Indiana University Bloomington
THE COLLEGE OF ARTS + SCIENCES
Department of Physics

National Nuclear Physics Summer School 2024
July 15 – July 26, 2024
Bloomington, IN

The Electron-Ion Collider (EIC)

- Lec. 1: EIC & Fundamentals of QCD
- Lec. 2: Probing Emergent Properties and Structure of Hadrons without seeing Quark/Gluon?
  – *breaking the hadron*!
- Lec. 3: Probing Structure of Hadrons without breaking them?
  – *Spin as another knob*
- Lec. 4: Dense Systems of gluons
  – *Nuclei as Femtosize Detectors*

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U.S. DEPARTMENT OF ENERGY | Office of Science
JSA

Physical Observables

Cross sections with identified hadron(s)
are
non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is NOT
a perturbative scale

Look for two-types physical observables:

- ❑ **Purely infrared safe quantities**
- ❑ **Observables with identified hadron(s), but, factorizable in QCD**

An Instructive Exercise for Factorization

- **Consider a cross section:**

$$\sigma(Q^2, m^2) = \sigma_0 \left[1 + \alpha_s I(Q^2, m^2) + \mathcal{O}(\alpha_s^2)\right]$$

- **Leading order quantum correction:**

$$I(Q^2, m^2) = \int_0^\infty dk^2 \, \frac{1}{k^2 + m^2} \, \frac{Q^2}{Q^2 + k^2}$$

- **Leading power contribution in O(m^2/Q^2):**

$$I(Q^2, m^2) = \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2 + m^2} + \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2 + k^2} + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

- **Leading power contribution to the cross section:**

$$\sigma(Q^2, m^2) = \left[1 + \alpha_s \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2 + m^2}\right] \left[1 + \alpha_s \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2 + k^2}\right] + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

$$\equiv \phi \otimes \hat{\sigma} + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Long-distance distribution (ϕ)

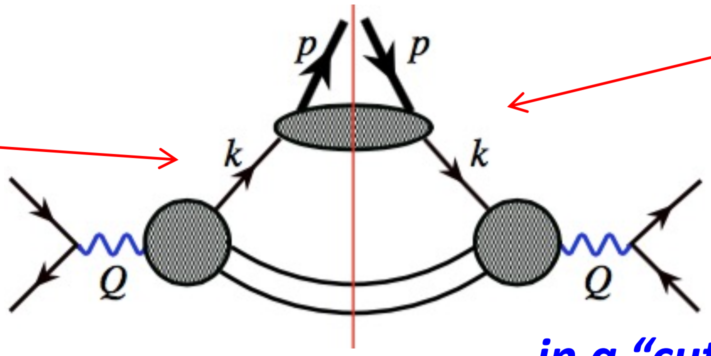
Short-distance hard part ($\hat{\sigma}$)

Observables with ONE identified hadron

- Creation of an identified hadron:

Not necessary to be dominated by one parton, which is always virtual!

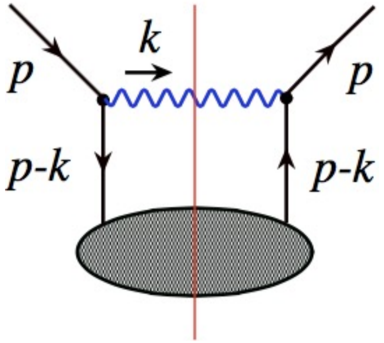
Non-perturbative!



– in a "cut-diagram" notation

- "Square" of the diagram with a *"unobserved gluon"*:

"Cut-line" – final-state



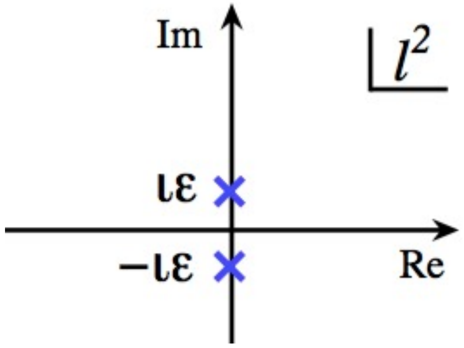
Amplitude

Complex conjugate of the Amplitude

$$\propto \int \mathcal{T}(p-k,Q)\,\frac{1}{(p-k)^2+i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}\,d^4k\,\delta(k^2)_+$$

$$\propto \int \mathcal{T}(l,Q)\,\frac{1}{l^2+i\epsilon}\,\frac{1}{l^2-i\epsilon}\,dl^2$$

$$\Rightarrow \infty$$



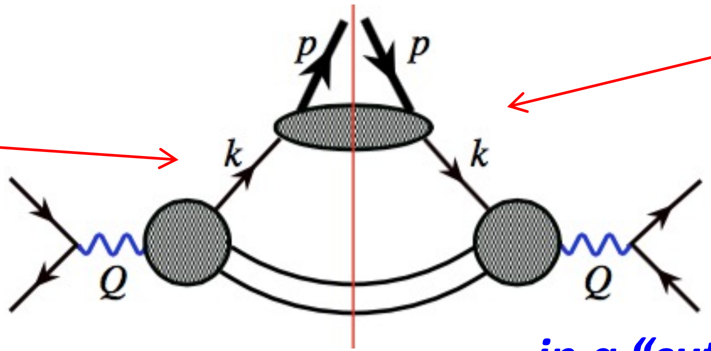
Pinch singularity & pinch surface
Two parts connected by a "classical" parton

Observables with ONE identified hadron

- **Creation of an identified hadron:**

Not necessary to be dominated by one parton, which is always virtual!

Non-perturbative!



- **On-shell approximation:**

– in a "cut-diagram" notation

$$\sigma_{e^+e^-\to h(p)X} \approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,k;\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

On-shell $\hat{k}^2 = 0$

$$\approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,\hat{k};\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right) + \ldots$$

Collinear $1 = \int dz\, \delta(z - \frac{p^+}{k^+})$

$$\approx \sum_f \int dz\, \mathcal{H}_{e^+e^-\to f(k)}(Q,\frac{p}{z};\sqrt{S}) \int \frac{d^4k}{(2\pi)^4} \delta(z - \frac{p\cdot n}{k\cdot n}) \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

$$\approx \sum_f \int dz\; \hat{\sigma}_{e^+e^-\to f(k)}(Q,z;\sqrt{S})\; D_{f(k)\to h(p)X}(z,p;\Lambda_{\rm QCD}) + \ldots$$

Hard collision to produce an on-shell parton – Perturbatively calculable!

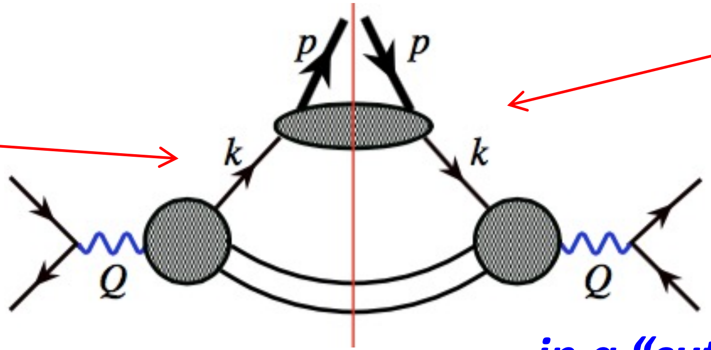
FF: Probability for the parton to become the observed hadron – Non-perturbative, universal!

Observables with ONE identified hadron

❑ Creation of an identified hadron:

Not necessary to be dominated by one parton, which is always virtual!

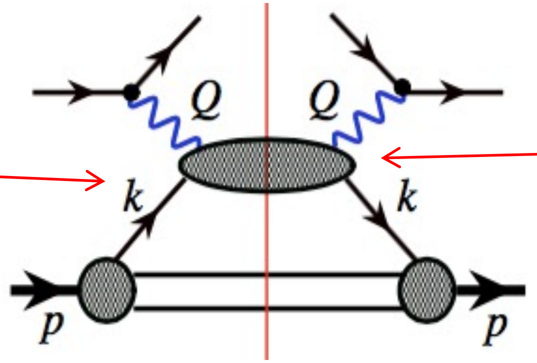
Non-perturbative!



– in a “cut-diagram” notation

❑ Identified initial hadron:

Pinch in k^2

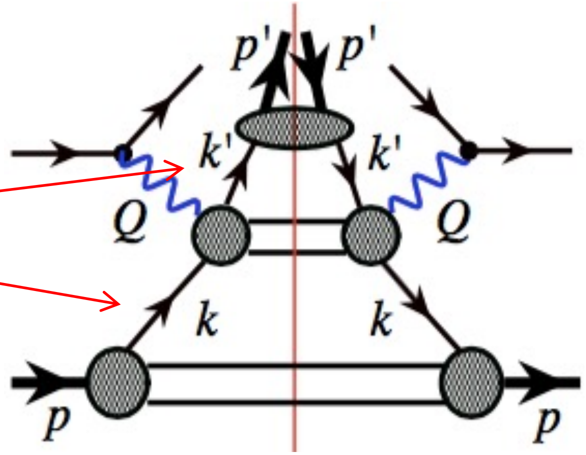


Perturbative!

Non-perturbative!

❑ Identified initial + created hadron(s):

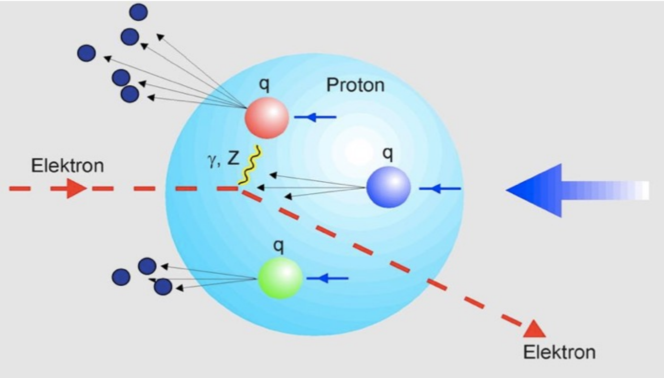
Pinch in both k^2 and k'^2



Quantum interference between dynamics at the HARD and hadronic scales is suppressed by power of (1/R)/Q !

But, the interaction between hadrons is not necessarily suppressed!

Inclusive Lepton-Hadron DIS (at EIC) – One Identified Hadron



$$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})}$$

Identified initial-state hadron – proton or nucleus!

- **Localized probe:**

$$Q^2 = -(l - l')^2 \gg 1 \text{ fm}^{-2} \quad \Rightarrow \quad \frac{1}{Q} \ll 1 \text{ fm}$$

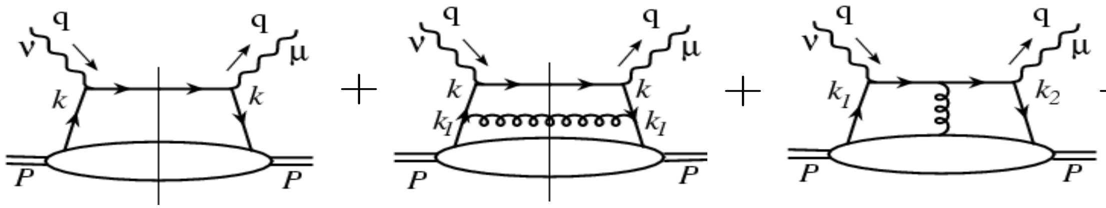
- **Two variables (hadron rest frame):**

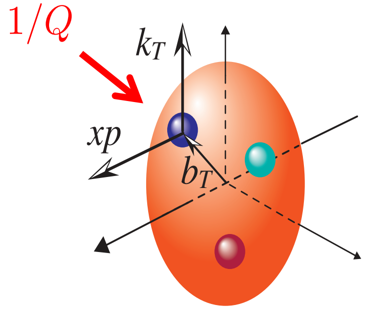
$$Q^2 = 4EE' \sin^2(\theta/2) \qquad y = \frac{s}{x_B Q^2}$$

$$x_B = \frac{Q^2}{2m_N \nu} \qquad \nu = E - E'$$

❑ **DIS cross section is infrared divergent, and nonperturbative!**

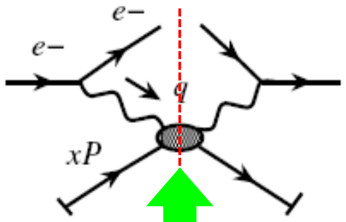
$$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})} \propto$$



