Effective Field Theories for Physics Beyond the Standard Model. Lecture 1

Emanuele Mereghetti

National Nuclear Physics Summer School, Indiana University

for any questions: emereghetti@lanl.gov

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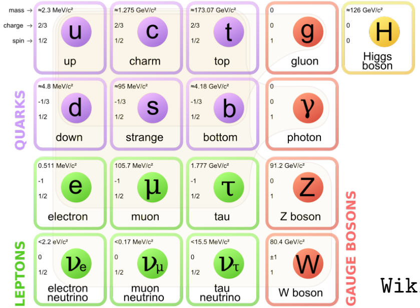
Introduction

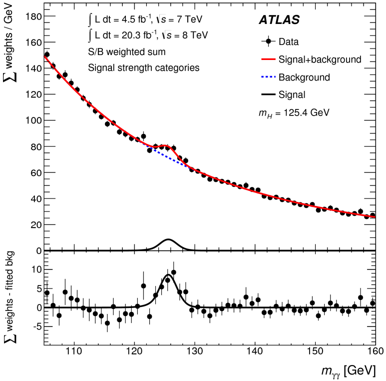


Wikipedia

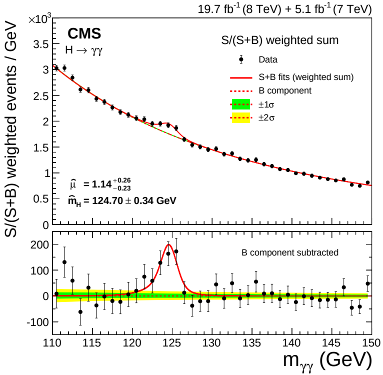
The Standard Model of Particle Physics

1. describes nature in a economic and elegant way
   (spontaneously broken) local gauge symmetry
2. validated over a wide variety of scales

Introduction



ATLAS collaboration, '14



CMS collaboration, '14

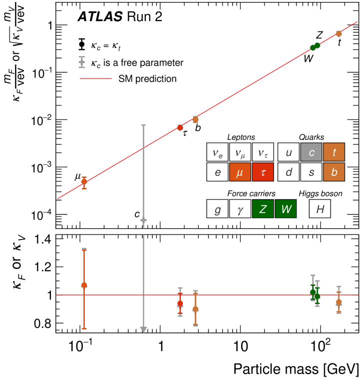
- Higgs boson discovered at the LHC Run I

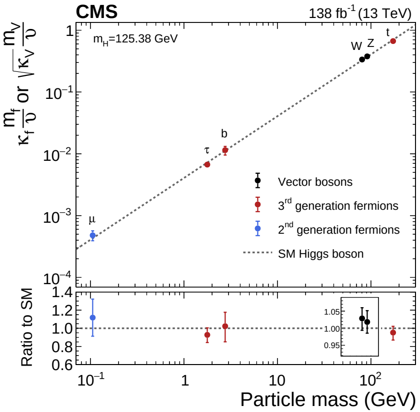
$$m_H = 125.25 \pm 0.17$$

last free parameter in SM!

- is the SM the final theory of nature?

Higgs couplings and electroweak symmetry breaking





1. is the Higgs the SM Higgs? What drives Electroweak Symmetry Breaking?

- couplings to gauge-bosons, 3$^{\mathrm{rd}}$ generation fermions are SM-like
- first hints of coupling to 2$^{\mathrm{nd}}$ generation
- measuring h^3, $h^2 W^+ W^-$ and $h^2 ZZ$ important for EWSB mechanism

The matter-antimatter asymmetry

see J. Singh's lecture

2. why is there more matter than antimatter?

Sakharov conditions A. Sakharov, '67

- C and CP violation
- baryon number violation
- deviation from thermal equilibrium

The matter-antimatter asymmetry

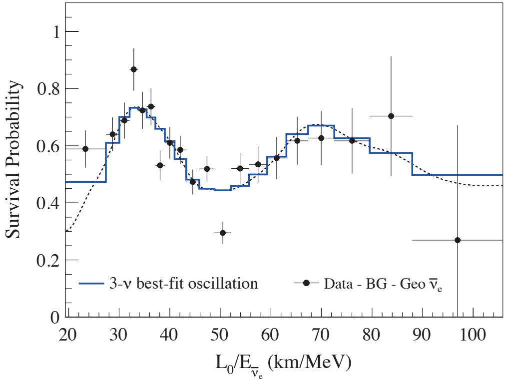
see J. Singh's lecture

2. why is there more matter than antimatter?

Sakharov conditions A. Sakharov, '67

- C and CP violation — × yes, but not enough
- baryon number violation — ✓
- deviation from thermal equilibrium — × not for $m_H = 125$ GeV

The nature of massive neutrinos



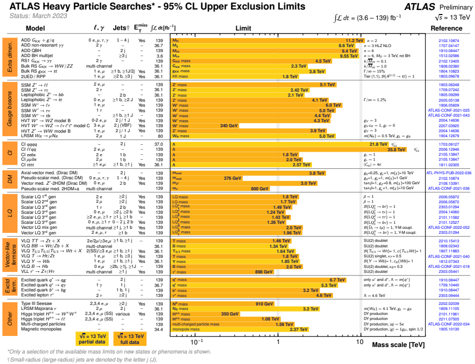
$$P(\bar{\nu}_e \to \bar{\nu}_e) \sim 1 - A \sin^2\left(\frac{\Delta m^2_{21} L}{E}\right)$$

KamLAND Collaboration, '11

3. what are the mass and nature of neutrinos?

- are neutrinos Dirac or Majorana particles?
- is there a relation between neutrinos and baryogenesis?

