



Effective Field Theories for Physics Beyond the Standard Model.

Lecture 3

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Lecture 3: Construction of the low-energy EFTs for BSM probes

From quarks to hadrons: chiral perturbation theory and chiral EFT

BSM processes dominated by one-body operators

Chiral EFT for EDMs

BSM processes dominated by two-body operators: $0\nu\beta\beta$

Light new physics: $0\nu\beta\beta$ with sterile neutrinos

Phenomenology

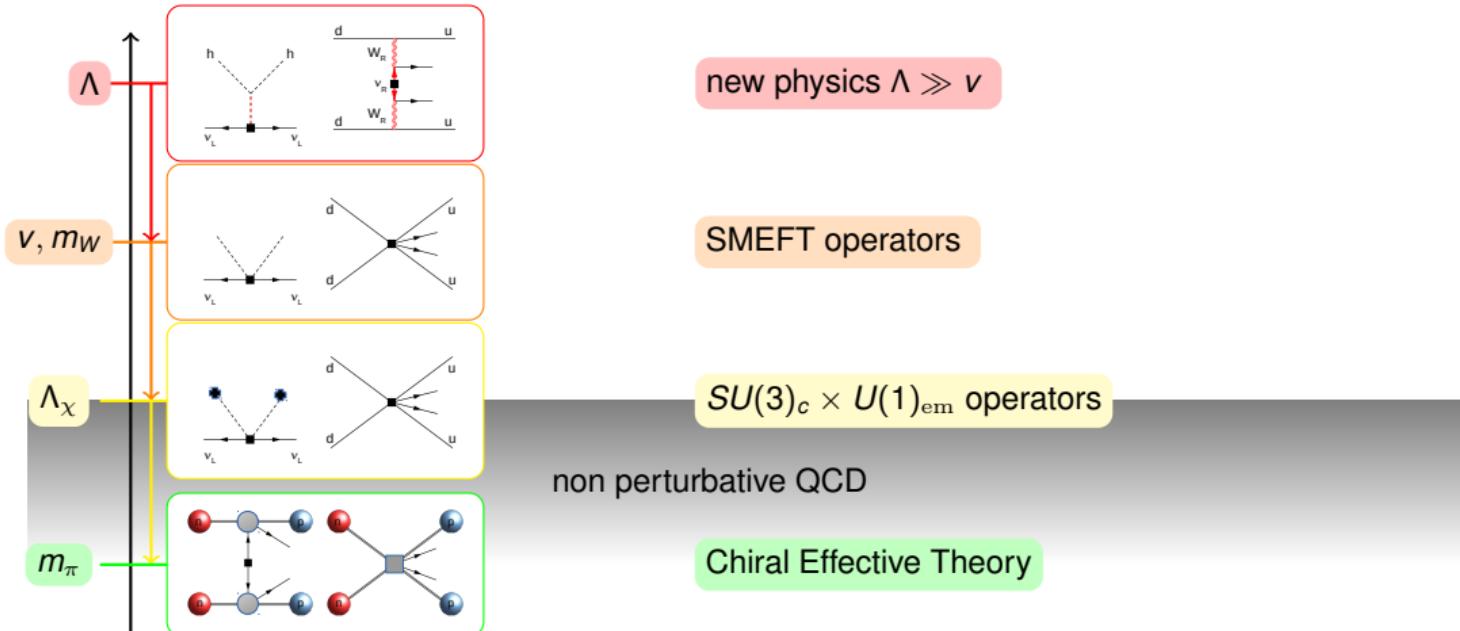
Neutrinoless double beta decay

Electric dipole moments

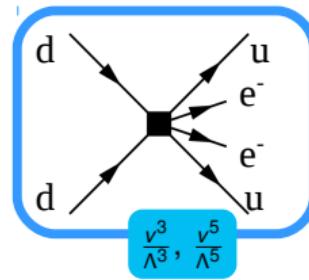
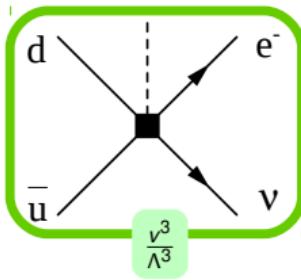
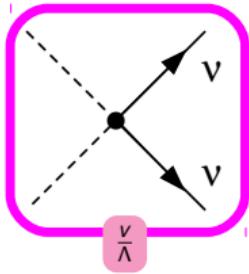
CKM unitarity



From quarks to hadrons: chiral EFT



From quarks to hadrons



- LEFT operators are expressed in terms of quark fields
e.g. consider the LNV operators discussed at the end of the last lecture

$$\mathcal{L}_{\Delta L=2}(\nu, e, u, d) = -\frac{1}{2}(m_\nu)_{ij}\nu^{Tj}C\nu^i + C_\Gamma \nu^T C \Gamma e \mathcal{O}_\Gamma + C_{\Gamma'} e^T C \Gamma' e \mathcal{Q}_{\Gamma'}$$

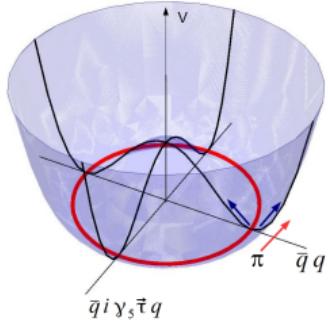
quark bilinear: $\bar{q}\Gamma q$

four-quark: $\bar{q}\Gamma_1 q \bar{q}\Gamma_2 q$

- we need to consider processes with nucleons and nuclei

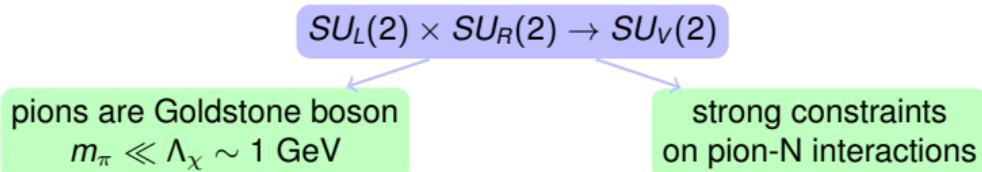
can we match onto an EFT for hadrons? $\mathcal{L} = \mathcal{L}(\pi, N, \dots)$?

Chiral Perturbation Theory



$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L$$

- at the moment, we cannot compute many nuclear observables directly from QCD
- use symmetry once again!