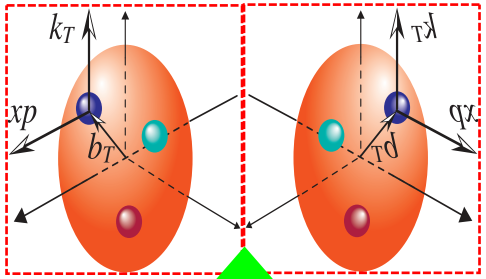


❑ **QCD factorization (approximation!)**

$$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})} =$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

Physical Observable

Controllable Probe

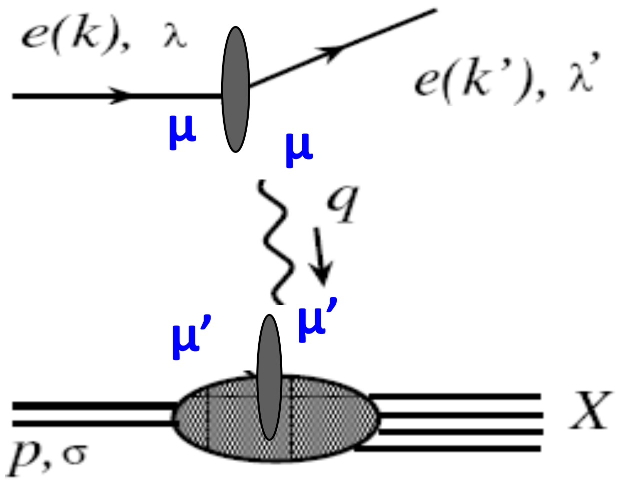
Quantum Probabilities Structure

Color entanglement Approximation

Inclusive Lepton-Hadron DIS (at EIC) – One Identified Hadron

❑ Scattering amplitude:

$$\mathrm{M}(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_{\mu}\right]u_{\lambda}(k)$$
$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$
$$* \left\langle X\middle|eJ^{em}_{\mu'}(0)\middle|p,\sigma\right\rangle$$



❑ Cross section:

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|\mathrm{M}(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X}\frac{d^3 l_i}{(2\pi)^3\, 2E_i}\right]\frac{d^3k'}{(2\pi)^3\, 2E'}(2\pi)^4\delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

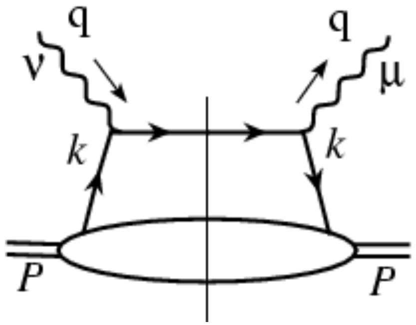
❑ Leptonic tensor:

– known from QED: $L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} - k\cdot k' g^{\mu\nu}\right)$

DIS Structure Functions

- **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\,\langle p,S|J^\dagger_\mu(z)J_\nu(0)|p,S\rangle$$



- **Symmetries:**
  - ✧ **Parity invariance (EM current)** ⟶ $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
  - ✧ **Time-reversal invariance** ⟶ $W_{\mu\nu} = W^*_{\mu\nu}$ real
  - ✧ **Current conservation** ⟶ $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

- **Structure functions – infrared sensitive:**

$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

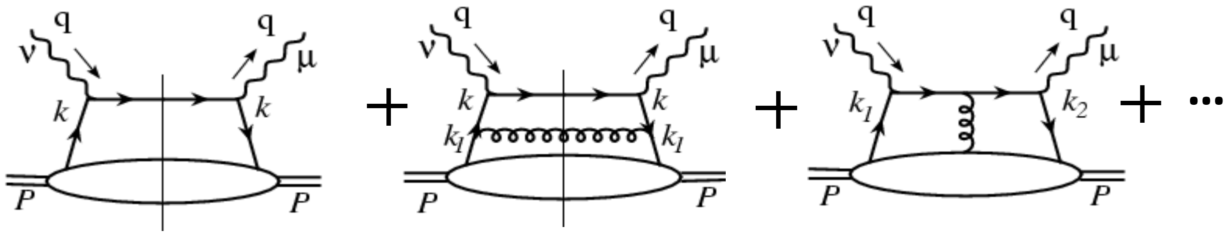
No QCD parton dynamics used in above derivation!

Jefferson Lab

Long-Lived Parton States

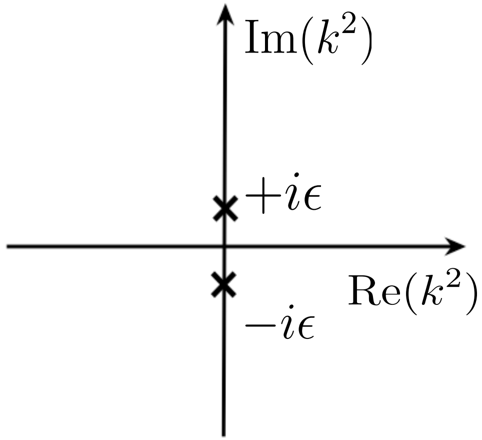
- **Feynman diagram representation of the hadronic tensor:**

$W^{\mu\nu} \propto$



- **Perturbative pinched poles:**

$$\int d^4k\, \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}$$



- **Perturbative factorization:**

Light-cone coordinate:

$$v^\mu = (v^+, v^-, v^\perp), \qquad v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$$

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

$$\int \frac{dx}{x} d^2k_T\, \mathrm{H}(Q,k^2=0)\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

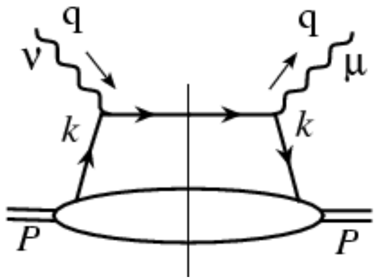
Short-distance

Nonperturbative matrix element

Collinear Factorization – Further Approximation

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T, \sqrt{k^2}$

– **Lowest order:**



$$W^{\mu\nu}_{\gamma^* p} = \sum_f \int \frac{d^4k}{(2\pi)^4} \sum_{ij} (\gamma^\mu \gamma\cdot(k+q)\gamma^\nu)_{ij}\, (2\pi)\delta((k+q)^2) \int d^4y e^{iky} \langle p|\overline{\psi}_j(0)\psi_i(y)|p\rangle + \ldots$$

$$\equiv \sum_f \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,k)\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

$$\delta((k+q)^2) = \tfrac{1}{2P\cdot q}\delta(x-\xi) = \tfrac{1}{2P\cdot q}\delta\left(x - \tfrac{k^+}{P^+}\right)$$

$$\approx \sum_f \int dx\ \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q, k\approx xp) \int \frac{d^4k}{(2\pi)^4}\delta(x - \frac{k\cdot n}{p\cdot n})\mathcal{F}_{f/p}(k,p)\right] + \mathcal{O}(\frac{\langle k^2\rangle}{Q^2}, \frac{\langle k_T^2\rangle}{Q^2}) + \ldots$$

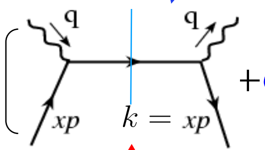
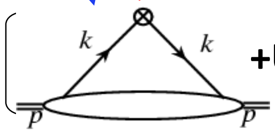
– **Collinear Approx.**

$$\approx \sum_f \int \frac{dx}{x}\ \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,xp)\frac{1}{2}\gamma\cdot(xp)\right] \int \frac{d^4k}{(2\pi)^4}\delta(x-\frac{k\cdot n}{p\cdot n})\mathrm{Tr}\left[\frac{\gamma\cdot n}{2p\cdot n}\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

– **Spin decomposition**

$$\approx \sum_f \int \frac{dx}{x}\ \widehat{W}^{\mu\nu}_{\gamma^* f}(x, Q^2/\mu^2)\ \phi_{f/p}(x,\mu^2) + \ldots$$

$$\int \frac{dx}{x} \qquad \frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x - \frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$

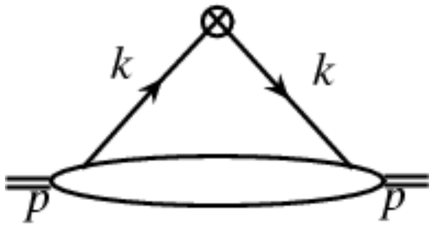
$\approx$  $+O\left(\frac{k_T^2}{Q^2}\right) \otimes$  **+UVCT(μ)**

$$\frac{1}{2}\gamma\cdot(xp) \qquad \widehat{W}^{\mu\nu}_{\gamma^* f}(x, Q^2/\mu^2) = \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,xp)\frac{1}{2}\gamma\cdot(xp)\right]$$

> $\int d^4k$ **in line 1 is limited, no UV**
> **But, factorization allows** $\int d^4k$ **to generate UV – Need UVCT(μ) to define parton distribution!**

Parton Distribution Functions (PDFs)

❑ **PDFs as matrix elements of two parton fields – twist 2 operators:**



– **combine the amplitude & its complex-conjugate**

$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\, \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle$$

The state $|h(p)\rangle$ can be a hadron, or a nucleus, or a parton state!

Twist = Dim. of the operator – its spin

But, it is NOT gauge invariant!

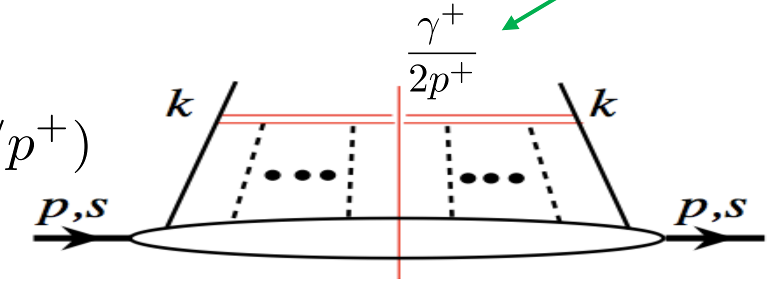
$$\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x) \qquad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$$

– **need a gauge link:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+} \psi_q(y^-)|h(p)\rangle\, \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

– **corresponding diagram in momentum space:**

$$\int \frac{d^4k}{(2\pi)^4}\, \delta(x - k^+/p^+)$$



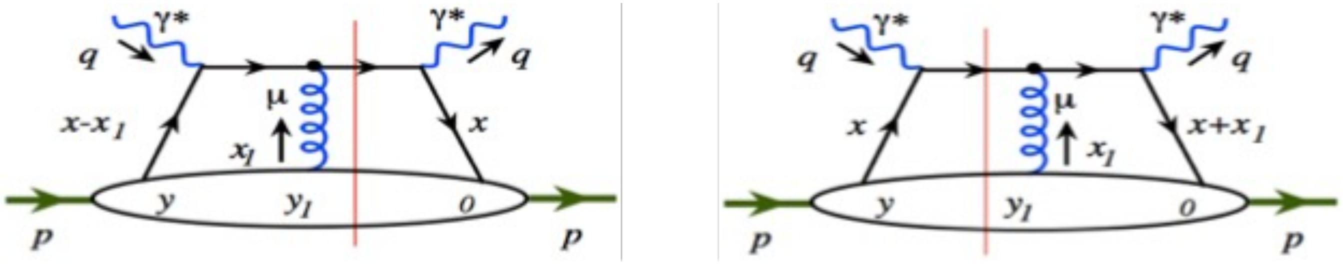
$$+\,\mathrm{UVCT}(\mu^2)$$

μ-dependence of the distribution

Universality – process independence – predictive power

Gauge Link – 1st order in coupling "g"

- Longitudinal gluon:



- Left diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1 p^+(y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1 \frac{1}{-x_1+i\epsilon}\, \mathrm{e}^{ix_1 p^+(y_1^- - y^-)}\right]\mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

- Right diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1 p^+ y_1^-} n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon}(+igt^a)\frac{\gamma\cdot p}{p^+}\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1 \frac{1}{x_1-i\epsilon}\, \mathrm{e}^{ix_1 p^+ y_1^-}\right]\mathcal{M} = ig\int_{0}^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

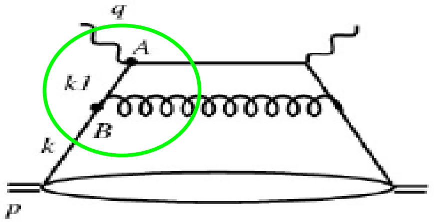
- Total contribution:

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\mathrm{LO}}$$

O(g)-term of the gauge link!