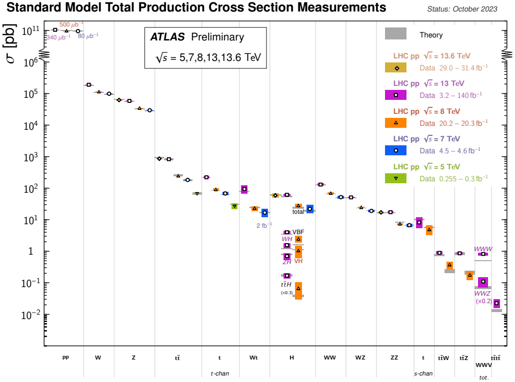
Finding BSM: the energy frontier



1. smash protons as hard as you can, and see what comes out

- create new particles

Finding BSM: the energy frontier



1. smash protons as hard as you can, and see what comes out

- create new particles
- **and/or** study their effects on rare processes

Finding BSM: the precision frontier

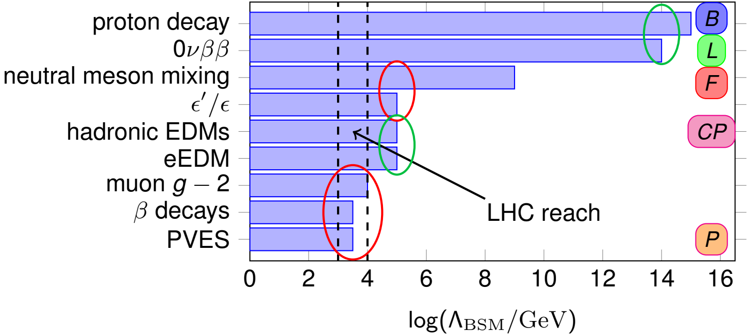
Majorana demonstrator

2. search for tiny indirect effects in processes with no (or very precisely known) SM background

- electric dipole moments
- neutrinoless double β decay
- lepton flavor violation $\mu \to e\gamma$
- muon and electron $g - 2$
- B and kaon physics
- precision β decays

Finding BSM: The precision frontier

$O = (Q/\Lambda_{\mathrm{BSM}})^n$



competitive/complementary to energy frontier

1. observables w. SM background
   need precise SM background to claim discovery
2. observables w/o (w. negligible) SM background (usually violating SM symmetries)
   need precision to extract microscopic symmetry violation params ($\bar{\theta}$, $m_{\beta\beta}$,...)

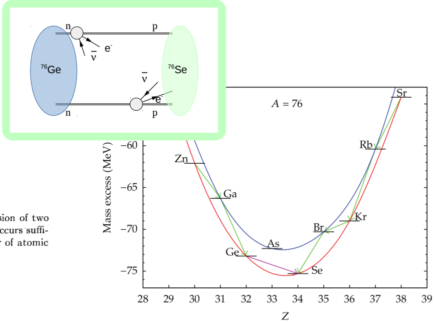
most of these probes use nuclei as targets!

A few sensitive nuclear observables: neutrinoless double beta decay

Double Beta-Disintegration

M. Goeppert-Mayer, *The Johns Hopkins University*
(Received May 20, 1935)

From the Fermi theory of β-disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over 10^{17} years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.



- Double beta decay is a rare doubly weak process
- lepton-number-conserving 2ν channel

see B. Jones' lectures

$$^{76}_{32}\mathrm{Ge} \rightarrow ^{76}_{34}\mathrm{Se} + e^- e^- \bar{\nu}_e \bar{\nu}_e \qquad T^{2\nu}_{1/2} = 1.9 \times 10^{21}\ \mathrm{yr}$$

rarest observed nuclear process

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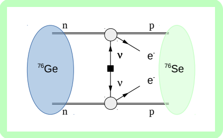

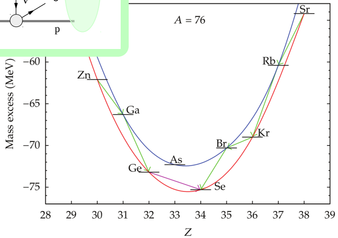

- Double beta decay is a rare doubly weak process — see B. Jones' lectures
- lepton-number-conserving 2ν channel

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rarest observed nuclear process

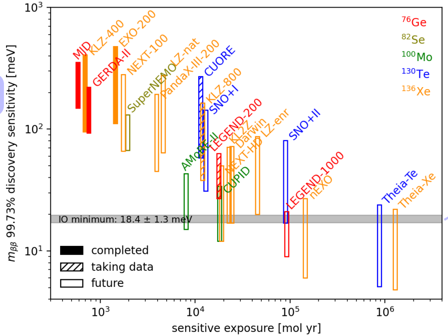
- if neutrinos are Majorana, a lepton-number-violating 0ν channel becomes possible

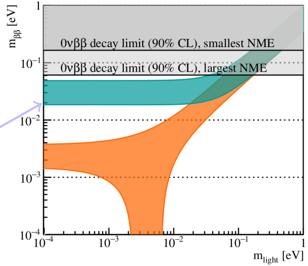
$$^{76}_{32}\mathrm{Ge} \rightarrow ^{76}_{34}\mathrm{Se} + e^- e^- \qquad T^{0\nu}_{1/2} > 1.8 \times 10^{26}\ \mathrm{yr}$$

BSM physics!