The 1-nucleon sector: (Heavy) Baryon Chiral Perturbation Theory

1. degrees of freedom: pions (Goldstone bosons) and nucleons
2. symmetries: global $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$
4. power counting: chiral symmetry & spontaneous breaking allow for an expansion in Q/Λ_χ

$$Q \in \{p, m_\pi\}, \quad \Lambda_\chi \sim 4\pi F_\pi \sim m_N$$

3. interactions: realize the symmetry non-linearly, encode the pions into a matrix & build “chiral covariant” objects

S. Weinberg, '79

- can be applied only to low-energy processes, $Q \ll 1 \text{ GeV!}$
- to have consistent power counting, is convenient to use non-relativistic formulation
- but HB χ PT is not a unique choice

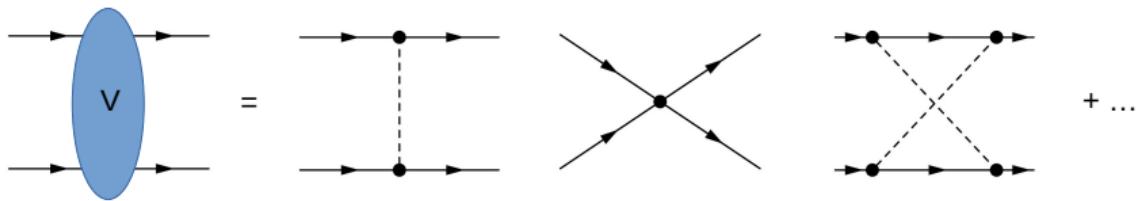
E. Jenkins and A. Manohar, '90

infrared regularization T. Becher and H. Leutwyler, '99,
extended on-mass-shell scheme T. Fuchs, J. Gegelia, G. Japaridze, S. Scherer, '03

see supplemental slides for examples



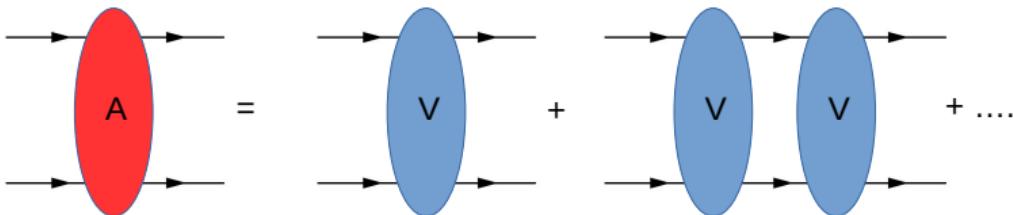
EFT in the 2 nucleon sector: Weinberg's recipe



S. Weinberg '90, S. Weinberg '91

- the fact that the nucleon energy is $E \ll m_\pi$ plays a role!
Diagrams with almost on-shell intermediate nucleons are enhanced
- 1. identify “irreducible diagrams”
 - do not have a purely A -nucleon intermediate state
 - internal nucleon energies $E_N \sim Q \sim m_\pi$
- 2. the potential V is the sum of irreducible diagrams
 - can be calculated perturbatively in a power expansion in Q/Λ_χ following χ PT counting rules

EFT in the 2 nucleon sector: Weinberg's recipe

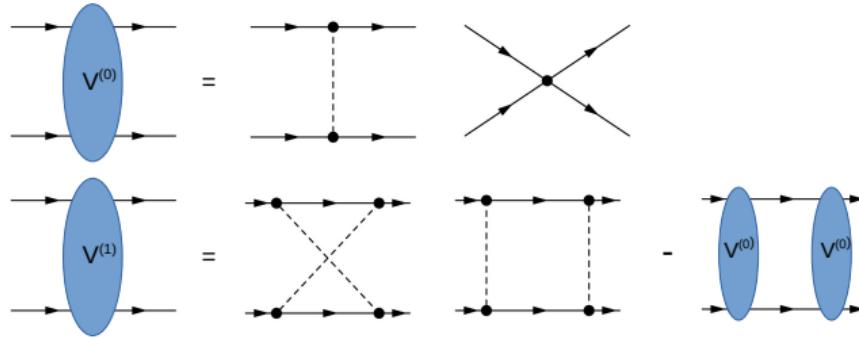


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- 3. calculate the full amplitude by “stitching” together
irreducible diagrams with A -nucleon Green’s functions
- equivalent to solving the Schrödinger or Lippmann-Schwinger equation with V

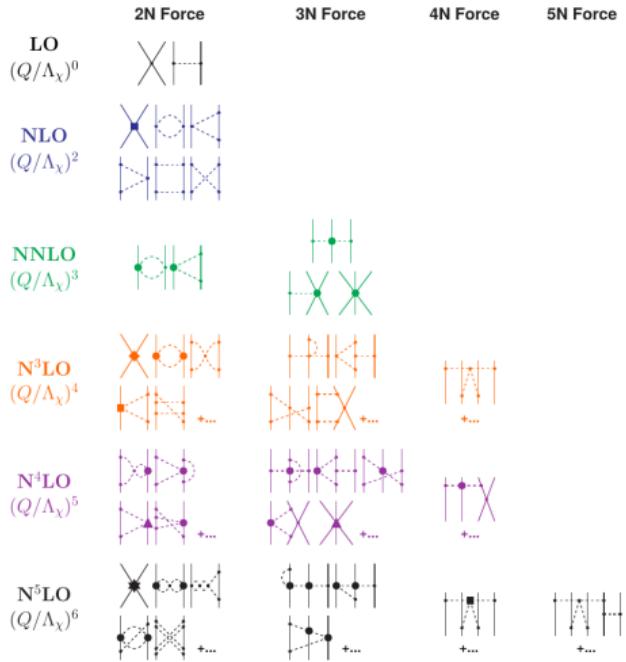


Weinberg's recipe



- **steps 1 and 2** are equivalent to integrating out “soft” and “potential” modes and matching onto a theory with nucleons interacting via instantaneous potentials (chiral EFT)
happens in several other EFTs with > 1 heavy particles: NRQCD, NRQED
- the same recipe can be applied to operators that mediate BSM processes
⇒ calculate matrix elements of BSM operators between nuclear wavefunctions
- the scaling of short-range operators **assumes** Weinberg's ν (naive dimensional analysis)

Chiral Potential in Weinberg's power counting



Incredible progress
in calculation of chiral potentials!

from D. R. Entem, R. Machleidt and Y. Nosyk, '17

see also:

P. Reinert, H. Krebs, E. Epelbaum, '18

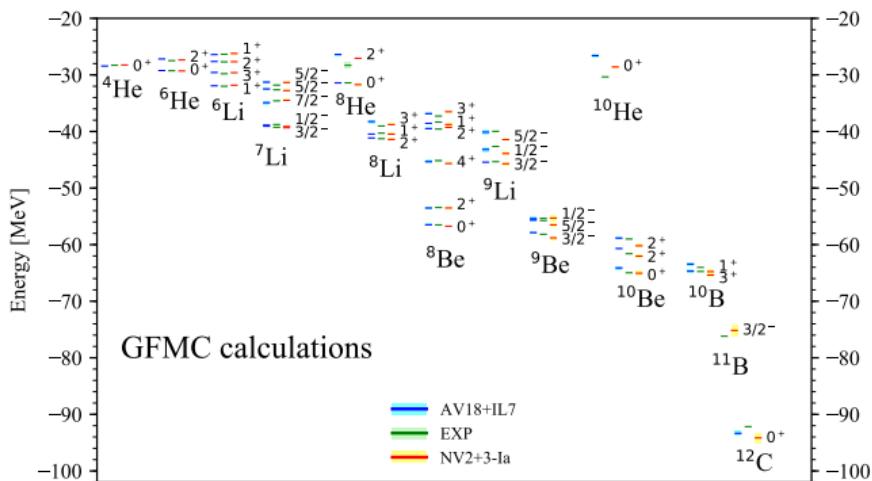
M. Piarulli *et al.*, '16

M. Piarulli and I. Tews, '19

- LECs are fit to data in 2- and 3-nucleon systems



Chiral Potential in Weinberg's power counting



and in predicting nuclear properties
(coupled to exact/semiexact methods
for solving Schrödinger eq.)