QCD High Order Corrections

- NLO partonic diagram to structure functions:

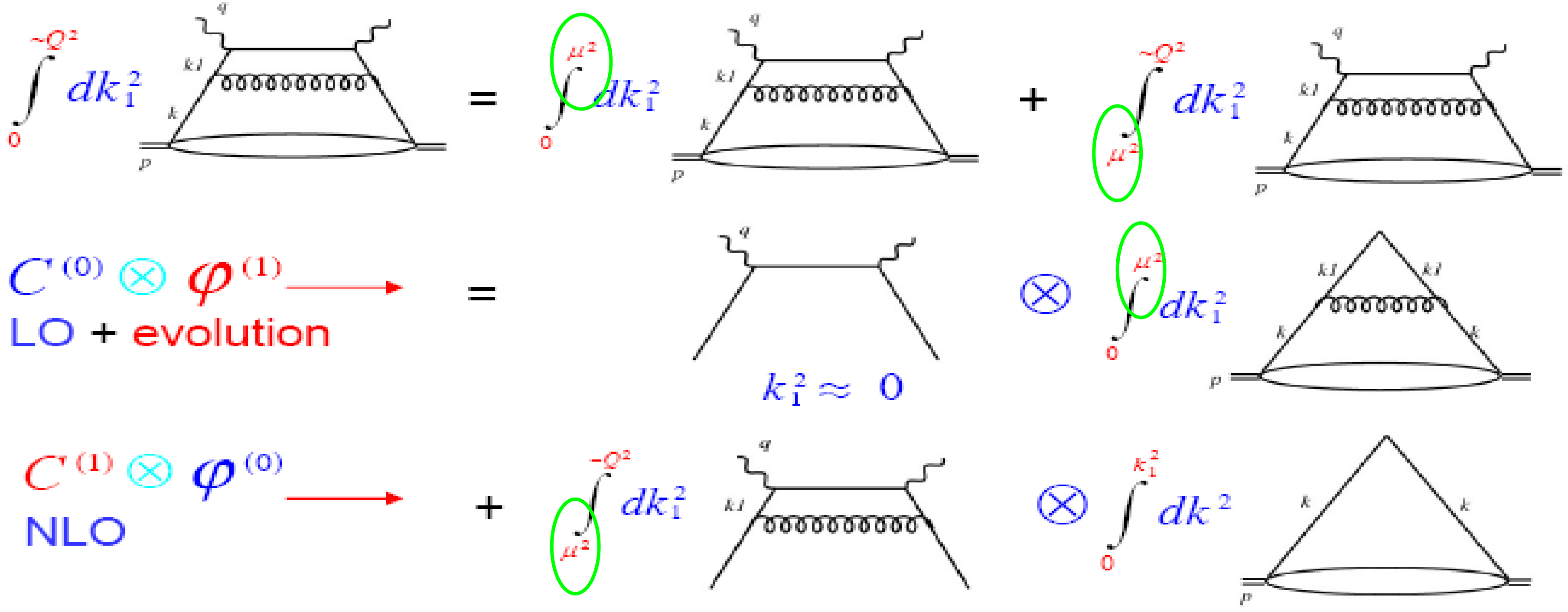


$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by $\begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$

Diagram has both long- and short-distance physics

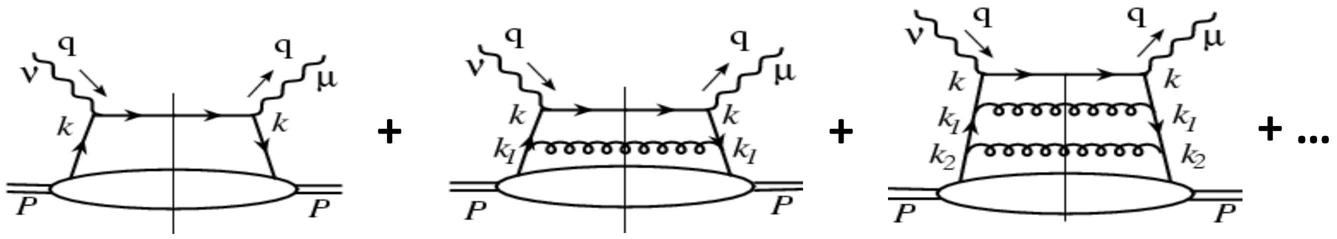
- Factorization, separation of short- from long-distance:



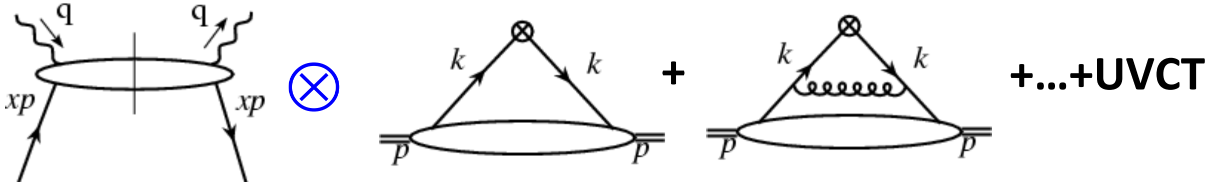
Same idea as the Instructive Exercise for Factorization

QCD Leading Power Factorization

- QCD corrections: pinch singularities in $\int d^4 k_i$



- Logarithmic contributions into parton distributions:



$$F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes f(x, \mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

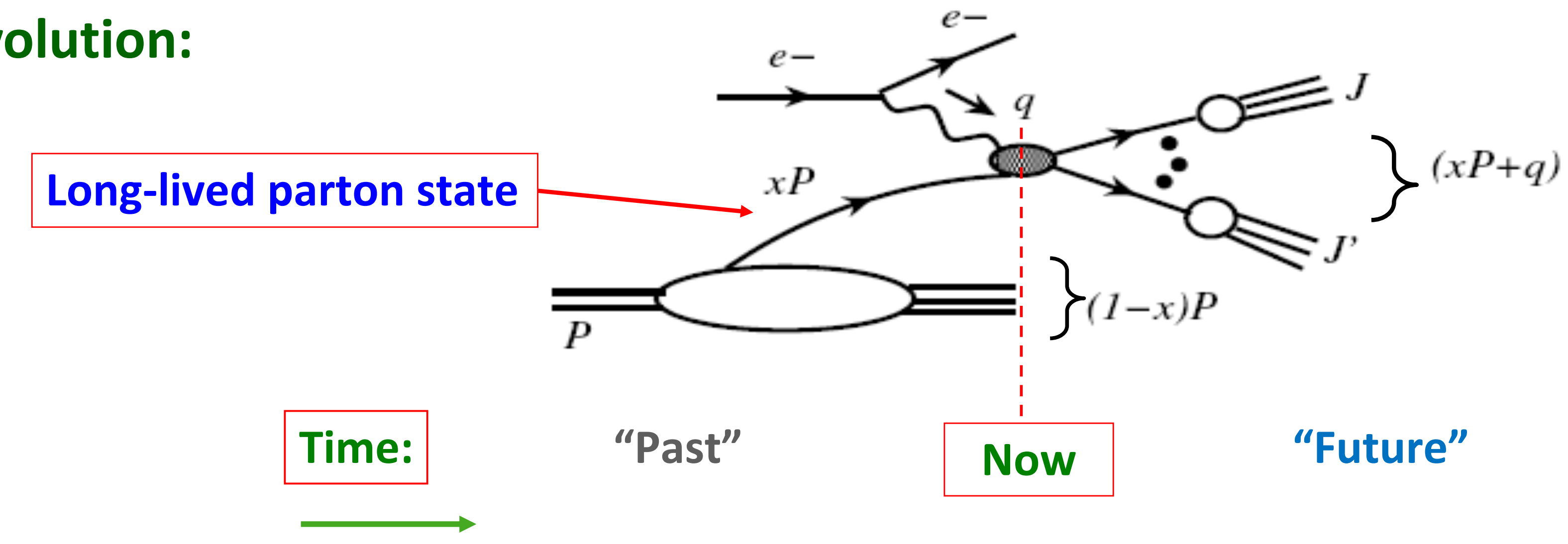
- Factorization scale: μ_F^2

To separate the collinear from non-collinear contribution

Recall: renormalization scale to separate local from non-local contribution

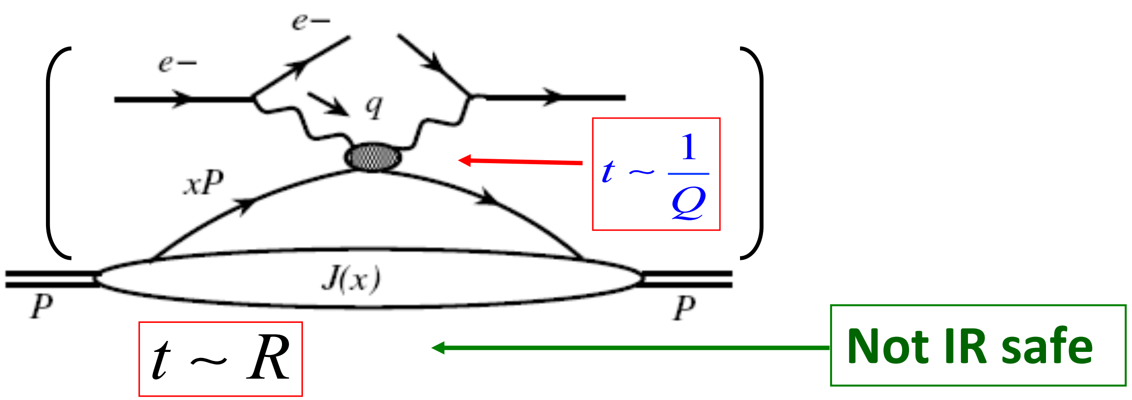
Physical Picture of Factorization for DIS

- ❑ **Time evolution:**



- ❑ **Unitarity – summing over all hard jets:**

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im}$$



Interaction between the "past" and "now" are suppressed!

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How to Calculate the Perturbative Parts?

- Use DIS structure function F_2 as an example:

$$F_{2h}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

✧ Apply the factorized formula to a parton state: $h \rightarrow q$

Feynman diagrams → $F_{2q}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$ ← Feynman diagrams

✧ Express both SFs and PDFs in terms of powers of α_s:

0th order: $F^{(0)}_{2q}(x_B,Q^2) = C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)$

$$\Rightarrow \boxed{C^{(0)}_q(x) = F^{(0)}_{2q}(x)} \qquad \varphi^{(0)}_{q/q}(x) = \delta_{qq}\,\delta(1-x)$$

1th order: $F^{(1)}_{2q}(x_B,Q^2) = C^{(1)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right) + C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)$

$$\Rightarrow \boxed{C^{(1)}_q(x,Q^2/\mu^2) = F^{(1)}_{2q}(x,Q^2) - F^{(0)}_{2q}(x,Q^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)}$$

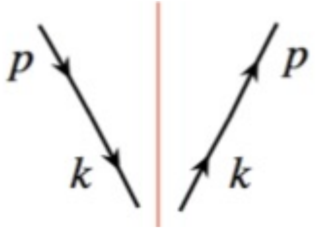
PDFs of a Parton

- **Change the state without changing the operator:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2} U^n_{[0,y^-]}\, \psi_2(y^-)|h(p)\rangle$$

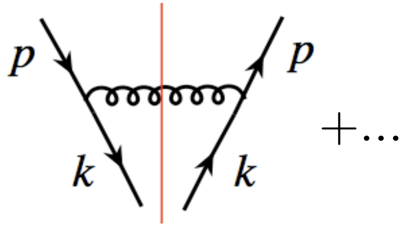
$|h(p)\rangle \;\Rightarrow\; |\text{parton}(p)\rangle$ ➡ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

- **Lowest order quark distribution:**
  - **From the operator definition:**



$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right] \delta\left(x - \frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k) \\ &= \delta_{qq'}\ \delta(1-x)\end{aligned}$$

- **Leading order in α_s quark distribution:**
  - **Expand to $(g_s)^2$ – logarithmic divergent:**



$$\phi^{(1)}_{q/q}(x) = C_F \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] + \text{UVCT}$$

UV and CO divergence

Choice of regularization

Partonic Cross Sections

❑ Projection operators for SFs:

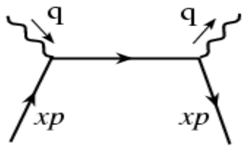
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x, Q^2\right) + \frac{1}{p \cdot q}\left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) F_2\left(x, Q^2\right)$$

$$F_1(x, Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

$$F_2(x, Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

❑ 0^{th} order:

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W_{\mu\nu,q}^{(0)} = x g^{\mu\nu} \left[\frac{1}{4\pi} \right.$$



$$\left. \right]$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2} \gamma \cdot p \gamma_\mu \gamma \cdot \left(p + q\right) \gamma_\nu\right] 2\pi \delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1 - x)$$

$$C_q^{(0)}(x) = e_q^2 x \delta(1 - x)$$

NLO Coefficient Function – a Complete Example

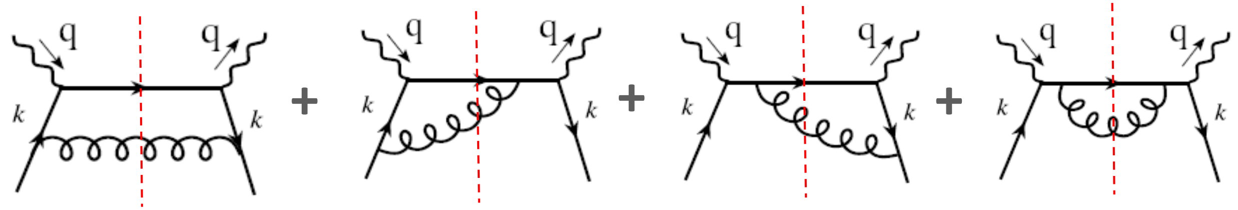
$$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

- **Projection operators in n-dimension:** $g_{\mu\nu} g^{\mu\nu} = n \equiv 4 - 2\varepsilon$

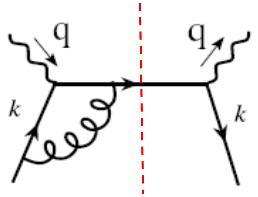
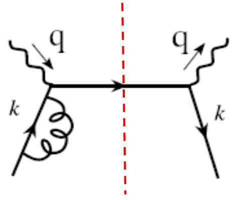
$$\left(1-\varepsilon\right) F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}$$

- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$



} **Real**

Virtual { +  + c.c. [+  + c.c. + UV CT]

- **Calculation:**

$$-g^{\mu\nu} W_{\mu\nu,q}^{(1)} \quad \text{and} \quad p^\mu p^\nu W_{\mu\nu,q}^{(1)}$$

Contribution from the Trace of $W_{\mu\nu}$

❑ Lowest order in n-dimension:

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

❑ NLO virtual contribution:

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

❑ NLO real contribution:

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

Contribution from the trace of $W_{\mu\nu}$

- **The "+" distribution:**

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ell n(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx\,\frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx\,\frac{f(x)-f(1)}{1-x} + \ell n(1-z)\,f(1)$$

- **One loop contribution to the trace of $W_{\mu\nu}$:**

$$-g^{\mu\nu}W^{(1)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon}P_{qq}(x) + P_{qq}(x)\,\ell n\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$
$$+C_F\left[\left(1+x^2\right)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x)\right.$$
$$\left.\left. +3-x-\left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

- **Splitting function:**

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

One-Loop Contribution to Partonic F2 and Quark-PDF:

- One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

$$(1-\varepsilon) F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}$$

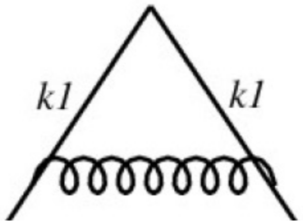
- One loop contribution to F_2 of a quark:

$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\rm CO} P_{qq}(x)\left(1+\varepsilon \ell n(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \;\; \varepsilon \to 0$$

- One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV} + \left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

NLO Coefficient Function for Inclusive DIS (at EIC):

❑ Common UV-CT terms:

✧ **MS scheme:**

$$\text{UV-CT}\Big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$$

✧ $\overline{\textbf{MS}}$ **scheme:**

$$\text{UV-CT}\Big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon\ell n(4\pi e^{-\gamma_E})\right)$$

✧ **DIS scheme: choose a UV-CT, such that**

$$C_g^{(1)}(x,Q^2/\mu^2)\Big|_{\text{DIS}} = 0$$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

$$C_q^{(1)}(x,Q^2/\mu^2) = e_q^2 x\frac{\alpha_s}{2\pi}\left\{P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right)\right.$$

$$\left.+C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Renormalization Group Improvement

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0 \qquad F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s) \otimes \phi_f(x, \mu_F^2)$$

➡ Evolution (differential-integral) equation for PDFs

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\rm QCD}^2\right)$

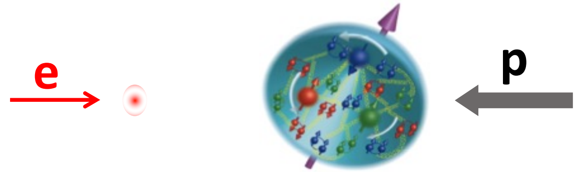
Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

➡ DGLAP evolution equation:

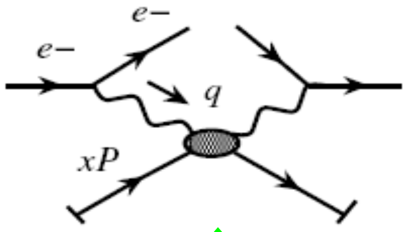
$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

From One Hadron to Two Hadrons

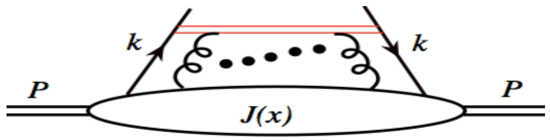
❑ One hadron:



$$\sigma_{\rm tot}^{\rm DIS} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

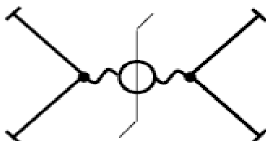
Hard-part
Probe

Parton-distribution
Structure

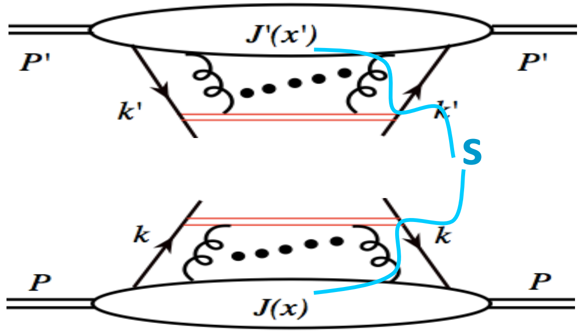
Power corrections
Approximation

❑ Two hadrons:

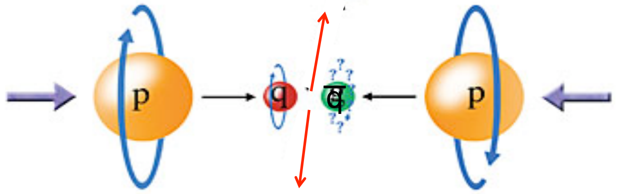
$$\sigma_{\rm tot}^{\rm DY} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$



Predictive power: Universal Parton Distributions
Calculable coefficient functions

Drell-Yan Process – Two Identified Hadrons

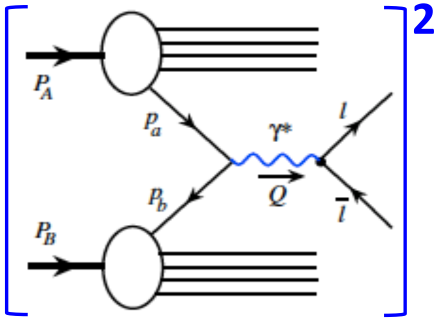
❑ Drell-Yan mechanism:

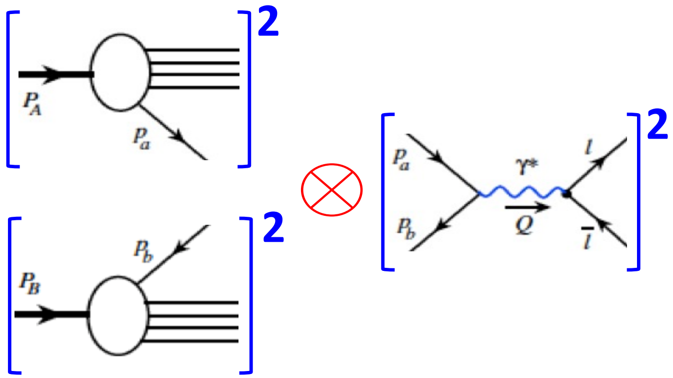
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)
Before QCD

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X \quad \text{with} \quad q^2 \equiv Q^2 \gg \Lambda_{\rm QCD}^2 \sim 1/{\rm fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:





$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2dy} = \frac{4\pi\alpha_{em}^2}{3Q^4}\sum_{p,\bar{p}} x_A\phi_{p/A}(x_A)\, x_B\phi_{\bar{p}/B}(x_B) \qquad x_A = \frac{Q}{\sqrt{S}}e^{y} \quad x_B = \frac{Q}{\sqrt{S}}e^{-y}$$

No color yet!

Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

Right shape – But – not normalization

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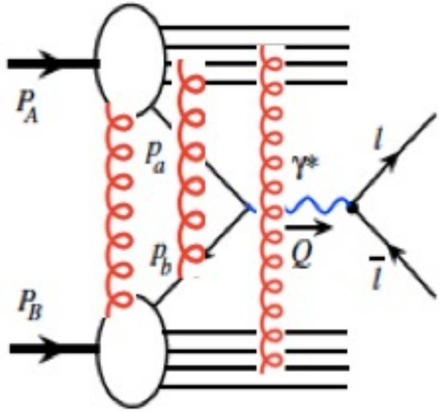
Drell-Yan Process in QCD – Factorization

Collins, Soper and Sterman, Review in QCD, edited by AH Mueller 1989

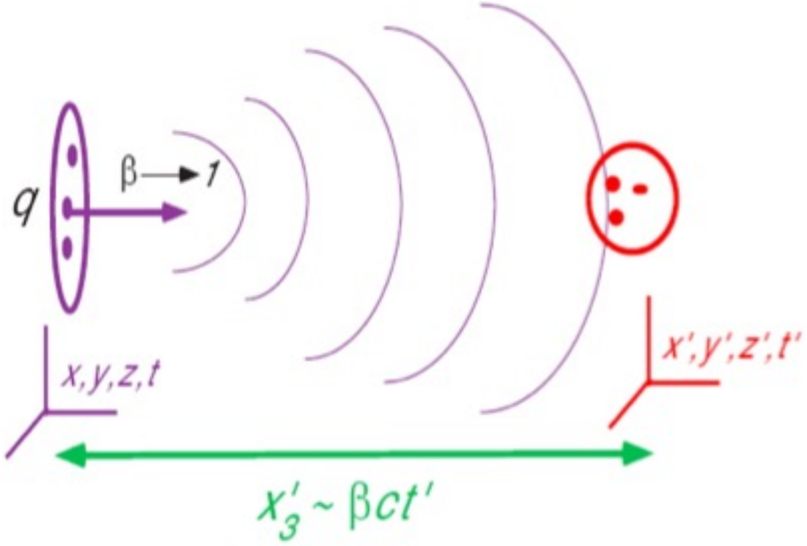
- Beyond the lowest order:



✧ Soft-gluon interaction takes place all the time
✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

- Factorization – power suppression of soft gluon interaction:



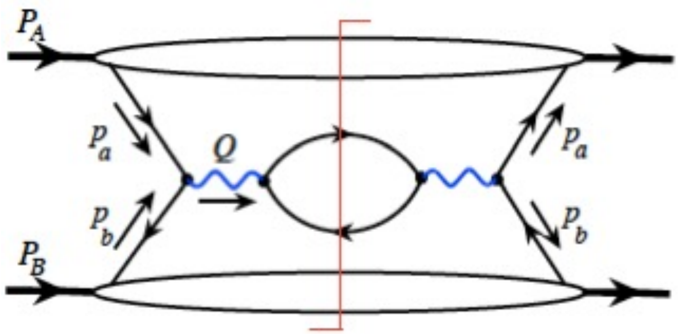
x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$ $\Rightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$ $\Rightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

Drell-Yan Process in QCD – Factorization

❑ Factorization – approximation:

✧ **Require the suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ $1/\Lambda_{QCD}$) physics**

➡ **Need "long-lived" active parton states linking the two hadrons**



$$\int d^4p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \rightarrow \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ **Active parton is effectively on-shell for the hard collision**

$$p_a^\mu = (p_a^+, p_a^-, p_{a\perp}) \sim Q(1, \lambda^2, \lambda) \quad \text{with} \quad \lambda \sim M/Q$$

