langle {\rm RS}|\mathcal{O}(z)|{\rm RS}\rangle}$$

Renormalization scheme = different choice of the state $|{\rm RS}\rangle$

RI-MOM for quasi-PDFs: $|{\rm RS}\rangle = \text{An off-shell parton state}$ arXiv:1705.11193 arXiv:1709.04933

Pseudo-PDFs: $|{\rm RS}\rangle = |P_z = 0\rangle$ arXiv:1706.05373

Vacuum-state: $|{\rm RS}\rangle = |\Omega\rangle$ arXiv:1810.00048 arXiv:2006.12370

Jefferson Lab

Lattice QCD Calculation of Hadron Structure

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ Short-distance factorization approach:

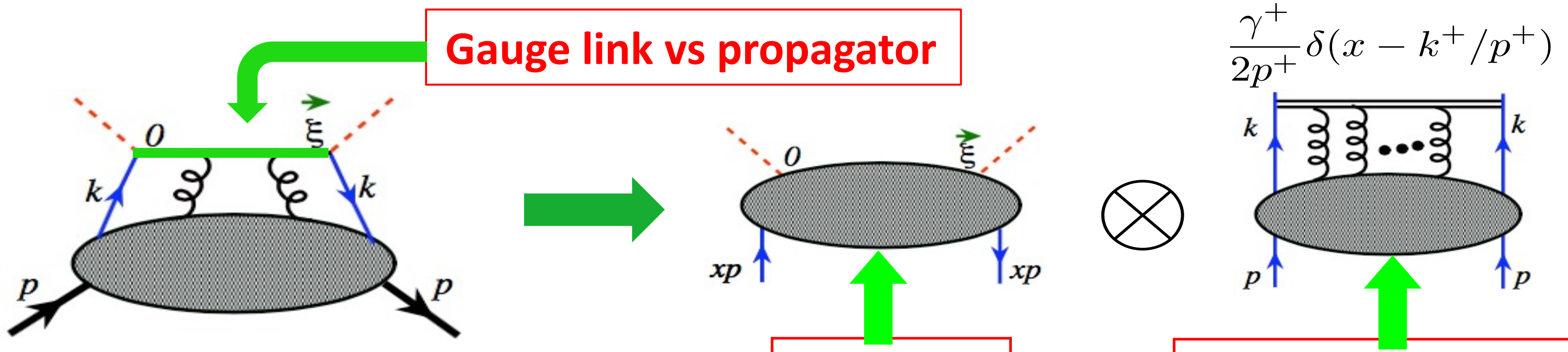
- **Single hadron matrix element:** $\sigma_n(\omega, \xi^2, P^2) = \langle P | T\{\mathcal{O}_n(\xi)\} | P \rangle$

  with Ioffe time: $\omega \equiv P \cdot \xi, \;\; \xi^2 \neq 0, \;\; \text{and } \xi_0 = 0;$

- **Two-parton correlator:** $\mathcal{O}_q(\xi) = Z_q^{-1}(\xi^2)\overline{\psi}_q(\xi)\,\gamma\cdot\xi\Phi(\xi,0)\,\psi_q(0)$

  Same operator for quasi-PDFs $\Phi(\xi,0) = \mathcal{P}e^{-ig\int_0^1 \xi\cdot A(\lambda\xi)\,d\lambda}$

- **Two-current correlator:** $\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}^{-1}\, Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$



$+ \; \mathcal{O}(\xi^2\Lambda_{\mathrm{QCD}}^2)$

Hard-part Probe | **Parton-distribution Structure** | **Corrections Approximation**

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\mathrm{QCD}}^2)$$

Extracting PDFs requires solving the inverse problem