M. Piarulli *et al.*, '17

- and predict light-nuclear observables
- chiral potentials as successful as high-quality phenomenological potentials (AV18, CD Bonn)
- and nowadays standard input for *ab initio* calculations



Non perturbative renormalization and scaling of short-distance operators

1. chiral potentials have a singular short-range behavior

$\delta(\vec{r})$, $1/r^3$ potentials

2. which requires the introduction of regulators to solve the Schroedinger equation
3. the physics should not depend on these regulators, up to the order in Q/Λ_χ we are working at
4. there are cases in which renormalization conflicts with the naive power counting scaling of short-distance operators



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E.g. LO phase shift in the 1S_0 channel

$$p \cot \delta(p) \approx \frac{4\pi}{m_N} \frac{1}{C_1 S_0(\Lambda)} + \frac{\Lambda}{\sqrt{2\pi}} + \frac{m_\pi^2}{M_{NN}} \log \Lambda + \dots$$

D. Kaplan, M. Savage, M. Wise, '96;

- the linear Λ dependence can be absorbed by renormalizing $C_1 S_0$
- but, in Weinberg's counting, $C_1 S_0$ is independent on the pion mass \Rightarrow cannot absorb the log



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- but, in Weinberg's counting, $C_1 {}_{S_0}$ is independent on the pion mass \Rightarrow cannot absorb the log
- a N²LO mass dependent operator needs to be promoted to LO

$$\mathcal{L}_{NN}^{(2)} = -m_\pi^2 [D_2]_{{}^1S_0} \left(N^T P_{{}^1S_0} N \right)^\dagger N^T P_{{}^1S_0} N$$



Non perturbative renormalization and scaling of short-distance operators

In the construction of chiral EFT operators for BSM physics

1. assume Weinberg's scaling for short-distance operators
 2. check in simple (2- or 3-nucleon) systems that the results can be made regulator independent (at a given order in Q/Λ_χ)
 3. if not, adjust the scaling
- failure of doing so leads to underestimate of the theory error

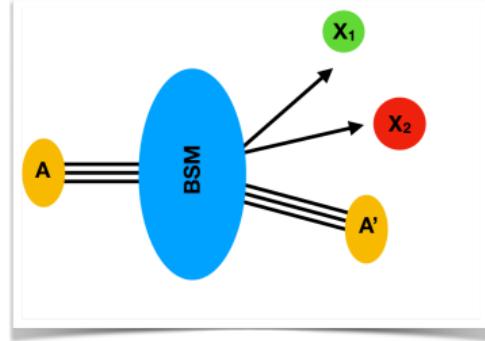
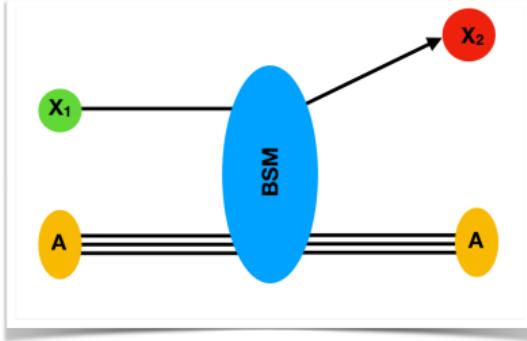
we'll claim $\mathcal{O}((Q/\Lambda_\chi)^{n+1})$ errors while there are missing pieces at $\mathcal{O}((Q/\Lambda_\chi)^n)$



Nuclear EFTs for BSM physics



BSM processes dominated by 1-body currents



$$\mathcal{L}^{\text{BSM}} = \bar{q}\Gamma\tau^aq\chi_a + \bar{q}\Gamma q\chi_0 + \bar{s}\Gamma s\chi_s, \quad q = (u, d)^T, \quad \Gamma = \{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$$

- non-standard charged-currents in β decays
- coherent neutrino-nucleus scattering (CE ν NS)
- $\mu \rightarrow e$ conversion in nuclei
- dark-matter - nucleus scattering
- neutron EDM from qEDM, molecular electric dipole moments

$$\chi_+ = \bar{e}\Gamma\nu$$

$$\chi_{u,d,s} = \bar{\nu}\gamma_\mu\nu$$

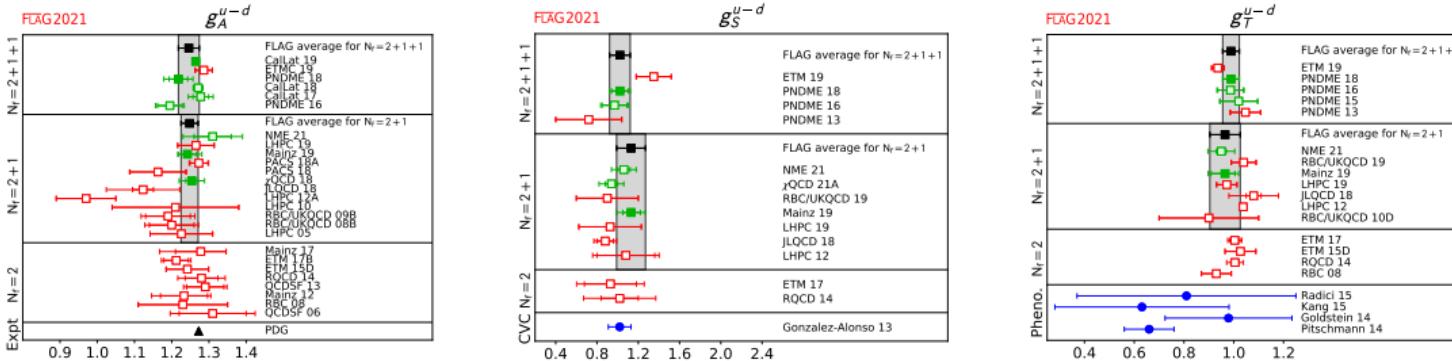
$$\chi_{u,d,s} = \bar{e}\Gamma\mu$$

$$\chi_{u,d,s} = \bar{\chi}\Gamma\chi$$

$$\chi_{u,d,s} = \bar{e}\gamma_5 e, \tilde{F}_{\mu\nu}$$



One-body operators



Flavor Lattice Averaging Group nucleon matrix elements

- we typically need

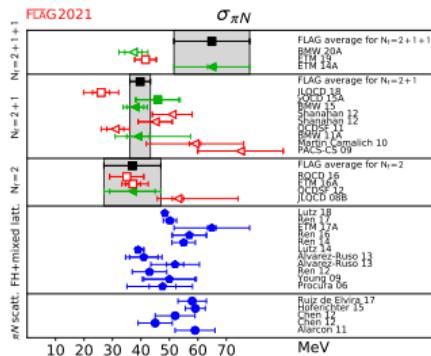
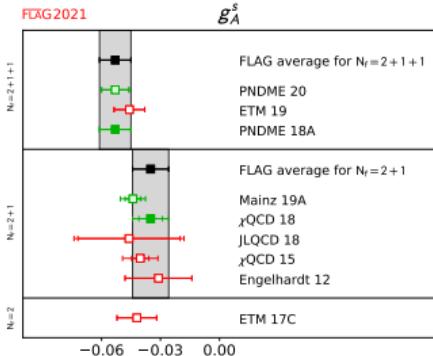
$$\langle N | \bar{q} \Gamma \{ \tau^a, 1 \} q, \bar{s} \Gamma s | N \rangle \implies \langle N | \bar{N} \Gamma \{ g_\Gamma^{(1)} \tau^a, g_\Gamma^{(0)}, g_\Gamma^{(s)} \} N | N \rangle,$$