➡ $p_a^2 \sim M^2 \ll Q^2$

✧ **Maintain the universality of PDFs:**

Long-range soft gluon interaction has to be power suppressed

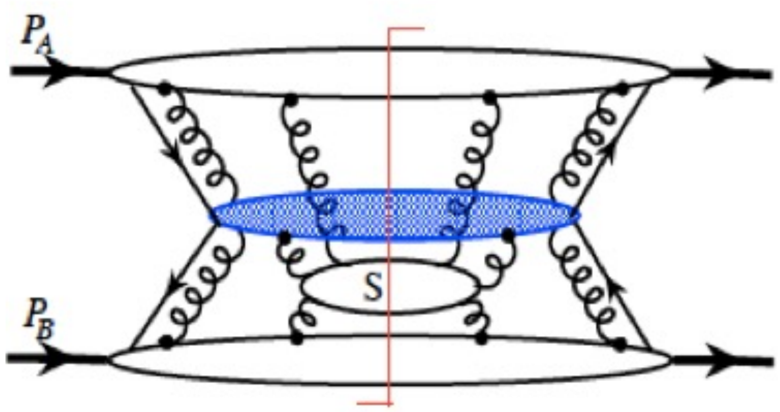
on-shell:	$p_a^2,\ p_b^2 \ll Q^2$;
collinear:	$p_{aT}^2,\ p_{bT}^2 \ll Q^2$;
higher-power:	$p_a^- \ll q^-$; and $p_b^+ \ll q^+$

✧ **Infrared safe of partonic parts:**

Cancelation of IR behavior
Absorb all CO divergences into PDFs

Drell-Yan Process in QCD – Factorization

- Leading singular integration regions (pinch surface):



Hard: all lines off-shell by Q

Collinear:
- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

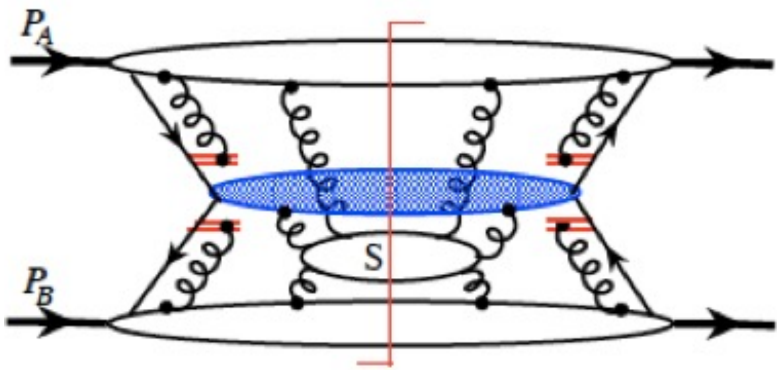
Soft: all components are soft

- Collinear gluons:

✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$

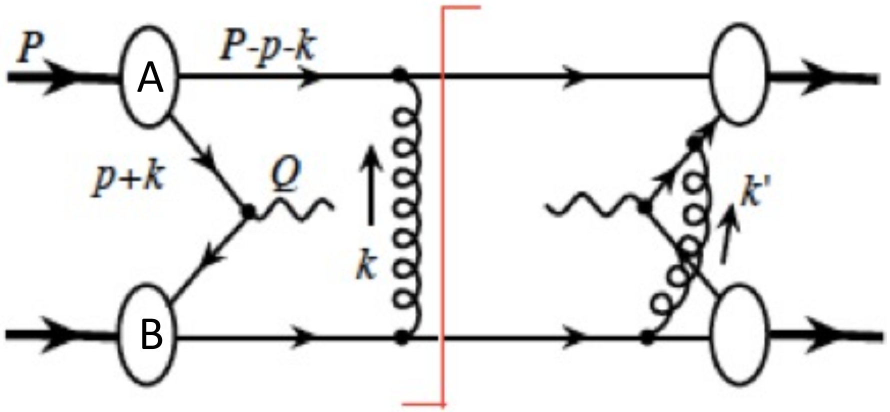
✧ The sum of the effect can be represented by the eikonal lines,

which are needed to make the PDFs gauge invariant!



Drell-Yan Process in QCD – Factorization

❑ **Trouble with soft gluons:**



Soft gluon: $k^\mu = (k^+, k^-, k_\perp) \sim (M, M, M)$

with $M \ll Q$

Glauber pinch: $(xP + k)^2 + i\epsilon \propto k^- + i\epsilon$

$((1-x)P - k)^2 + i\epsilon \propto k^- - i\epsilon$

Same for k^+ when it flows through B

➡ **Soft gluon is forced in the Glauber region**

$k^\mu \sim (\lambda M, \lambda M, M)$ with $\lambda \sim M/Q$

✧ **Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B**

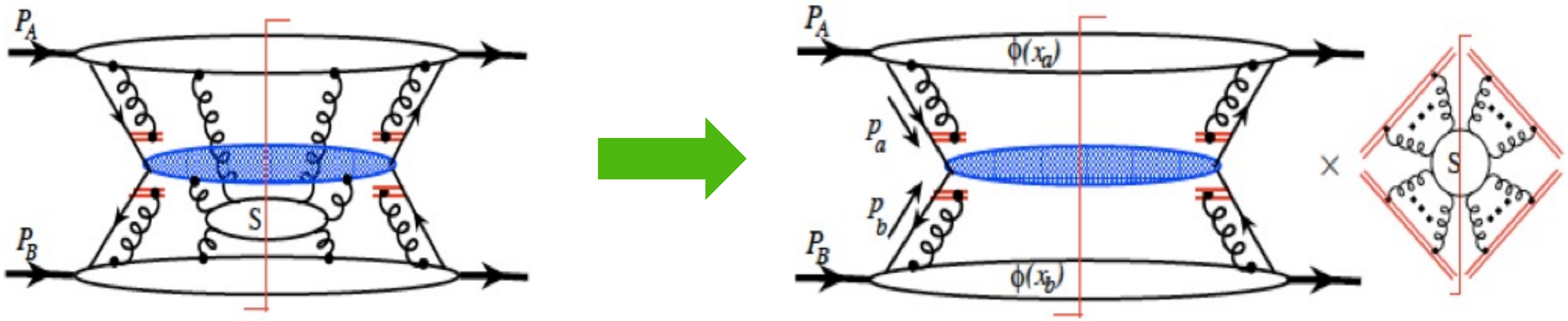
✧ **The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small.**
But, $k^\pm$ could be trapped in "too small" region due to the pinch from spectator interaction:

$$k^\pm \sim M^2/Q \ll k_\perp \sim M$$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan Process in QCD – Factorization

❑ **Most difficult part of factorization:**



- ✧ **Sum over all final states to remove all poles in one-half plane – no more pinch poles**
- ✧ **Deform the $k^{\pm}$ integration out of the trapped soft region** ➡ $k^{\mu} \sim M(\lambda, \lambda, \lambda)$
- ✧ **Leading power from "k^{-} flows into A (moving in + direction) & k^{+} flows into B (moving in – direction)"**
- ✧ **Eikonal approximation** ➡ **soft gluons to eikonal lines – gauge links**
- ✧ **Collinear factorization: Unitarity** ➡ **soft factor = 1**

All identified leading integration regions are factorizable!

Factorized Drell-Yan Cross Section

- Collinear factorization – single hard scale ($q_\perp \sim Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a,\mu) \int dx_b\, f_{b/B}(x_b,\mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

for $q_\perp \sim Q$ or $q_\perp$ integrated Drell-Yan cross sections: $d^4q = dQ^2\, dy\, d^2q_T$

- TMD factorization ($q_\perp \ll Q$) – active parton is still pinched to be on-shell:

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp}) + \mathcal{O}(q_\perp/Q)$$

$$x_A = \frac{Q}{\sqrt{s}}\, e^{y} \qquad x_B = \frac{Q}{\sqrt{s}}\, e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- Spin dependence:

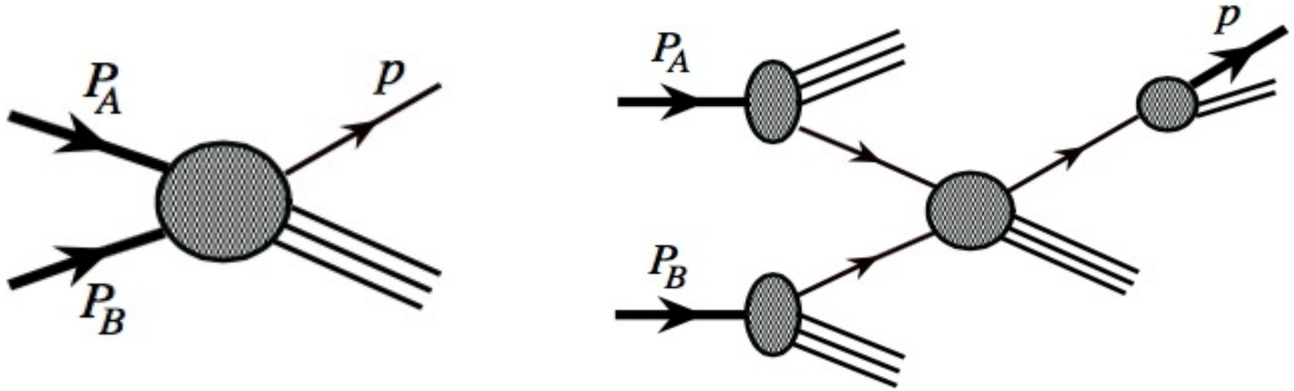
The factorization arguments at the leading power are independent of the spin states of the colliding hadrons

➡ Same formula with polarized PDFs for γ*,W/Z, H^0…

Factorization for more than Two Hadrons

- **Factorization for high p_T single hadron:**

Nayak, Qiu, Sterman, 2006



$\gamma, W/Z, \ell(s), \text{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O (1/P$_T^2$)

$p_T \gg m \gtrsim \Lambda_{\text{QCD}}$

$$\frac{d\sigma_{AB\to C+X}(p_A,p_B,p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x,\mu_F^2) \otimes \phi_{B\to b}(x',\mu_F^2)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x,x',z,y,p_T^2,\mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z,\mu_F^2)$$

Same arguments work for more final-state hadrons if every pair of hadrons have an invariant mass >> Λ_{QCD}

- **Fragmentation function:** $D_{c\to C}(z,\mu_F^2)$
- **Choice of the scales:** $\mu_{\text{Fac}}^2 \approx \mu_{\text{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Predictive Power of QCD Factorization

- **Universality of non-perturbative hadron structure + calculable matching coefficients:**
  - lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{\rm EXP}_{l+P\to l+X} = C_{l+k\to l+X} \otimes {\rm PDF}_P + \mathcal{O}(Q_s^2/Q^2)$$

$$\sigma^{\rm EXP}_{l+P\to l+H+X} = C_{l+k\to l+k+X} \otimes {\rm PDF}_P \otimes {\rm FF}_H + \mathcal{O}(Q_s^2/Q^2)$$

  - hadron-hadron reactions (LHC)

$$\sigma^{\rm EXP}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes {\rm PDF}_P \otimes {\rm PDF}_P + \mathcal{O}(Q_s^2/Q^2)$$

  - lepton-lepton reactions (Belle)

$$\sigma^{\rm EXP}_{l+\bar{l}\to H+X} = C_{l+\bar{l}\to k+X} \otimes {\rm FF}_H + \mathcal{O}(Q_s^2/Q^2)$$

Plus other factorizable observables – cross sections Plus LQCD calculable & factorizable hadron matrix elements

- **Hadron structure = Theory + Experiment + Phenomenology:**
  - **Factorization – Identify "Good" observables (Theory)**
  - **Measurement – Get "Reliable" data (Experiment)**
  - **Global analysis – Extract "Universal" structure information (Phenomenology) by solving an inverse problem**

QCD Global Analysis of Experimental Data (as well as Lattice Data)

- **It is an inverse problem:**

Experimental data

Future EIC, Jefferson Lab, KEK, CERN, Brookhaven National Laboratory, DESY, Fermilab

QCD Factorization Theorem

$= \int \cdots \int$ **Analytical Coefficient functions derived from QCD perturbation theory** $\otimes$ **Quantum Correlation Functions of quarks and gluons Unknown** $+$ **Systematic error from theory**

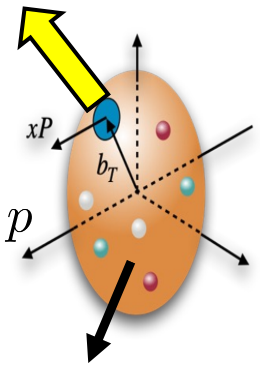
Flavors: u, d, s, …; g
Spin (quark, gluon hadron)
Momentum/position
(Before/after the collision)