Neutrinoless double beta decay

$$m_{\beta\beta} = \sum U_{ei}^2 m_i$$

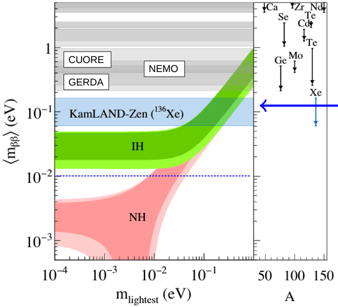


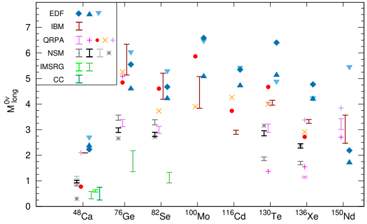


arXiv:2212.11099 M. Agostini, G. Benato, J. Detwiler, J. Menendez, F. Vissani '22

- $0\nu\beta\beta$ experiments have the best chance to reveal the neutrinos' Majorana nature
- next-generation tonne-scale experiments will improve existing bounds by 1-2 orders of magnitude
- and rule out the inverted ordering (in minimal scenario)

Neutrinoless Double Beta Decay





M. Agostini, G. Benato, J. Detwiler, J. Menendez, F. Vissani '22

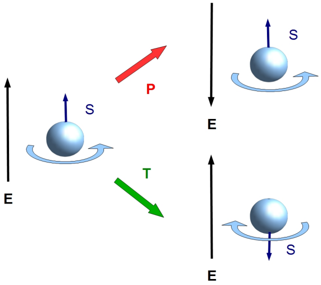
- observation clear signal of BSM physics but interpretation needs theory!
- E.g. in "standard scenario", rate is prop. to the neutrino Majorana mass × nuclear matrix element

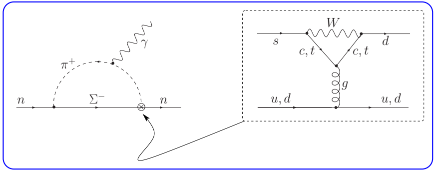
$$\Gamma \propto \sum_i U_{ei}^2 m_i \times M^{0\nu}$$

need to get hadronic and nuclear physics right
to extract neutrino mass & interplay with oscillation experiments

Electric dipole moments

$H = -d_n \mathbf{S} \cdot \mathbf{E}$





- signal of T and P violation (CP)
- insensitive to CP violation in the SM
- BSM CP violation needed for baryogenesis

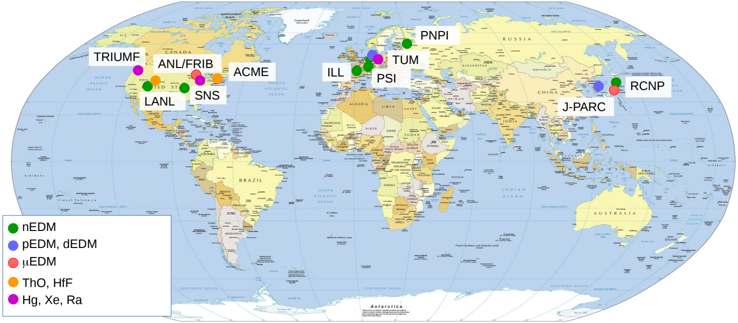
see J. Singh's lectures

neutron $d_n|_{\mathrm{SM}} \sim 10^{-32}\, e\,\mathrm{cm}$ $\ll$ $|d_n|_{\mathrm{exp}} < 1.8 \cdot 10^{-26}\, e\,\mathrm{cm}$

M. Pospelov and A. Ritz, '05; C. Y. Seng, '14; nEDM Collaboration, '20

large window & strong motivations for new physics!

Electric dipole moments



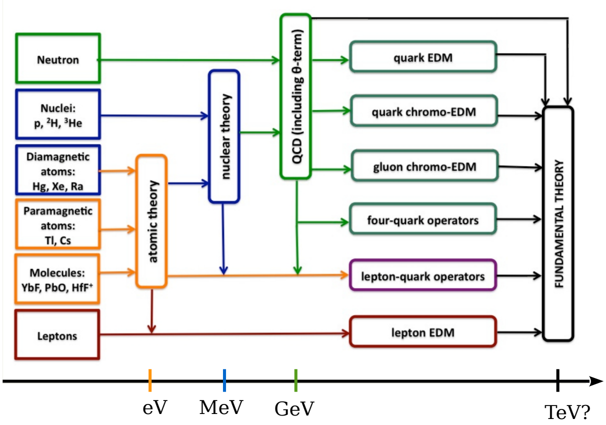
- large worldwide experimental program

$$d_e < 4.0 \cdot 10^{-30}\ e\,\text{cm} \qquad d_n < 1.8 \cdot 10^{-26}\ e\,\text{cm}$$
$$d_{^{225}\mathrm{Ra}} < 1.2 \cdot 10^{-23}\ e\,\text{cm} \qquad d_{^{199}\mathrm{Hg}} < 6.2 \cdot 10^{-30}\ e\,\text{cm}$$

$\Lambda_{\mathrm{naive}} \sim$ 10-100 TeV

- orders of magnitude improvements & new systems in next generation

EDMs and BSM physics

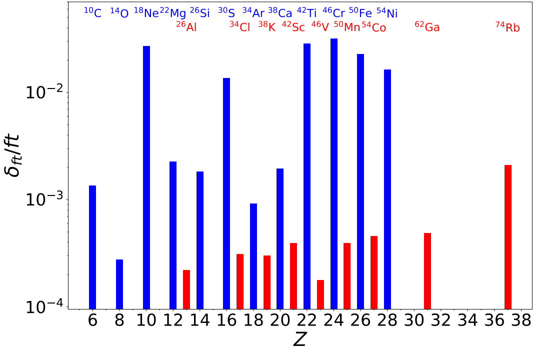


Non-zero EDM in **any** system will be revolutionary

thanks to J. de Vries

- different systems crucial to pinpoint BSM
- need atomic, nuclear, hadronic theory to identify CPV at quark level
- need to correlate with flavor physics and LHC to identify BSM