- the r.h.s can be systematically constructed in HB χ PT
- but the one-body low-energy constants need data/Lattice QCD
- isovector LECs are known with good accuracy

see Huey-Wen Lin's Lectures



One-body operators



Flavor Lattice Averaging Group [nucleon matrix elements](#)

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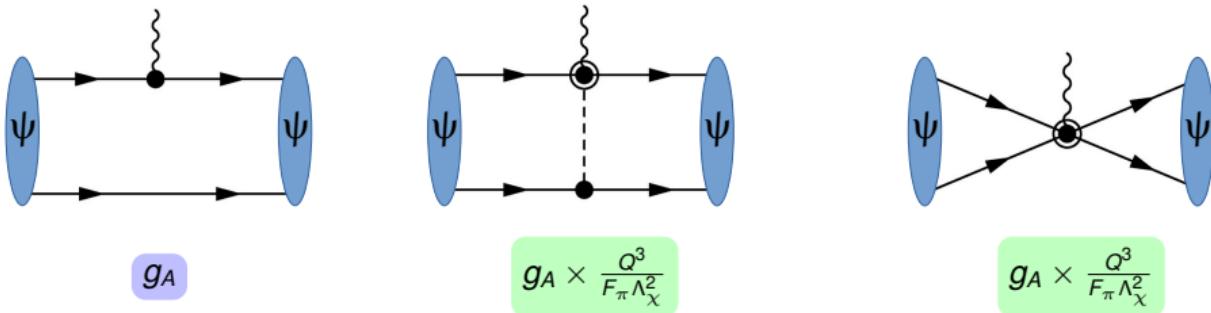
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- the r.h.s can be systematically constructed in HB χ PT
- but the one-body low-energy constants need data/Lattice QCD
- isovector LECs are known with good accuracy
- isoscalar and strange matrix elements are more uncertain

see Huey-Wen Lin's Lectures



Two nucleon contributions: axial current

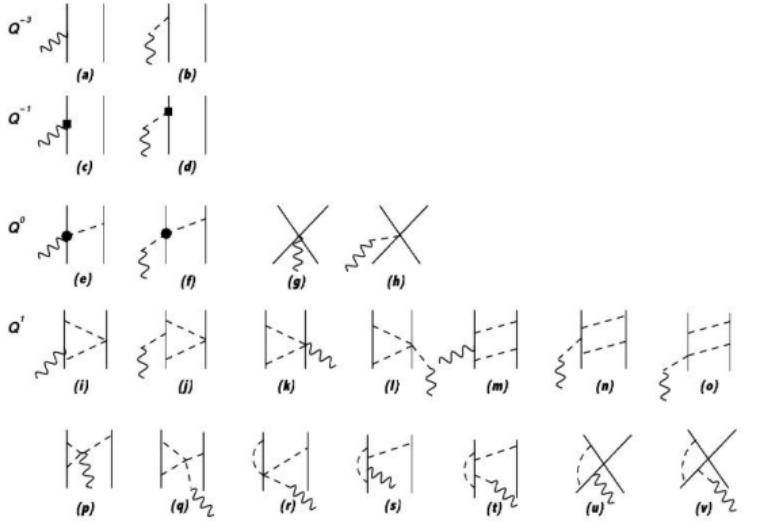


- we can include subleading corrections, such as two-body currents
- these arise from pion-exchange diagrams and contact interactions

$$\mathcal{L}_{NN}^p = \frac{c_D}{2\Lambda_\chi F_\pi^2} \bar{N} \sigma^i \tau N \bar{N} N \cdot \left(\frac{1}{F_\pi} \nabla_i \pi - \mathbf{a}_i \right)$$

- c_D needs to be extracted from data (no LQCD available)
- and contribute at $N^2\text{LO}/N^3\text{LO}$ (depending a bit on the counting scheme)

Axial current

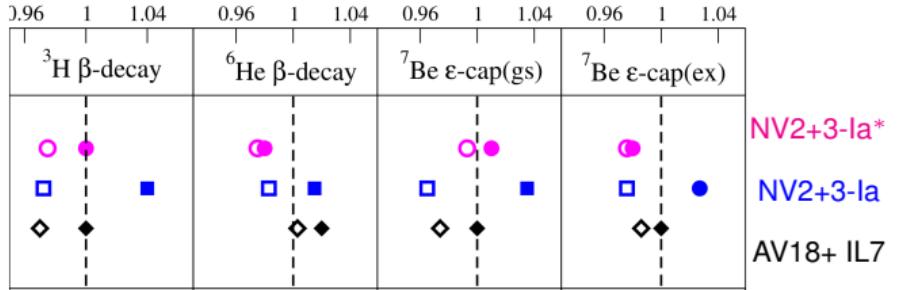


A. Baroni, S. Pastore, R. Schiavilla, M. Viviani, '16
H. Krebs, E. Epelbaum, U. Meissner, '16

from A. Baroni et al, '16

- calculation of SM currents have been carried out at very high order
- like for the potential, need to subtract the “reducible” components

Axial current: impact of two-body currents



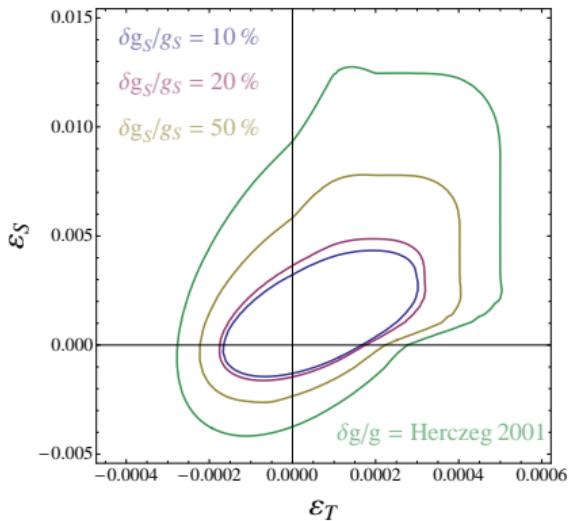
G. King, L. Andreoli, S. Pastore, M. Piarulli, R. Schiavilla, '20