Bayesian inference
Theory/Phenomenology
AI/ML, Computation

Extract QCFs

PDF_P, FF_H, …

Quark Polarization



Nucleon Polarization

- **Input for QCD Global analysis/fitting:**

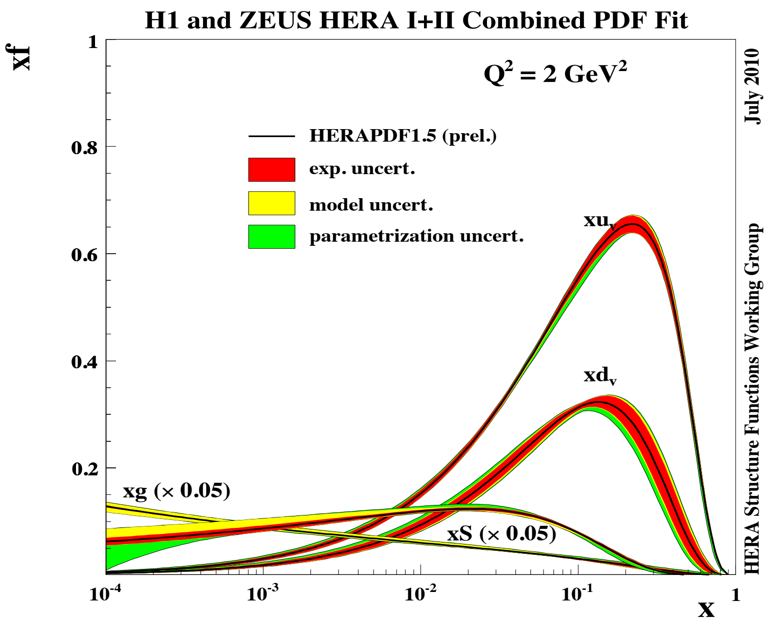
PDFs, FFs at an input scale:

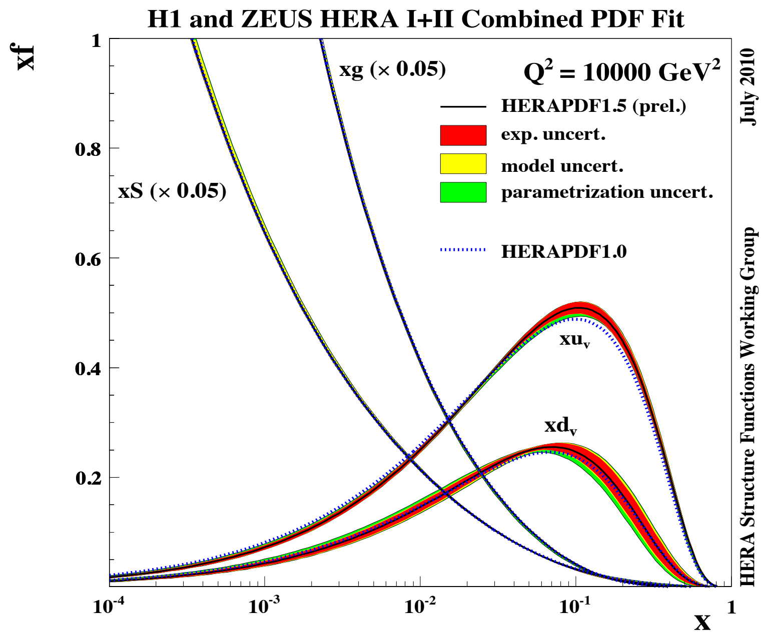
$$\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$$

Input scale ~ GeV — **Fitting paramters**

PDFs from Global Analysis

❑ Q2-dependence is a prediction of pQCD calculation:





❑ Physics interpretation of PDFs:

$f(x, Q^2)$: **Probability density to find a parton of flavor "f" carrying momentum fraction "x", probed at a scale of "Q²"**

✧ **Number of partons:** $$\int_0^1 dx\, u_v(x,Q^2) = 2, \quad \int_0^1 dx\, d_v(x,Q^2) = 1$$

✧ **Momentum fraction:** $$\langle x(Q^2)\rangle_f = \int_0^1 dx\, x\, f(x,Q^2) \;\Rightarrow\; \sum_f \langle x(Q^2)\rangle = 1$$

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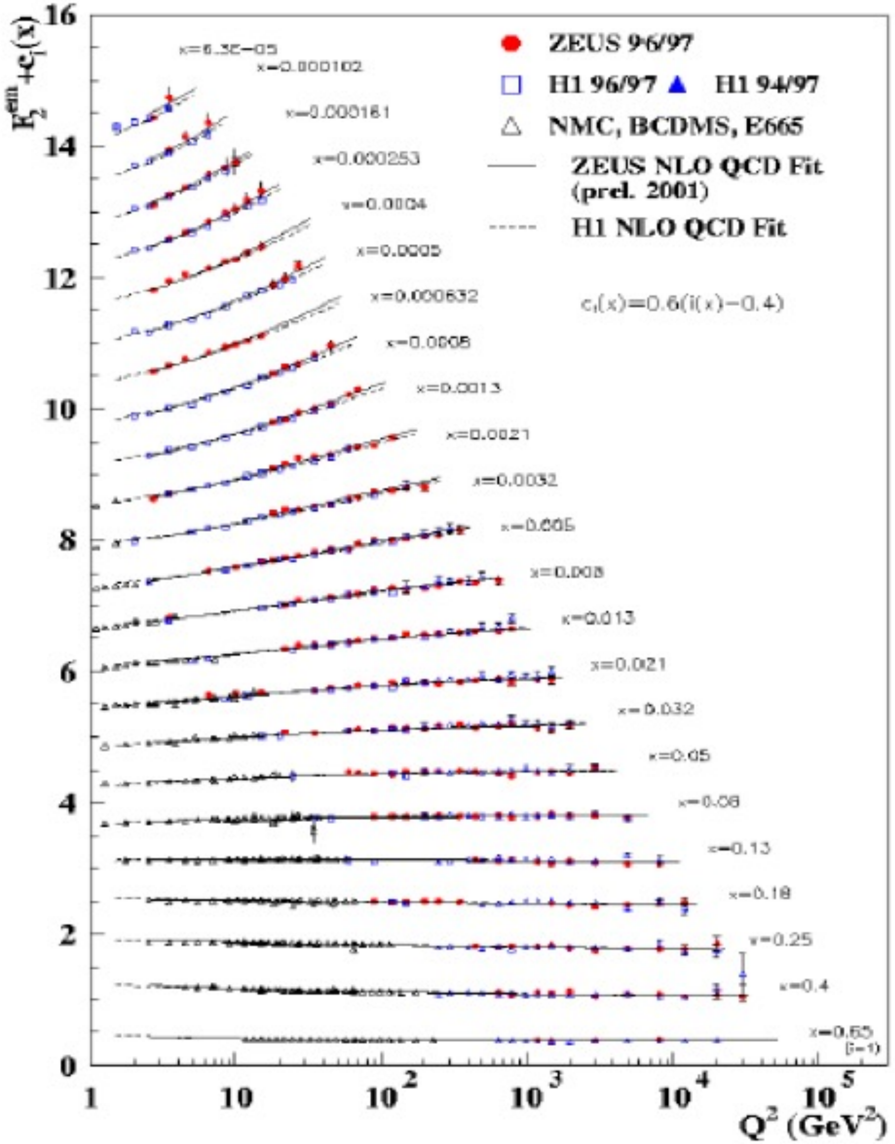
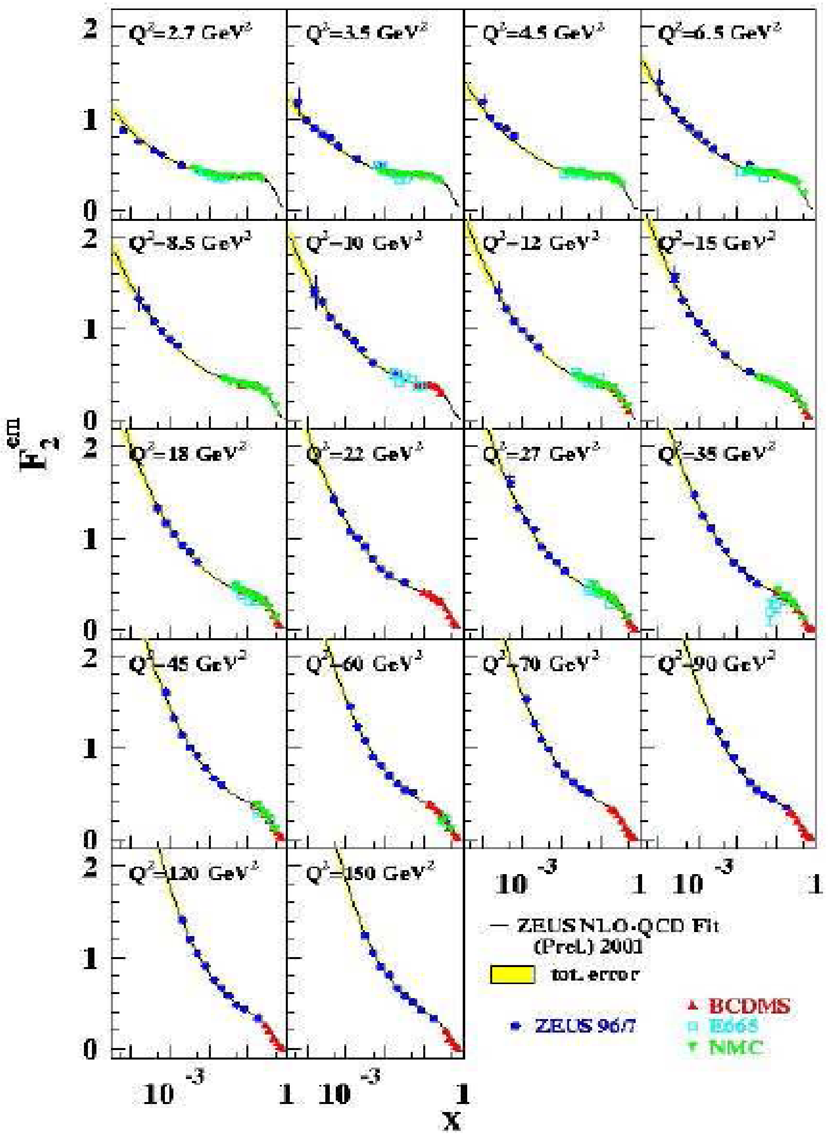
Scaling and Scaling Violation





Fit Quality:

$\chi^2/\mathrm{dof} \sim 1 \Rightarrow$ Non-trivial check of QCD

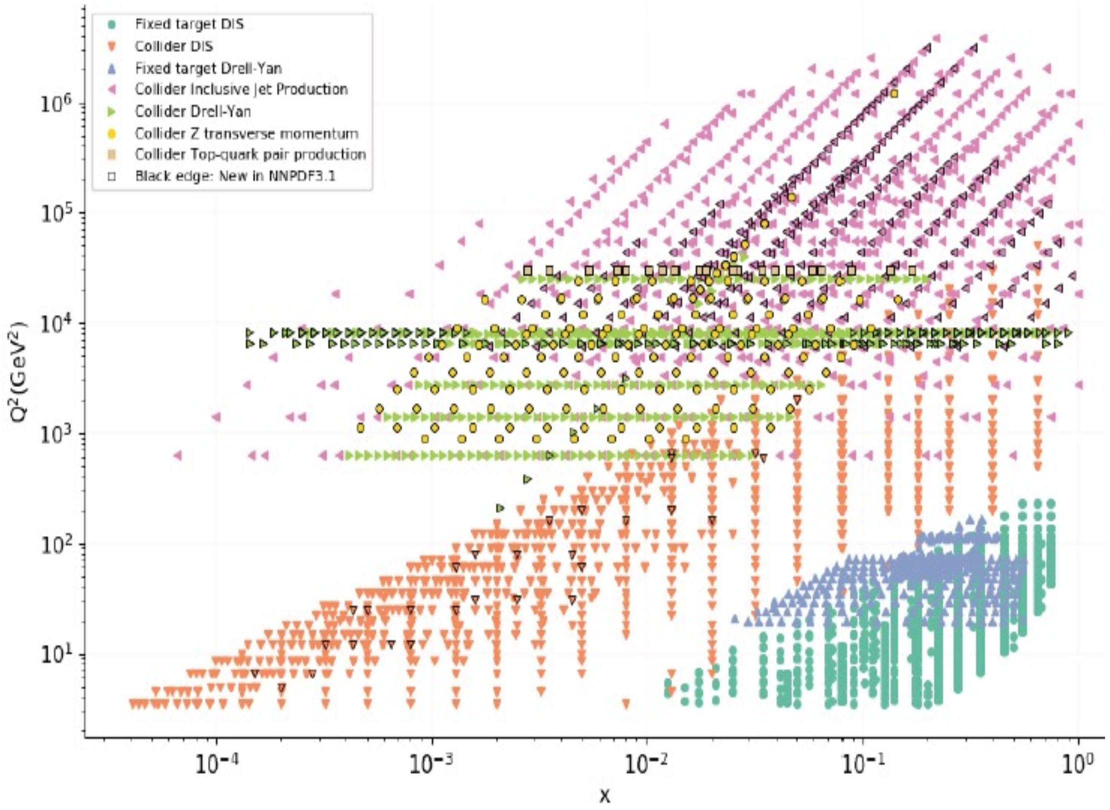
Q^2-dependence is a prediction of pQCD calculation