Precision β decay



$0^+ \to 0^+$ transitions

data from J. C. Towner and I. S. Hardy, '20

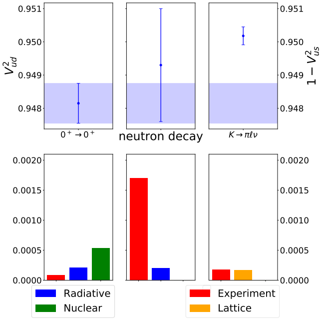
- lifetime measured with better than 0.1% accuracy in several systems — see A. Holley's lectures

$$\frac{1}{ft} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2}(1 + \Delta_R^V)(1 + \delta_R')(1 + \delta_{\rm NS} - \delta_C)$$

- allow to extract the V_{ud} CKM element with high accuracy

  provided we can control % level electromagnetic corrections (Δ_R^V, δ_R', $\delta_{\rm NS}$, δ_C)

Precision β decay

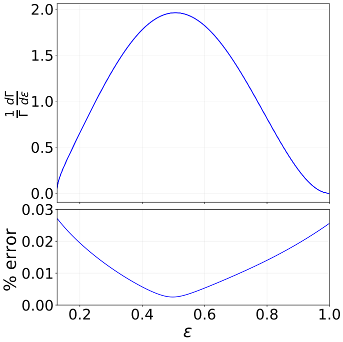


- SM predicts the CKM mixing matrix to be unitary: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- extraction of V_{ud} and V_{us} from nuclear β decays, K and π decays require theory input
- with current experiment and theory

$$\Delta = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -(15 \pm 5) \cdot 10^{-4}$$

$\sim 3\sigma$ deviation, new physics at the 10 TeV scale?
dominated by theory errors! need nuclear theory to claim BSM physics . . .

Precision β decay



Decay correlations and spectral shape

$$\omega(\langle J\rangle|E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu = \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \left\{ 1 + a\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e} + c\left[\frac{1}{3}\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} - \frac{(\mathbf{p}_e\cdot\mathbf{j})(\mathbf{p}_\nu\cdot\mathbf{j})}{E_e E_\nu}\right]\left[\frac{J(J+1)-3\langle(\mathbf{J}\cdot\mathbf{j})^2\rangle}{J(2J-1)}\right] + \frac{\langle\mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_\nu}{E_\nu} + D\frac{\mathbf{p}_e\times\mathbf{p}_\nu}{E_e E_\nu}\right]\right\}. \quad (2)$$

J. D. Jackson, S. B. Treiman, and H. W. Wyld, '57

- lowest-order th. uncertainties drops out in pure Fermi/Gamow-Teller decays e.g. $^6\mathrm{He} \to {}^6\mathrm{Li}$

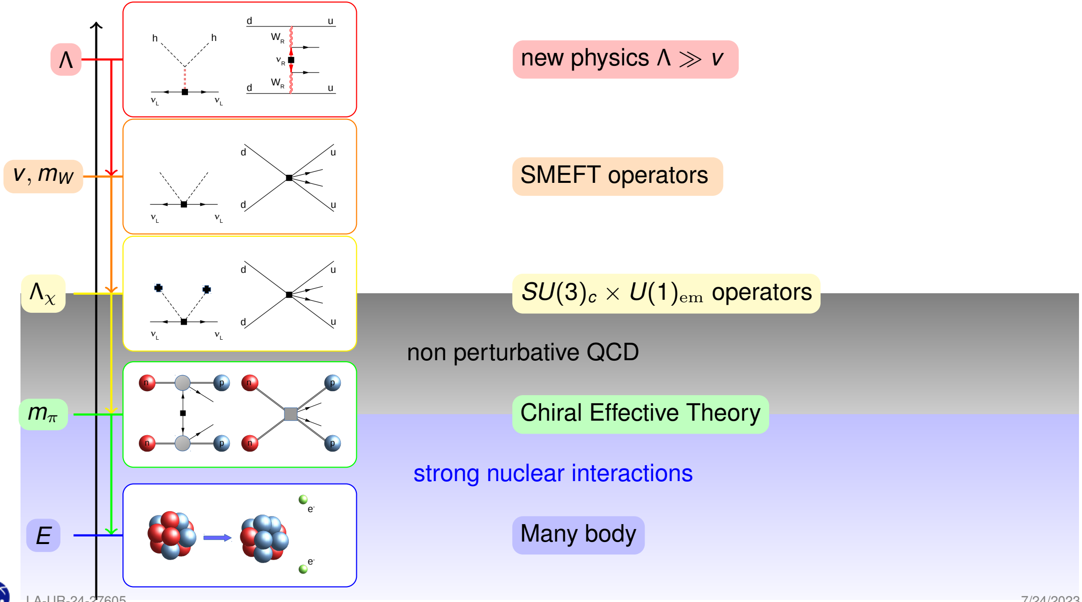
$$a_{\mathrm{GT}} = -1/3 + \mathcal{O}(E_e/m_N) + \mathcal{O}(\alpha), \qquad b_{\mathrm{GT}} = 0 + \mathcal{O}(E_e/m_N) + \mathcal{O}(\alpha)$$

- permille level spectral measurements probe non V-A currents at the $\sim$ 10 TeV scale

$$b_{BSM} = -8\frac{g_T \varepsilon_T}{g_A} + b_{\mathrm{SM}}$$

- but need to control radiative and recoil corrections

Nuclear Physics for Physics Beyond the Standard Model



Why Effective Field Theories?

a. At high energy, $Q \gtrsim 2$ GeV

- parametrize BSM physics **without** relying on explicit models

  "bottom-up" approach, data will tell us the right model

- well developed tools (renormalization group equation, matching) to connect observables at the electroweak scale with the precision frontier.

b. At the transition between perturbative and nonperturbative QCD, $Q \sim$ 1-2 GeV

- construct and organize hadronic interactions compatible with QCD symmetries
- determination of EFT couplings requires nonperturbative techniques