- percent level predictions of β decay observables!
- LO currents (empty symbols) and N³LO currents (full symbol) differ by a few percent
- results still sensitive to extraction of c_D from triton decay (magenta) or from trinucleon binding energy and nd scattering (blue)

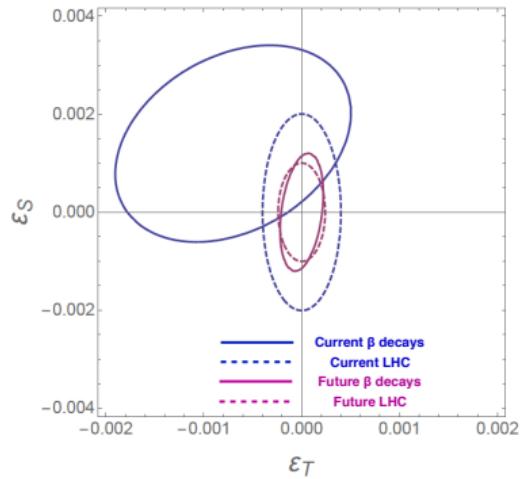
indication of sensitivity to the next order

- for SM background, % level corrections are important, but they won't affect the BSM contrib. that much

Impact of better hadronic matrix elements



projections from [T. Bhattacharya et al, '11](#)



- projection on the extraction of scalar and tensor couplings from β decay experiments
- assuming quark model estimates of g_S and g_T vs. target precisions of LQCD calculations
- factor of 2-3 needed to keep up with LHC!

EFT for electric dipole moments

Electric dipole moments

- one dim-4 operator: QCD $\bar{\theta}$ term

$$\mathcal{L}_\theta = \frac{g_s^2}{32\pi^2} \bar{\theta} \tilde{G}_{\mu\nu}^a G^{a\mu\nu} - m_q \bar{q}_L q_R - m_q^* \bar{q}_R q_L$$

a. quark bilinears:

$$\mathcal{L} = \sum_{q=u,d,s} \text{Im} L_{q\gamma} \bar{q} \sigma^{\mu\nu} \gamma_5 q F_{\mu\nu} + L_{\Gamma\Gamma'} \bar{e} \Gamma e \bar{q} \Gamma' q$$

b. quark-gluon chiral-breaking operators

$$\mathcal{L} = \sum_{q=u,d,s} \text{Im} L_{qg} \bar{q} \sigma^{\mu\nu} \gamma_5 t^a q G_{\mu\nu}^a$$

c. gluon chiral invariant operators

$$\mathcal{L} = L_{\tilde{G}} f^{abc} \tilde{G}_{\mu\nu}^a G_\rho^{b\nu} G^{c\mu\rho}$$

d. chiral-breaking four-fermion

$$\mathcal{L} = L_{uddu}^{\text{V1LR}} (\bar{u}_L \gamma^\mu d_L) (\bar{d}_R \gamma_\mu u_R) + \dots$$

how do we use measurements of CP-violation in different systems
to identify the underlying mechanism?



Electric dipole moments: theory input.

1. HfF , ThO and YbF

- depend mostly on the electron EDM and scalar semileptonic operators
- at lowest order, same single nucleons parameters as for the scalar charge $\sigma_{\pi N}$, σ^s
- calculation of precession frequency mostly an atomic/molecular physics problem
... small uncertainties ...

2. neutron EDM

- sensitive to $L_{q\gamma}$ and hadronic operators
- for $L_{q\gamma}$, just need the tensor charges

$$d_n = \langle n | \bar{q} \sigma^{\mu\nu} q | n \rangle \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \propto g_T$$

... small uncertainties ...

- for fully hadronic operators, typically need

$$\langle n | \bar{q} Q_q \gamma^\mu q \int d^4x \mathcal{O}_{CPV}(x) | n \rangle$$

challenging in Lattice QCD



Electric dipole moments: theory input

3. EDMs of light ions (deuteron, ${}^3\text{He}$)

- receives contributions from the EDMs of the constituent nucleons
- and from corrections to the wavefunctions induced by CP-violating NN potentials

$$\vec{d}_A = \langle A | \vec{\sigma} (d_n P_n + d_p P_p) | A \rangle + \sum_n \frac{\langle A | \vec{r} | n \rangle \langle n | V_{\text{CP}} | A \rangle}{E_A - E_n}$$

- the relative size of the two depends on specific CP-odd operator

4. EDMs of diamagnetic atoms: ${}^{199}\text{Hg}$, ${}^{129}\text{Xe}$, ${}^{225}\text{Ra}$

- the EDMs of the constituent nucleons are screened
- depend on the Schiff operator

$$\vec{d}_A = \sum_n \frac{\langle A | \vec{r} (r^2 - \frac{5}{3} \langle r^2 \rangle_{\text{ch}}) | n \rangle \langle n | V_{\text{CP}} | A \rangle}{E_A - E_n}$$

Need:

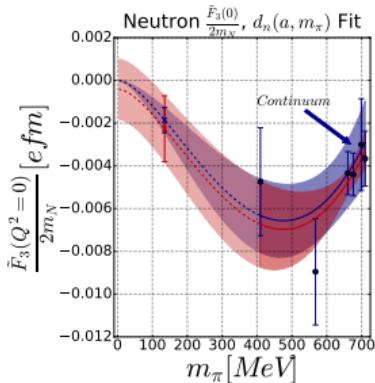
1. $d_{n,p}(C_{\text{LEFT}})$

2. $V_{\text{CP}}(C_{\text{LEFT}})$

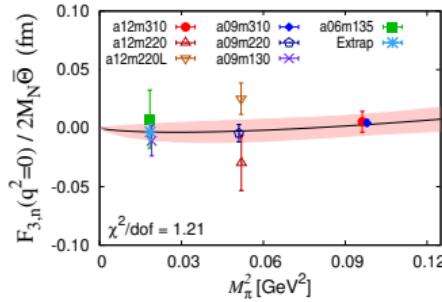
3. nuclear matrix elements



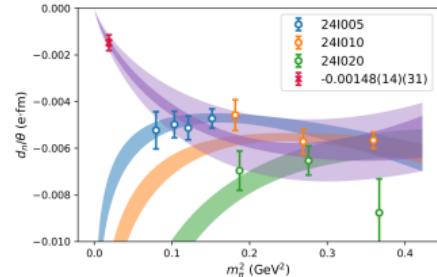
Lattice QCD calculations of nEDM. $\bar{\theta}$ term