QCD Factorization Works to the Precision

❑ Data sets for Global Fits:

	Process	Subprocess	Partons	x range
Fixed Target	$\ell^{\pm}\{p,n\}\to\ell^{\pm}+X$	$\gamma^* q\to q$	$q,\bar{q},g$	$x\gtrsim 0.01$
	$\ell^{\pm}n/p\to\ell^{\pm}+X$	$\gamma^* d/u\to d/u$	d/u	$x\gtrsim 0.01$
	$pp\to\mu^+\mu^-+X$	$u\bar{u},d\bar{d}\to\gamma^*$	$\bar{q}$	$0.015\lesssim x\lesssim 0.35$
	$pn/pp\to\mu^+\mu^-+X$	$(u\bar{d})/(u\bar{u})\to\gamma^*$	$\bar{d}/\bar{u}$	$0.015\lesssim x\lesssim 0.35$
	$\nu(\bar{\nu})N\to\mu^-(\mu^+)+X$	$W^* q\to q'$	$q,\bar{q}$	$0.01\lesssim x\lesssim 0.5$
	$\nu N\to\mu^-\mu^+ +X$	$W^* s\to c$	s	$0.01\lesssim x\lesssim 0.2$
	$\bar{\nu}N\to\mu^+\mu^- +X$	$W^*\bar{s}\to\bar{c}$	$\bar{s}$	$0.01\lesssim x\lesssim 0.2$
Collider DIS	$e^{\pm}p\to e^{\pm}+X$	$\gamma^* q\to q$	$g,q,\bar{q}$	$0.0001\lesssim x\lesssim 0.1$
	$e^+p\to\bar{\nu}+X$	$W^+\{d,s\}\to\{u,c\}$	d,s	$x\gtrsim 0.01$
	$e^{\pm}p\to e^{\pm}c\bar{c}+X$	$\gamma^* c\to c,\gamma^* g\to c\bar{c}$	c,g	$10^{-4}\lesssim x\lesssim 0.01$
	$e^{\pm}p\to e^{\pm}b\bar{b}+X$	$\gamma^* b\to b,\gamma^* g\to b\bar{b}$	b,g	$10^{-4}\lesssim x\lesssim 0.01$
	$e^{\pm}p\to \text{jet}+X$	$\gamma^* g\to q\bar{q}$	g	$0.01\lesssim x\lesssim 0.1$
Tevatron	$p\bar{p}\to\text{jet}+X$	$gg,qg,qq\to 2j$	g,q	$0.01\lesssim x\lesssim 0.5$
	$p\bar{p}\to(W^{\pm}\to\ell^{\pm}\nu)+X$	$ud\to W^+,\bar{u}\bar{d}\to W^-$	$u,d,\bar{u},\bar{d}$	$x\gtrsim 0.05$
	$p\bar{p}\to(Z\to\ell^+\ell^-)+X$	$uu,dd\to Z$	u,d	$x\gtrsim 0.05$
	$p\bar{p}\to t\bar{t}+X$	$qq\to t\bar{t}$	q	$x\gtrsim 0.1$
LHC	$pp\to\text{jet}+X$	$gg,qg,q\bar{q}\to 2j$	g,q	$0.001\lesssim x\lesssim 0.5$
	$pp\to(W^{\pm}\to\ell^{\pm}\nu)+X$	$u\bar{d}\to W^+,d\bar{u}\to W^-$	$u,d,\bar{u},\bar{d},g$	$x\gtrsim 10^{-3}$
	$pp\to(Z\to\ell^+\ell^-)+X$	$q\bar{q}\to Z$	$q,\bar{q},g$	$x\gtrsim 10^{-3}$
	$pp\to(Z\to\ell^+\ell^-)+X,p_{\perp}$	$gq(\bar{q})\to Zq(\bar{q})$	$g,q,\bar{q}$	$x\gtrsim 0.01$
	$pp\to(\gamma^*\to\ell^+\ell^-)+X$, Low mass	$q\bar{q}\to\gamma^*$	$q,\bar{q},g$	$x\gtrsim 10^{-4}$
	$pp\to(\gamma^*\to\ell^+\ell^-)+X$, High mass	$q\bar{q}\to\gamma^*$	$\bar{q}$	$x\gtrsim 0.1$
	$pp\to W^+\bar{c},W^-c$	$sg\to W^+c,\bar{s}g\to W^-\bar{c}$	$s,\bar{s}$	$x\sim 0.01$
	$pp\to t\bar{t}+X$	$gg\to t\bar{t}$	g	$x\gtrsim 0.01$
	$pp\to D,B+X$	$gg\to c\bar{c},b\bar{b}$	g	$x\gtrsim 10^{-6},10^{-5}$
	$pp\to J/\psi,\Upsilon+pp$	$\gamma^*(gg)\to c\bar{c},b\bar{b}$	g	$x\gtrsim 10^{-6},10^{-5}$
	$pp\to\gamma+X$	$gq(\bar{q})\to\gamma q(\bar{q})$	g	$x\gtrsim 0.005$

❑ Kinematic Coverage:



❑ Fit Quality: $\chi^2/\text{dof}\sim 1$ NNPDF

	LO	NLO	NNLO
All data sets	3706 / 2763	3267 / 2996	2717 / 2663

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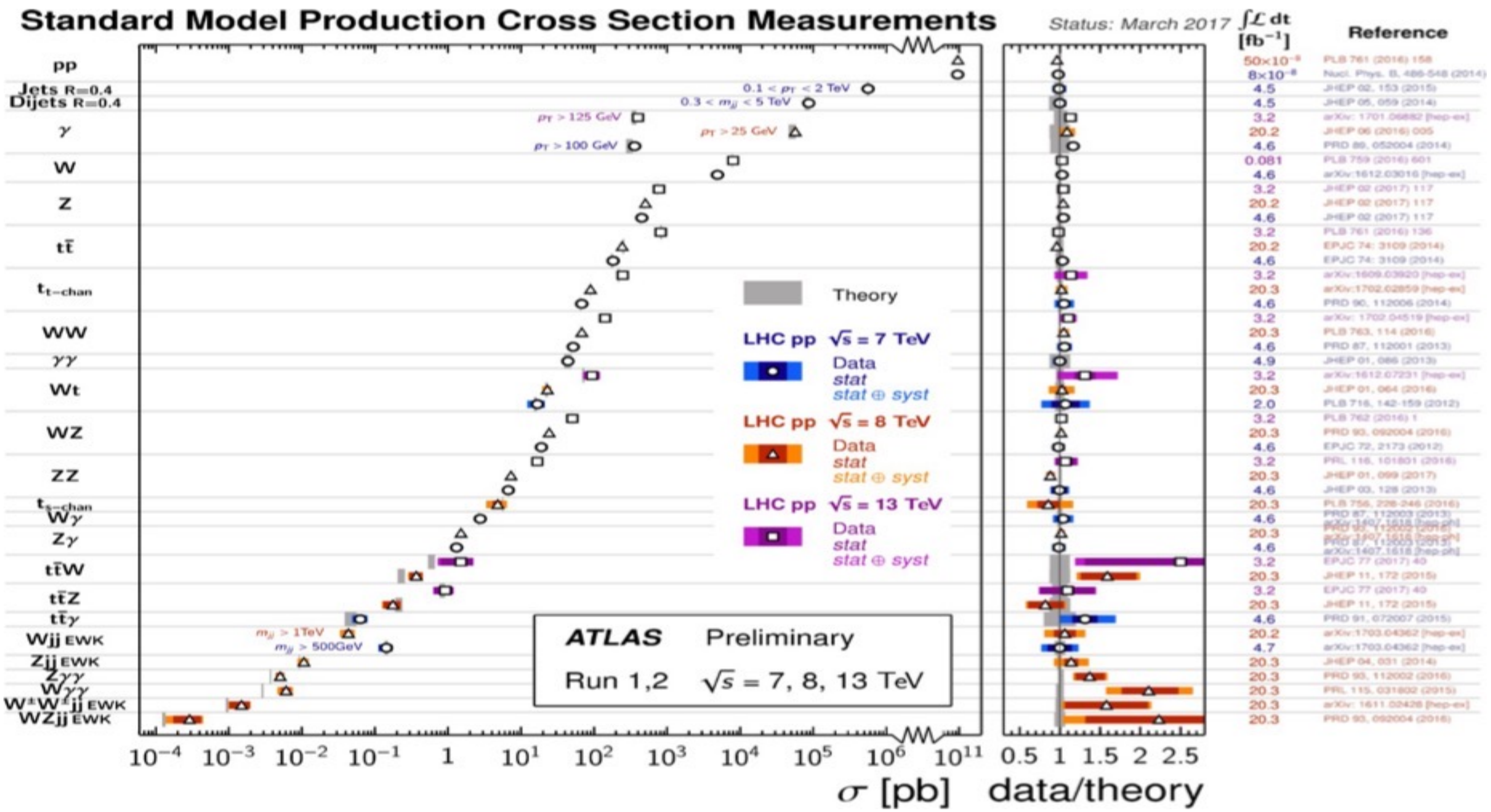
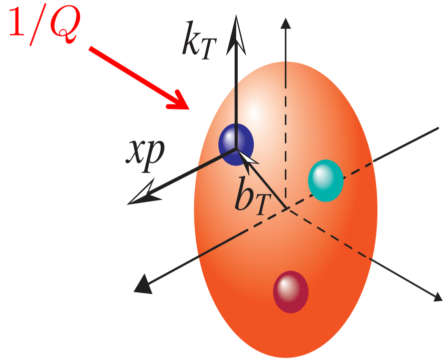
Unprecedent Success of QCD and Standard Model



SM: Electroweak processes + QCD perturbation theory + PDFs works!

Probes for 3D Hadron Structure

- **Single scale hard probe is too "localized":**



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> 1/Q

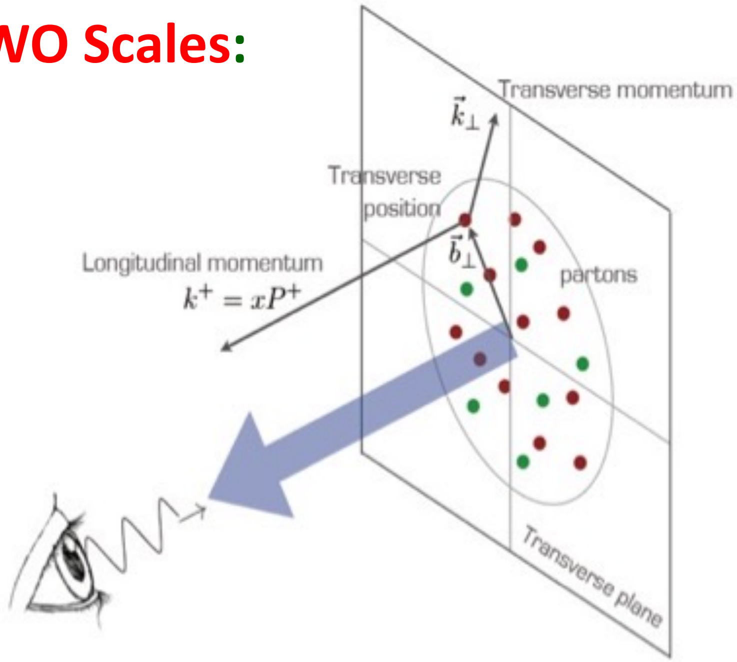
- **Need new type of "Hard Probes" – Physical observables with TWO Scales:**

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

Hard scale: Q_1 To localize the probe particle nature of quarks/gluons

"Soft" scale: Q_2 could be more sensitive to the hadron structure ~ 1/fm

Hit the hadron "very hard" without breaking it, clean information on the structure!