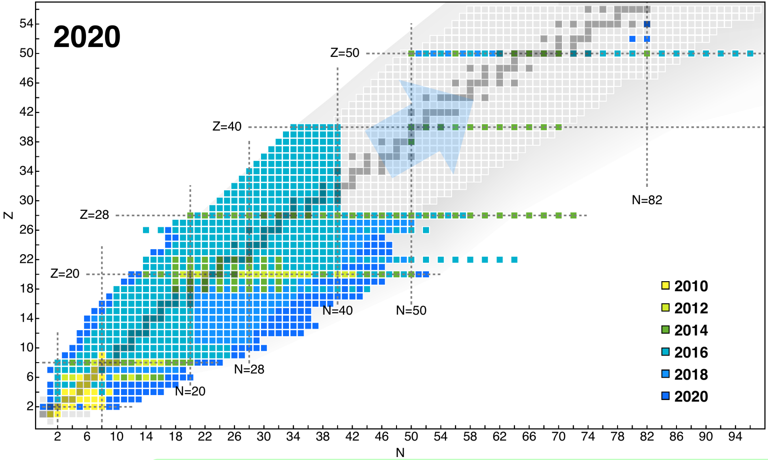
  strong connection with Lattice QCD

c. In the nuclear regime, $Q \lesssim 0.5$ GeV

- *ab initio* methods achieved great progress and can reach regions unimaginable 10 yrs ago
- can be coupled with nuclear interactions and BSM operators from EFT

EFTs as bridge to transfer info from pQCD/LQCD into many-body calculations and provide fully *ab initio* predictions for BSM probes

Effective Field Theories for Nuclear Physics

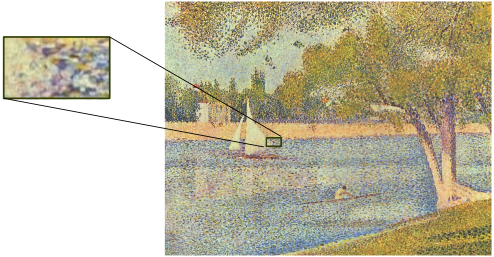


H. Hergert, ‘20

progress in *ab initio* many-body methods

- use nucleon degrees of freedom,
- have nuclear interactions valid across the nuclear landscape,
- solve the nuclear many-body problem with controlled and improvable approximations

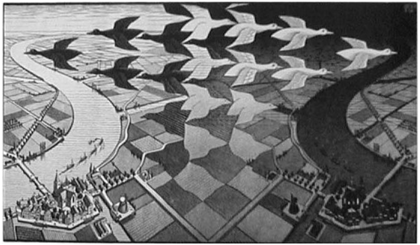
What is an Effective Field Theory?



The EFT paradigm

1. focus relevant scales & degrees of freedom

What is an Effective Field Theory?

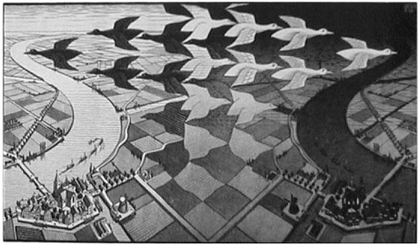


The EFT paradigm

1. focus relevant scales & degrees of freedom
2. identify symmetries of the theory
3. write all the possible interactions allowed by symmetries

everything that is not forbidden is allowed!

What is an Effective Field Theory?



The EFT paradigm

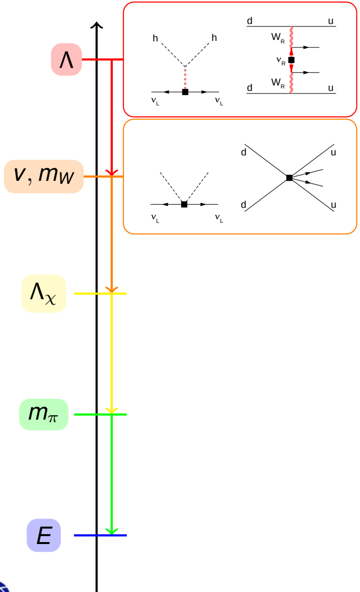
1. focus relevant scales & degrees of freedom
2. identify symmetries of the theory
3. write all the possible interactions allowed by symmetries

everything that is not forbidden is allowed!

4. an organizing principle (power counting)

expand in ratio of scales $Q/\Lambda \ll 1$

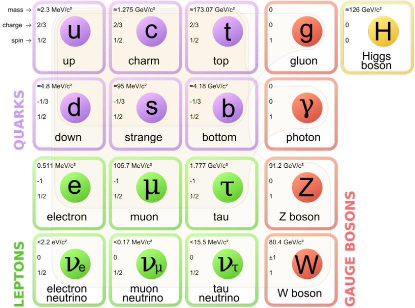
Effective Field Theories for BSM Physics: the Standard Model



new physics $\Lambda \gg v$

SMEFT operators

EFTs for BSM physics: the SM



1. degrees of freedom and scales

a. three generations of massive quarks and leptons

b. two massless gauge bosons (photon and gluons)

c. two massive gauge bosons (Z and W) and one massive scalar (H) with mass ~ 100 GeV

- LHC suggests there is a mass gap $m_X \gg m_H \sim 100$ GeV

EFTs for BSM physics: the SM

2. symmetries: $SU(2)_L \times U(1)_Y \times SU(3)_c$ gauge symmetry, spontaneously broken to $U(1)_{\rm em} \times SU(3)_c$

S. Weinberg, '67, A. Salam, '67
see S. Weinberg's reminiscences.

EFTs for BSM physics: the SM

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a. to explain μ and β decays, left- and right-handed particles are different under the weak interactions

- left-handed particles come in pairs, which mix under a gauge transformation

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix};$$

- right-handed particles, u_R, d_R and e_R are singlet under $SU(2)$ (notice, no ν_R!)
- L- and R-handed particles are assigned a $U(1)$ "hypercharge", related to the electric charge

$$\mathrm{y}_q = \frac{1}{6}, \quad \mathrm{y}_\ell = -\frac{1}{2}, \quad \mathrm{y}_u = \frac{2}{3}, \quad \mathrm{y}_d = -\frac{1}{3}, \quad y_e = -1, \qquad Q_\psi = \frac{\tau_3}{2} + y_\psi$$