J. Dragos, T. Luu, A. Shindler, et al '19



T. Bhattacharya, et al, '21



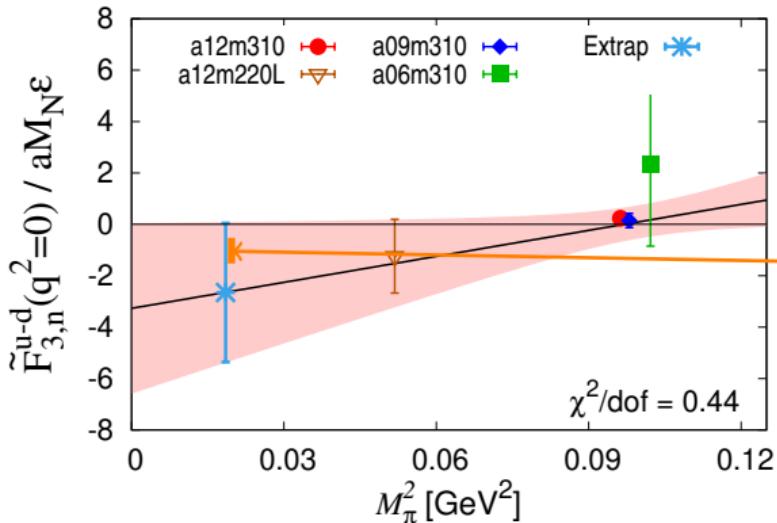
J. Liang, et al (χ QCD Coll.), '23

- baseline for all nEDM calculations
- EDM from QCD $\bar{\theta}$ term extremely challenging
 - vanishing signal at small m_π , large excited state contamination, ...
- published results compatible with zero at $\sim 2\sigma$
- approaching $d_n \sim 10^{-3} \bar{\theta}$ e fm, size of “chiral log”
- need more work to control all systematics

Crewther, Di Vecchia, Veneziano and Witten, '79



nEDM from dimension-6 operators



$\bar{q}\tau_3\sigma\tilde{G}q$

power div. subtracted

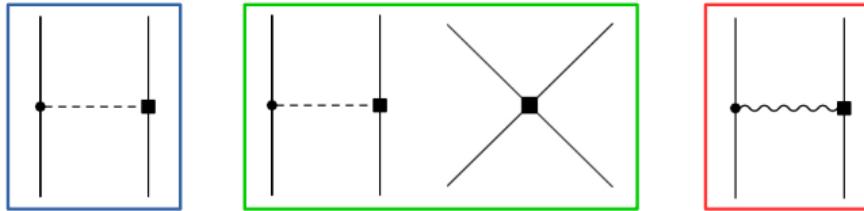
QCD sum rules

thanks to T. Bhattacharya and B. Yoon

- preliminary results for qCEDM and gCEDM
- error still a factor of 5 larger than QCD sum rule estimate
- no studies of 4-fermion operators yet

best results still from QCD sum rule calculations
Pospelov and Ritz, '05, Haisch and Hala, '19

Calculation of the CP-violating potential



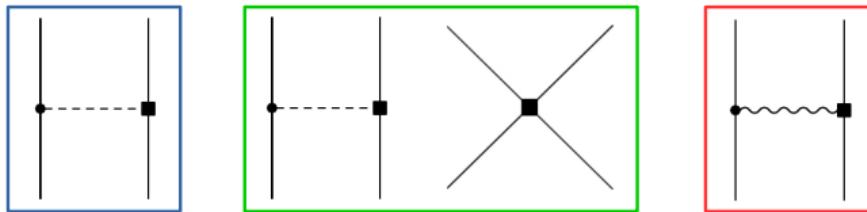
$$\mathcal{L}_{\pi N} = -\frac{\bar{g}_0}{F_\pi} \bar{N} \boldsymbol{\pi} \cdot \boldsymbol{\tau} N - \frac{\bar{g}_1}{F_\pi} \pi_3 \bar{N} N - \frac{\bar{g}_2}{F_\pi} \bar{N} \left(\pi_3 \tau_3 - \frac{1}{3} \right) \boldsymbol{\pi} \cdot \boldsymbol{\tau} N + \dots$$

- all pion-nucleon interactions break chiral symmetry,
- \bar{g}_1 and \bar{g}_2 also break isospin by 1 and 2 units
- we can write down 5 $S-P$ transition operators

- $\tilde{C}_{3S_1-1P_1}$ and $\tilde{C}_{1S_0-3P_0}^{(0)}$ conserve isospin (and chiral symmetry)
- $\tilde{C}_{3S_1-3P_1}$ and $\tilde{C}_{1S_0-3P_0}^{(1)}$ break isospin by 1 unit
- $\tilde{C}_{1S_0-3P_0}^{(2)}$ break isospin by 2 units



Calculation of the CP-violating potential



- the relative importance depends on the chiral symmetry properties of CPV operators
- $\bar{\theta}$: $\bar{g}_0 \gg \bar{g}_1$
- qCEDM: $\bar{g}_0 \sim \bar{g}_1$
- LL RR 4-fermion: $\bar{g}_0 \ll \bar{g}_1$
- 4-nucleon operators are usually neglected, but this is not always justified!
see [J. de Vries, A. Gnech, S. Shain, '20](#)
- only $\bar{g}_0(\bar{\theta})$ is known well, for other LEFT operators only order of magnitude estimates

EDMs of light nuclei: chiral calculations

	α_n	α_p	a_0 (e fm)	a_1 (e fm)	a_2 (e fm)
d	0.9	0.9	0	-0.100	0
${}^3\text{He}$	0.9	0	-0.027	-0.079	-0.060
${}^3\text{H}$	0	0.9	0.027	-0.079	0.060

using the calculation of [A. Gnech and M. Viviani, '19](#)

$$d_{AX} = \left(\alpha_n d_n + \alpha_p d_p + a_0 \frac{\bar{g}_0}{F_\pi} + a_1 \frac{\bar{g}_1}{F_\pi} + a_2 \frac{\bar{g}_2}{F_\pi} \right)$$

- for light ions, the nuclear theory input is under control (at the $\sim 10\%$ level)
- $\alpha_{n,p}$ agree with PC expectations
- a_0 and a_1 are a bit smaller than expected
- light nuclei can be important filters, singling out different isospin structures

e.g. $N = Z$ nuclei single out \bar{g}_1



Diamagnetic atoms and Schiff moment calculations

	A_{Schiff}	α_n	α_p	a_0 (e fm)	a_1 (e fm)	a_2 (e fm)
^{199}Hg	$-(2.40 \pm 0.24) \cdot 10^{-4}$	1.9 ± 0.1	0.20 ± 0.06	$0.13^{+0.5}_{-0.07}$	$0.25^{+0.89}_{-0.63}$	$0.09^{+0.17}_{-0.04}$
^{129}Xe	$-(0.364 \pm 0.025) \cdot 10^{-4}$	$-(0.29 \pm 0.10)$	—	$0.10^{+0.53}_{-0.037}$	$0.076^{+0.55}_{-0.038}$	
^{225}Ra	$(6.3 \pm 0.5) \cdot 10^{-4}$	—	—	2.5 ± 7.5	-65 ± 40	14 ± 6.5