From One-Scale to Two-Scale Observables

❑ Drell-Yan process in hadron-hadron collisions:

The process: $\sigma^{\text{EXP}}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes \text{PDF}_P \otimes \text{PDF}_P + \mathcal{O}(Q_s^2/Q^2)$

One-scale case:

$$\frac{d\sigma^{\text{DY}}_{hh'}(p_A, p_B, q)}{dQ^2} = \sum_{f,f'} \int_0^1 dx \int_0^1 dx' \, \phi_f(x) \, \frac{d\hat{\sigma}^{\text{el}}_{ff'}(xp_A, x'p_B, q)}{dQ^2} \, \phi_{f'}(x')$$

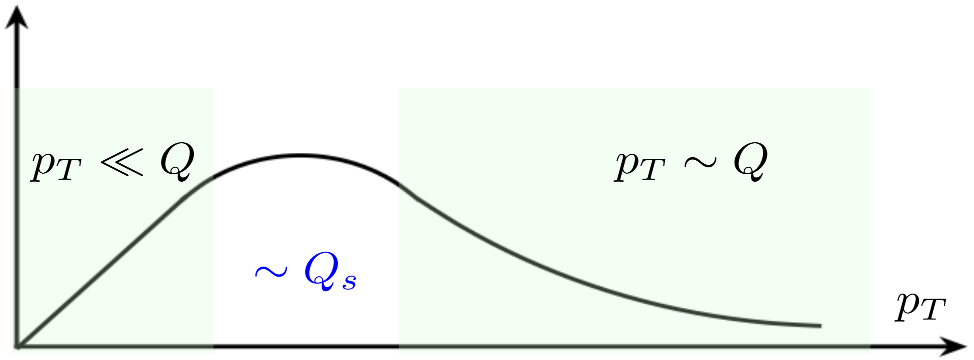
Hard scale – invariant mass of the lepton-pair: $Q^2 \equiv q^2 = (l+\bar{l})^2 \gg \Lambda^2_{\text{QCD}} \sim 1/R_h^2$

Two-scale case:

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

Hard scale: Q^2 **Soft scale:** q_T^2 **when** $Q^2 \gg q_T^2$ $\quad d^4q = dy\, dQ^2 dq_T^2 d\phi_q$

❑ Matching between TMD and Collinear Region:

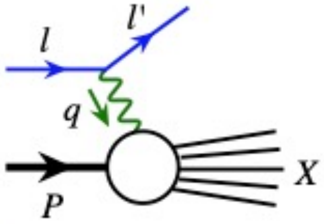


$$\frac{d\sigma^{\text{EXP}}}{dyd^2p_T dQ^2} = \frac{d\sigma^{\text{TMD}}}{dyd^2p_T dQ^2} + \frac{d\sigma^{\text{CO}}}{dyd^2p_T dQ^2} - \frac{d\sigma^{\text{ASY}}}{dyd^2p_T dQ^2}$$

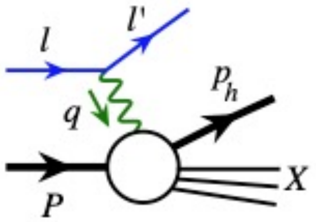
$$\frac{d\sigma^{\text{ASY}}}{dyd^2p_T dQ^2} \to \frac{d\sigma^{\text{TMD}}}{dyd^2p_T dQ^2} \quad \text{as } p_T \to Q$$

$$\frac{d\sigma^{\text{ASY}}}{dyd^2p_T dQ^2} \to \frac{d\sigma^{\text{CO}}}{dyd^2p_T dQ^2} \quad \text{as } p_T \to 0$$

Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering (SIDIS)

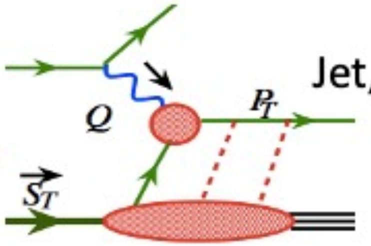


Scale: Q^2 - PDFs



$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!



Jet, π, J/ψ, …

$f(x, k_T, Q)$ - TMDs

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big]$$

$$+S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big]$$

$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)$$

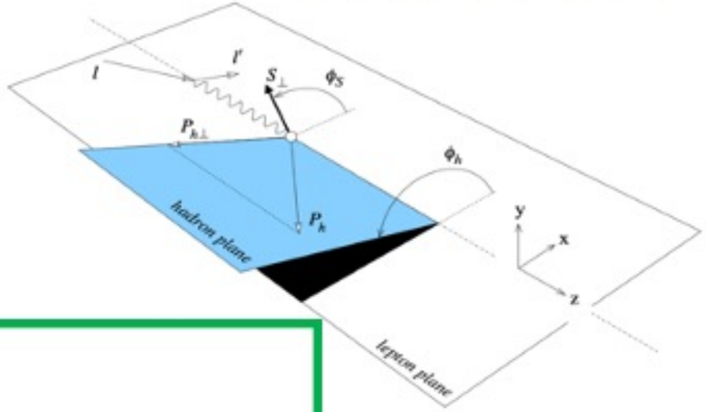
$$+\varepsilon\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

18 SIDIS Structure Functions



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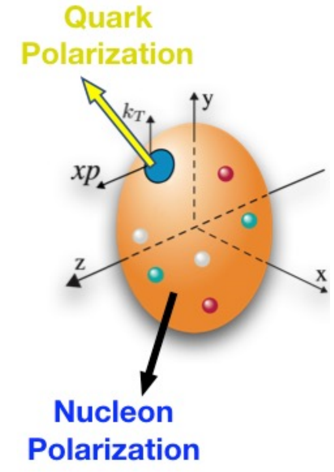
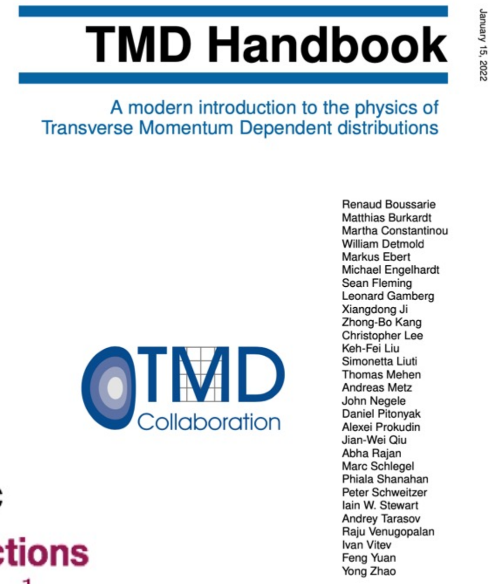
Transverse Momentum Dependent PDFs (TMDs)

❑ Quark TMDs with polarization:

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ – Boer-Mulders
	L		$g_1(x,k_T^2)$ – Helicity	$h_{1L}^{\perp}(x,k_T^2)$ – Long-Transversity
	T	$f_1^{\perp}(x,k_T^2)$ – Sivers	$g_{1T}(x,k_T^2)$ – Trans-Helicity	$h_1(x,k_T^2)$ – Transversity $h_{1T}^{\perp}(x,k_T^2)$ – Pretzelosity



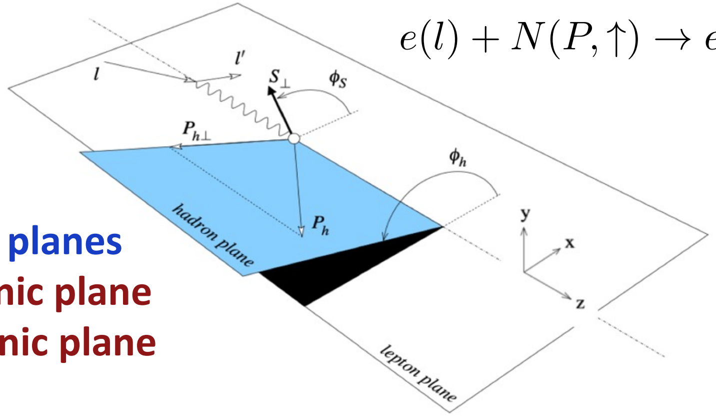


Analogous tables for:

- Gluons $f_1 \to f_1^g$ etc
- Fragmentation functions
- Nuclear targets $S \neq \frac{1}{2}$

❑ Polarized SIDIS:

$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$



Two planes
Leptonic plane
Hadronic plane

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

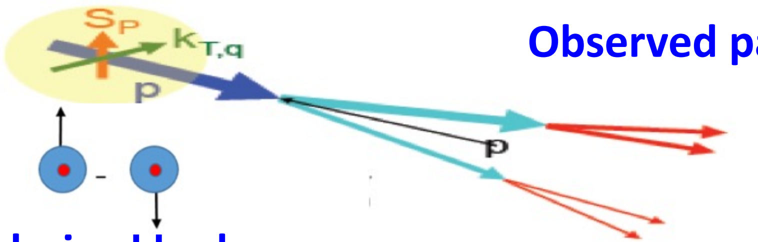
$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

Angular modulation provides the best way to separate TMDs

TMDs: Correlation between Hadron Property and Parton Flavor-Spin-Motion

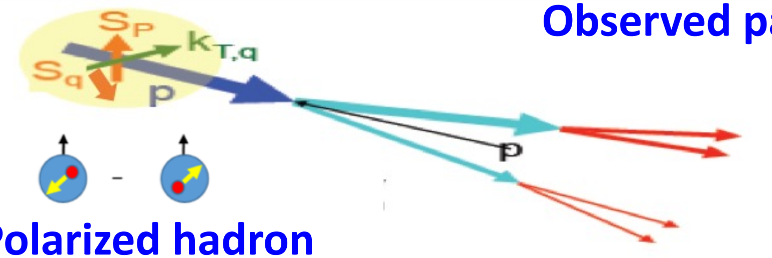
- Quantum correlation between hadron spin and parton motion:



Sivers effect – Sivers function

Hadron spin influences parton's transverse motion

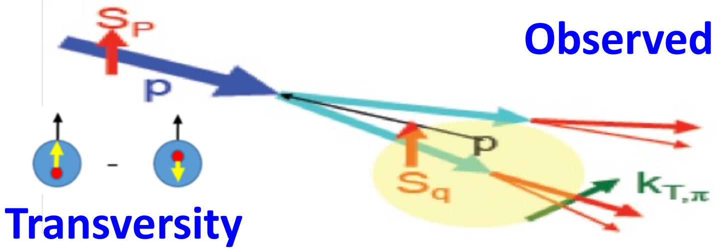
- Quantum correlation between hadron spin and parton spin:



Pretzelosity – model OAM

Hadron spin and parton spin influence parton's transverse motion

- Quantum correlation between parton's spin and its hadronization:



Collins effect – Collins function

Parton's transverse polarization influences its hadronization

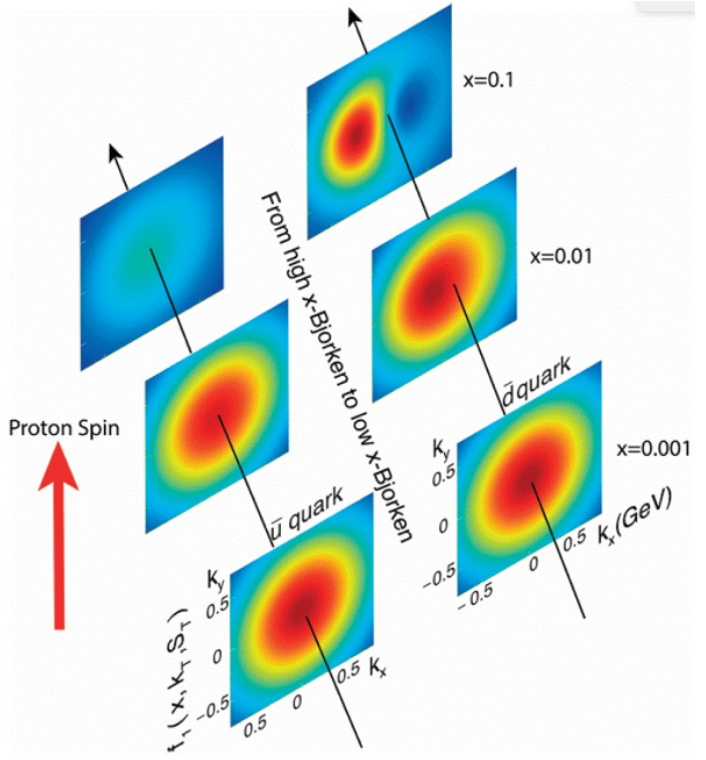


Fig. 2.7 NAS Report

The Electron-Ion Collider (EIC)

- Lec. 1: EIC & Fundamentals of QCD
- Lec. 2: Probing Structure of Hadrons without seeing Quark/Gluon?
  – *breaking the hadron*!
- Lec. 3: Probing Structure of Hadrons with polarized beam(s)
  – *Spin as another knob*
- Lec. 4: Probing Structure of Hadrons without breaking them? Dense Systems of gluons
  – *Nuclei as Femtosize Detectors*

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