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- under a $SU_L(2) \times U_Y(1)$ transformation, parametrized by the angles $\alpha_i(x)$ and $\beta(x)$

$$q_L'(x) = \exp i \left(\boldsymbol{\alpha}(x) \cdot \frac{\boldsymbol{\tau}}{2} + \mathrm{y}_q \beta(x) \right) q_L(x), \qquad u_R'(x) \; = e^{i \mathrm{y}_u \beta(x)} u_R(x), \quad d_R'(x) = e^{i \mathrm{y}_{\rm d} \beta(x)} d_R(x).$$

where $\boldsymbol{\tau}$ are the Pauli matrices acting on weak isospin space

EFTs for BSM physics: the SM

3. construct all interactions allowed by symmetry

- to be able to construct gauge invariant interactions, introduce the gauge fields W_i^μ and B^μ

$$\mathbf{W}'_\mu(x)\cdot\frac{\boldsymbol{\tau}}{2} = \exp\left(i\boldsymbol{\alpha}(x)\cdot\frac{\boldsymbol{\tau}}{2}\right)\left(\mathbf{W}_\mu(x)\cdot\frac{\boldsymbol{\tau}}{2}+\frac{i}{g}\partial_\mu\right)\exp\left(-i\boldsymbol{\alpha}(x)\cdot\frac{\boldsymbol{\tau}}{2}\right)$$

$$B'_\mu(x) = \exp\left(i\beta(x)\right)\left(B_\mu(x)+\frac{i}{g'}\partial_\mu\right)\exp\left(-i\beta(x)\right),$$

- and the covariant derivatives and "field strengths"

$$D_\mu\psi_L = \left[\partial_\mu - ig\mathbf{W}_\mu\cdot\frac{\boldsymbol{\tau}}{2} - ig'\mathrm{y}_{\psi_L}B_\mu\right]\psi_L \qquad D_\mu\psi_R = \left[\partial_\mu - ig'\mathrm{y}_{\psi_R}B_\mu\right]\psi_R$$

$$\mathbf{W}_{\mu\nu}\cdot\frac{\boldsymbol{\tau}}{2} = \left(\partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu\right)\cdot\frac{\boldsymbol{\tau}}{2} - ig\left[\mathbf{W}_\mu\cdot\frac{\boldsymbol{\tau}}{2},\mathbf{W}_\nu\cdot\frac{\boldsymbol{\tau}}{2}\right] \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

- These allow to build gauge covariant objects. Show that:

$$\left[D_\mu\psi_L\right]'(x) = \exp i\left(\boldsymbol{\alpha}(x)\cdot\frac{\boldsymbol{\tau}}{2}+\mathrm{y}_{\psi_L}\beta(x)\right)D_\mu\psi_L(x), \qquad \left[D_\mu\psi_R\right]'(x) = e^{i\mathrm{y}_{\psi_R}\beta(x)}D_\mu\psi_R(x)$$

$$\left[\mathbf{W}'_{\mu\nu}\cdot\frac{\boldsymbol{\tau}}{2}\right](x) = e^{i\boldsymbol{\alpha}(x)\cdot\frac{\boldsymbol{\tau}}{2}}\left[\mathbf{W}_{\mu\nu}\cdot\frac{\boldsymbol{\tau}}{2}\right](x)e^{-i\boldsymbol{\alpha}(x)\cdot\frac{\boldsymbol{\tau}}{2}}, \qquad B'_{\mu\nu}(x) = B_{\mu\nu}(x)$$

EFTs for BSM physics: the SM

- covariant derivatives and field strengths provide the basic building blocks to construct gauge invariant operators
- the Lagrangian of the theory will contain Lorentz-scalar, gauge-invariant operators

$$\mathcal{L} = \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R - \frac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$
$$+ C_{qu}\bar{q}_L q_L \bar{u}_R u_R + C_{qd}\bar{q}_L q_L \bar{d}_R d_R + C_W\left(\mathbf{W}_{\mu\nu}\times\mathbf{W}^{\nu\rho}\right)\cdot\mathbf{W}^\mu_\rho \ldots$$

4. power counting

- in general

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum C_k^{(5)}\mathcal{O}_k^{(5)} + \sum_j C_j^{(6)}\mathcal{O}_j^{(6)} + \ldots$$

- for $\mathcal{L}$ to have dimension 4, coefficients of d-dimensional operators have dimension of $[M]^{d-4}$
- $d > 4$ operators arise from particles not in the theory,
- if their masses $M \gg m_W$, operators of the lowest dimension are the most important and we can expand observables in powers of m_W/M

EFTs for BSM physics: the SM

- with the ingredient discussed so far, the complete list of dim-4 operators is

$$\mathcal{L}^{(4)} = \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{\ell}_L i\gamma^\mu D_\mu \ell_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + \bar{e}_R i\gamma^\mu D_\mu e_R$$
$$- \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} - \frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \bar{\theta} \frac{g_s^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma},$$

- inducing kinetic terms for quarks, leptons and gauge bosons, and a CP-odd QCD θ term,
- the three gauge couplings g, g' and g_s determine all the interactons,
- all particles in this theory are massless, as mass terms

$$m_u \bar{u}_L u_R, \qquad m_d \bar{d}_L d_R, \qquad m_W^2 \mathbf{W}^\mu \cdot \mathbf{W}_\mu$$

break $SU(2)_L \times U(1)_Y$ gauge invariance

not quite the right theory yet...