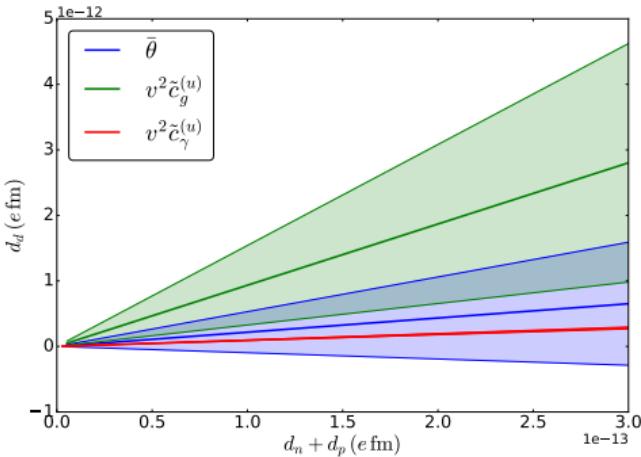
- Schiff moments have similar expressions (though the operators are more complicated)

$$d_A = A_{\text{Schiff}} \left(\alpha_n d_n + \alpha_p d_p + a_0 \frac{\bar{g}_0}{F_\pi} + a_1 \frac{\bar{g}_1}{F_\pi} + a_2 \frac{\bar{g}_2}{F_\pi} \right)$$

- for diamagnetic atoms, at the moment, no EFT calculations exists
- Schiff moments have been calculated with pheno models and have large nuclear theory errors
see [J. Engel, M. Ramsey-Musolf, U. van Kolck, '13, T. Chupp, P. Fierlinger, M. Ramsey-Musolf, J. Singh '17](#)
- similar hierarchies as for light nuclei, except large enhancement for Ra



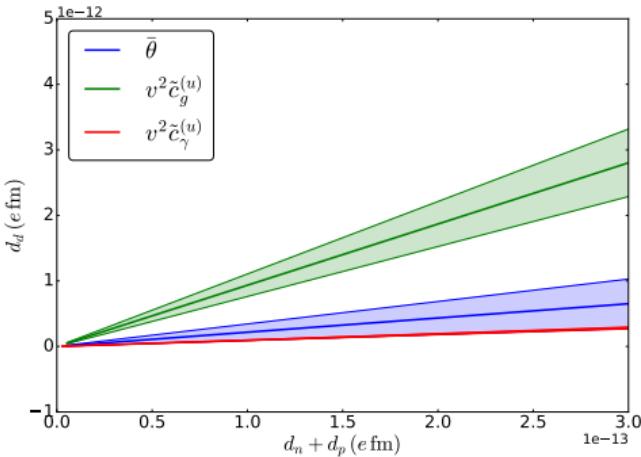
Disentangling CP-violating mechanisms



How do we use this info? Suppose we can measure d_n , d_p and d_d

- $d_d \gg d_n + d_p$ isospin-breaking sources
- $d_d \sim d_n + d_p$ QCD $\bar{\theta}$ term
- $d_d = d_n + d_p$ qEDM, Weinberg operator
... but swamped by current theory uncertainties

Disentangling CP-violating mechanisms



How do we use this info? Suppose we can measure d_n , d_p and d_d

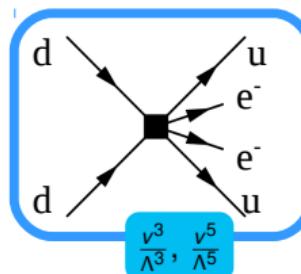
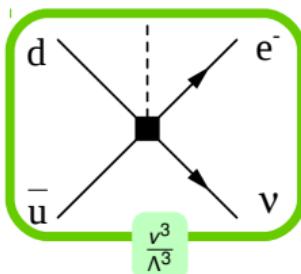
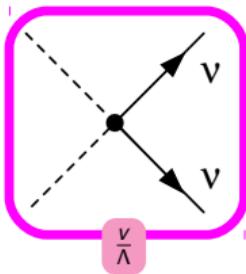
- $d_d \gg d_n + d_p$ isospin-breaking sources
- $d_d \sim d_n + d_p$ QCD $\bar{\theta}$ term
- $d_d = d_n + d_p$ qEDM, Weinberg operator
...but swamped by current theory uncertainties
- $\mathcal{O}(20\%)$ uncertainties sufficient to discriminate!



Processes dominated by two-body operators



Neutrinoless double beta decay in SMEFT + chiral EFT



- as discussed in Lecture 2, in SMEFT we can have several sources of LNV
 - a. Majorana masses of active and sterile neutrinos

$$M_L \nu_L^T C \nu_L, \quad M_R \nu_R^T C \nu_R,$$

- b. β decay operators with neutrinos vs antineutrinos

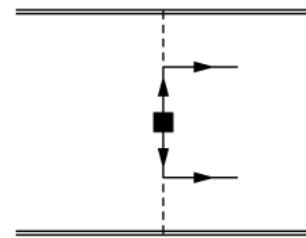
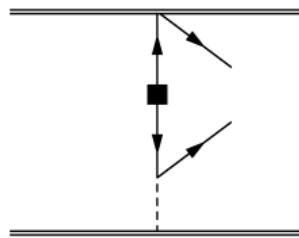
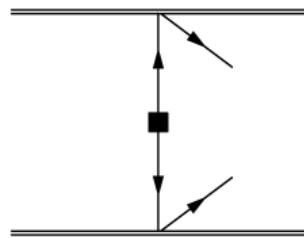
$$\bar{u} \Gamma d \nu_L^T C \Gamma' e, \quad \bar{u} \Gamma d \nu_R^T C \Gamma' e$$

- c. four-quark two-lepton operators

$$(\bar{u} \Gamma d) (\bar{u} \Gamma' d) e^T C e$$

accurate predictions for each mechanism?
differentiating between different mechanisms?

Neutrinoless double beta decay in chiral EFT. Standard Mechanism



- in the “standard mechanism”, $0\nu\beta\beta$ is induced by the exchange of Majorana neutrinos
- the lepton tensor combines in a form that looks like a boson propagator

$$L^{\mu\nu} \rightarrow g^{\mu\nu} \bar{e}_L e_L^c \frac{\sum U_{ei}^2 m_i}{q^2 - m_i^2 + i\varepsilon}$$

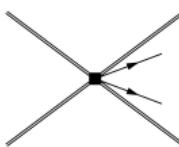
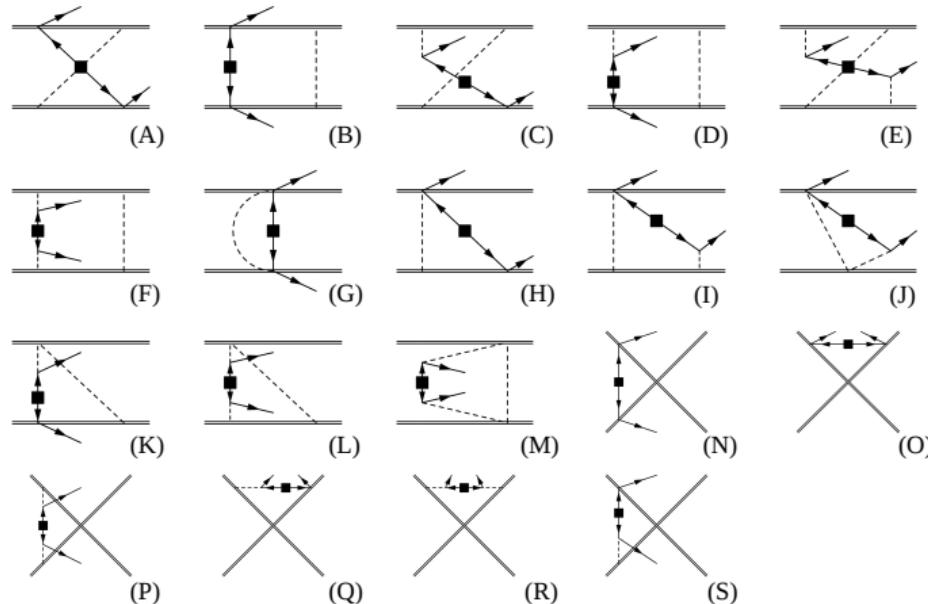
- at lowest order, neutrinos couple to the nucleons via the weak axial and vector currents
- the leading contrib. comes from “potential modes” $q_0 \sim \vec{q}^2/m_N \ll |\vec{q}|$
neutrino exchange gives rise to a weak potential, on top of the QCD potential