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- we have not used our scalar!
  Introduce a complex scalar φ, which is doublet under $SU(2)_L$ and has hypercharge $y_\varphi = 1/2$

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

Electroweak symmetry breaking

- up to dim-4, in the scalar sector we can write

$$\mathcal{L}_\varphi = (D_\mu \varphi)^\dagger D_\mu \varphi - V(\varphi^\dagger \varphi)$$
$$V(\varphi^\dagger\varphi) = -\mu^2 \varphi^\dagger \varphi + \frac{\lambda}{2}(\varphi^\dagger \varphi)^2$$

- μ is the only dimensionful parameter in the theory

will set the scale of EW physics

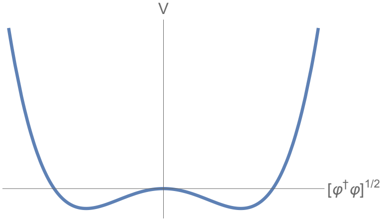
- in addition, we can write down scalar-fermion "Yukawa" interactions

$$\mathcal{L}_Y = -Y_d \delta^{jk} \bar{q}_L^j \varphi^k d_R - Y_u \varepsilon^{jk} \bar{q}_L^j (\varphi^k)^\dagger u_R - Y_e \varepsilon^{jk} \bar{\ell}_L^j (\varphi^k)^\dagger e_R$$

- Y_d, Y_u and Y_e are dimensionless 3×3 **complex** matrices

show that $\mathcal{L}_Y$ is gauge invariant

Electroweak symmetry breaking



- To generate fermion/boson masses EW symmetry “spontaneously broken”
  i.e. the ground state of the theory breaks the symmetry
- if $\mu > 0$ and $\lambda > 0$, the minimum of the Higgs potential is at

$$\langle \varphi^\dagger \varphi \rangle = \frac{v^2}{2} = \frac{\mu^2}{\lambda}$$

- which, if we don’t want the vacuum to spontaneously break charge, can be realized by

$$\langle \varphi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},$$

Electroweak symmetry breaking

- Expanding around the vev

$$(D_\mu\varphi)^\dagger\ D^\mu\varphi = -\frac{1}{2}\left[\left(\frac{gv}{2}\right)^2(W^{1\mu}W^1_\mu + W^{2\mu}W^2_\mu) + \left(\frac{\sqrt{g^2+g'^2}v}{2}\right)^2\left(\frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2+g'^2}}\right)\left(\frac{gW^{3\mu} - g'B^\mu}{\sqrt{g^2+g'^2}}\right)\right]$$

- two charged massive bosons

$$W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}, \qquad m_W = \frac{gv}{2}$$

- one neutral massive boson

$$Z_\mu = \frac{gW^{3\mu} - g'B^\mu}{\sqrt{g^2+g'^2}} = c_w W^{3\mu} - s_w B^\mu, \qquad m_Z = \frac{gv}{2c_w}$$

- one neutral massless boson

$$A_\mu = s_w W^{3\mu} + c_w B^\mu, \qquad m_\gamma = 0$$

- similar, expanding the scalar potential, one can find

$$m_H = \sqrt{\lambda}v$$

- where we introduced the Weinberg angle

$$c_w = \cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}, \qquad s_w = \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}},$$

Couplings to fermions

- the coupling to the massless boson

$$\mathcal{L} \supset \frac{gg'^{\,2}}{\sqrt{g^2+g'}}\left[\bar{q}_L\gamma^\mu\left(\frac{\tau_3}{2}+\mathrm{y}_\mathrm{q}\right)q_L + \mathrm{y}_u\bar{u}_R\gamma^\mu u_R + \mathrm{y}_d\bar{d}_R\gamma^\mu d_R\right]A_\mu$$

  which justifies

$$\frac{gg'}{\sqrt{g^2+g'^{\,2}}} \equiv e, \qquad Q_\psi \equiv \frac{\tau_3}{2}+\mathrm{y}_\psi.$$

- the couplings to the W remain simple, and purely left-handed

$$\mathcal{L} \supset \frac{g}{\sqrt{2}}\left(\bar{u}_L\gamma^\mu d_L W^+_\mu + \bar{d}_L\gamma^\mu u_L W^-_\mu\right) + \frac{g}{\sqrt{2}}\left(\bar{\nu}_L\gamma^\mu e_L W^+_\mu + \bar{e}_L\gamma^\mu \nu_L W^-_\mu\right)$$

- the coupling to the Z is more messy

$$\mathcal{L} \supset \frac{g}{c_w}\left[\bar{q}_L\gamma^\mu\left(\frac{\tau_3}{2}-Q_q s_w^2\right)q_L - Q_u s_w^2\bar{u}_R\gamma^\mu u_R - Q_d s_w^2\bar{d}_R\gamma^\mu d_R\right]Z_\mu$$

- the couplings are diagonal in generation space

4 indep. parameters, v, e, s_w and λ determine (almost...) everything

Fermion masses and the CKM matrix

- in the Higgs-scalar sector

$$\mathcal{L}_Y \to -\frac{v}{\sqrt{2}}\left[\bar{d}_L Y_d d_R + \bar{u}_L Y_u u_R + \bar{e}_L Y_e e_R\right] + \mathrm{h.c.}$$

  give rise to quark and charged-leptons masses

- Y_u, Y_d and Y_e are generic 3×3 complex matrices, don't need to be diagonal
- by rotating the quark and charged lepton fields with unitary matrices

$$u'_{L,R} = U^u_{L,R} u_{L,R}, \qquad d'_{L,R} = U^d_{L,R} d_{L,R}, \qquad e'_{L,R} = U^e_{L,R} e_{L,R},$$

  the mass term can be diagonalized, with real and positive eigenvalues

$$\frac{v}{\sqrt{2}}\left[U^u_L\right]^\dagger Y_u U^u_R = \mathrm{diag}(m_u, m_c, m_t), \qquad \frac{v}{\sqrt{2}}\left[U^d_L\right]^\dagger Y_d U^d_R = \mathrm{diag}(m_d, m_s, m_b)$$

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- the right-handed matrices $U^{u,d,e}_R$ disappear from every term in $\mathcal{L}$. E.g.

$$\bar{d}_R \gamma^\mu d_R Z_\mu \to \bar{d}'_R \left[U^d_R\right]^\dagger \gamma^\mu U^d_R d'_R = \bar{d}'_R \gamma^\mu d'_R Z_\mu$$

RH neutral current interactions are the same in “weak” and “mass” basis

Fermion masses and the CKM matrix

- the left-handed matrices $U_L^{u,d,e}$ drop out of the Z and A couplings

  at tree level, SM has no flavor-changing neutral currents
  e.g. $\bar{d}\gamma^\mu s\, Z_\mu$, $\bar{b}\gamma^\mu s\, A_\mu$, ...

- but not from the charged-currents

$$(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_L\gamma^\mu e_L)\, W^+_\mu \to \left(\bar{u}'_L\gamma^\mu \left[U_L^u\right]^\dagger U_L^d d'_L + \bar{\nu}_L\gamma^\mu U_L^e e'_L\right) W^+_\mu$$

- since in the minimal SM there is no neutrino Yukawa term, U_L^e can be absorbed in the ν_L field

  all leptonic interactions are diagonal in the mass basis! E.g. no e-ν_μ interactions

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- in the quark sector, this defines the Cabibbo-Kobayashi-Maskawa (CKM) matrix

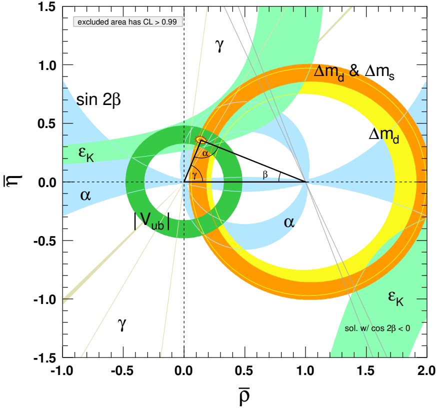
$$\left[U_L^u\right]^\dagger U_L^d \equiv V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- V_{CKM} is a 3×3 unitary matrix, parameterized by 3 real numbers and a complex phase
- the "Jarlskog invariant" J is a measure of CP-violation in the SM

C. Jarlskog

$$J = \mathrm{Im}[V_{ud} V_{ts} V_{us}^* V_{td}^*]$$

Determination of the CKM elements



$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$J = A^2\lambda^6\bar\eta$$

- the CKM matrix is fairly hierarchical

see PDG CKM review

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22501 \pm 0.00068 & 0.003732^{+0.000090}_{-0.000085} \\ 0.22487 \pm 0.00068 & 0.97349 \pm 0.00016 & 0.04183^{+0.00079}_{-0.00069} \\ 0.00858^{+0.00019}_{-0.00017} & 0.04111^{+0.00077}_{-0.00068} & 0.999118^{+0.000029}_{-0.000034} \end{pmatrix}$$

- CP-violation is small $J \sim 3.12 \cdot 10^{-5}$

SM summary

$$\mathcal{L}_{SM} = \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{\ell}_L i\gamma^\mu D_\mu \ell_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + \bar{e}_R i\gamma^\mu D_\mu e_R$$
$$- \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} - \frac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (D_\mu\varphi)^\dagger D_\mu\varphi - V(\varphi^\dagger\varphi)$$
$$- Y_d \delta^{jk} \bar{q}^j_L \varphi^k d_R - Y_u \varepsilon^{jk} \bar{q}^j_L (\varphi^k)^\dagger u_R - Y_e \varepsilon^{jk} \bar{\ell}^j_L (\varphi^k)^\dagger e_R + \bar{\theta}\frac{g_s^2}{64\pi^2}\varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma},$$

- the gauge symmetry of the SM imposes several constraints which can be looked for in the lab

  relations between m_Z and m_W, CKM unitarity, high-energy behavior of WW production, ...

Concerning discrete symmetries:

1. P & C (exchange L into R): broken by W and Z gauge interactions

   small at low energy only cause $m_{W,Z} \gg m_p, m_e$

2. CP: broken by $Y_{u,d}$ and $\bar{\theta}$

   no CP-violation if all quarks are massless

- using the freedom to redefine the fields, in the SM it boils down to 2 params, J and $\bar{\theta}$

SM Summary

$$
\begin{aligned}
\mathcal{L}_{SM} = & \ \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{\ell}_L i\gamma^\mu D_\mu \ell_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + \bar{e}_R i\gamma^\mu D_\mu e_R \\
& - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} - \frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (D_\mu \varphi)^\dagger D_\mu \varphi - V(\varphi^\dagger \varphi) \\
& - Y_d \delta^{jk} \bar{q}^j_L \varphi^k d_R - Y_u \varepsilon^{jk} \bar{q}^j_L (\varphi^k)^\dagger u_R - Y_e \varepsilon^{jk} \bar{\ell}^j_L (\varphi^k)^\dagger e_R + \bar{\theta} \frac{g_s^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} ,
\end{aligned}
$$

- for low-energy processes, CP is basically conserved

  no EDM, no baryogenesis

3. quark flavor: no flavor changing neutral currents at tree level

   K-$\bar{K}$ oscillations, B-$\bar{B}$ oscillations, $B \to X_s \gamma$, $K \to \pi\nu\nu$
   suppressed by loop factors and masses or CKM factors

4. L and B: lepton and baryon number are conserved (in perturbation theory)

   no $0\nu\beta\beta$, no proton decay

5. lepton flavor is conserved

   no $e \to \tau$, $e \to \mu$, $\mu \to \tau$ transitions

6. neutrinos are massless