$$V_{\nu,0}^{(a,b)} = \tau^{(a)+} \tau^{(b)+} \frac{1}{\vec{q}^2} \left\{ 1 - g_A^2 \left[\boldsymbol{\sigma}^{(a)} \cdot \boldsymbol{\sigma}^{(b)} - \boldsymbol{\sigma}^{(a)} \cdot \vec{q} \boldsymbol{\sigma}^{(b)} \cdot \vec{q} \frac{2m_\pi^2 + \vec{q}^2}{(\vec{q}^2 + m_\pi^2)^2} \right] \right\} .$$

- in Weinberg’s counting, this is the only LO contribution



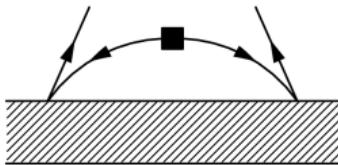
Neutrinoless double beta decay in chiral EFT



$$\mathcal{O}\left(\frac{1}{\Lambda_\chi^2}\right)$$

- beyond LO, we can consider pion-neutrino-nucleon loops + local counterterms

Usoft corrections to $0\nu\beta\beta$



- and contributions from neutrinos with very small momentum ($q_0, \vec{q} \sim Q$)
- they see the nucleus as a whole, and yield expressions very similar to “standard” QM

$$\begin{aligned} T_{\text{usoft}} &= -\frac{T_{\text{lept}}}{4} \sum_n \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{|\vec{k}|} \left[\frac{\langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle}{|\vec{k}| + E_2 + E_n - E_i - i\eta} + \frac{\langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle}{|\vec{k}| + E_1 + E_n - E_i - i\eta} \right] \\ &= T_{\text{lept}} \times \frac{1}{8\pi^2} \sum_n \langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle \left\{ (E_2 + E_n - E_i) \left(\log \frac{\mu_{\text{us}}}{2(E_2 + E_n - E_i)} + 1 \right) + (1 \rightarrow 2) \right\}, \end{aligned}$$

- where $E_{1,2}$ are the electron energies,
 E_i, E_f, E_n the initial, final and nuclear intermediate state energy
- the corrections scale as E/k_F , similar to “closure corrections” in pheno approaches

$0\nu\beta\beta$ at N2LO

1. correction to the one-body currents (magnetic moment, radii, ...)

$$\sim \mathcal{O}\left(\frac{Q^2}{\Lambda_\chi^2}\right)$$

2. pion-neutrino loops, local counterterms

$$\sim \mathcal{O}\left(\frac{Q^2}{\Lambda_\chi^2}\right)$$

3. ultrasoft contributions (“closure corrections”)

$$\sim \mathcal{O}\left(\frac{\Delta E}{4\pi k_F}\right)$$

3. two-body corrections to V and A currents

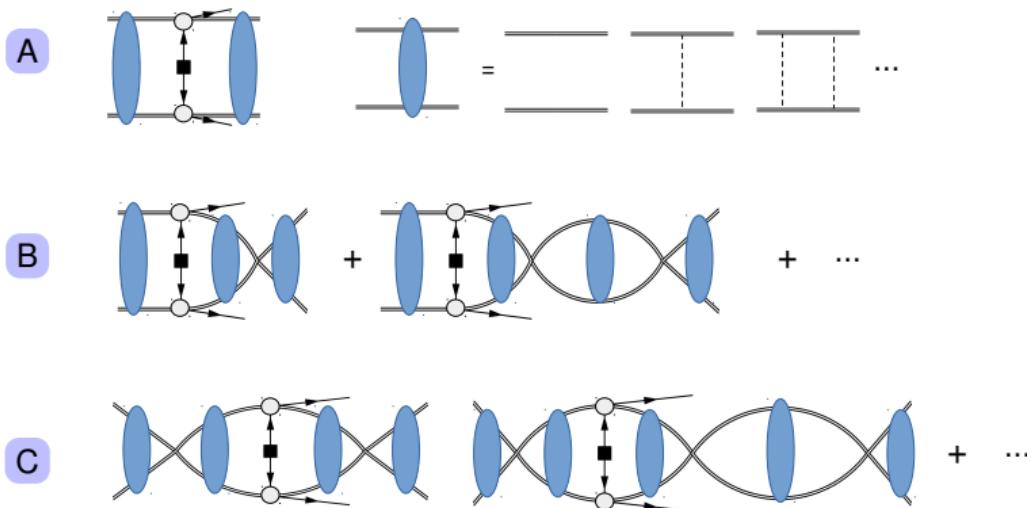
all these corrections should be fairly small



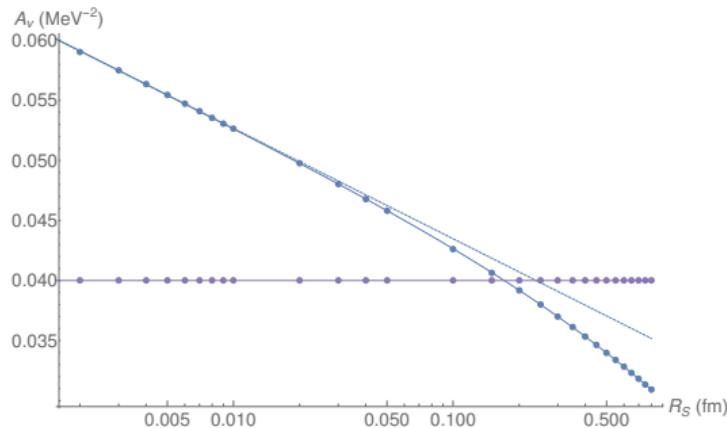
Neutrinoless double beta decay beyond Weinberg

- is this picture consistent?
- should check that the local counterterms follow Weinberg's counting
- the neutrino potential might cause problems similar to the OPE potential in 1S_0

Consider the 2-to-4 process $nn \rightarrow ppe^- e^-$



$0\nu\beta\beta$ in renormalized chiral EFT



- need to solve the Schrödinger equation with the LO chiral potential

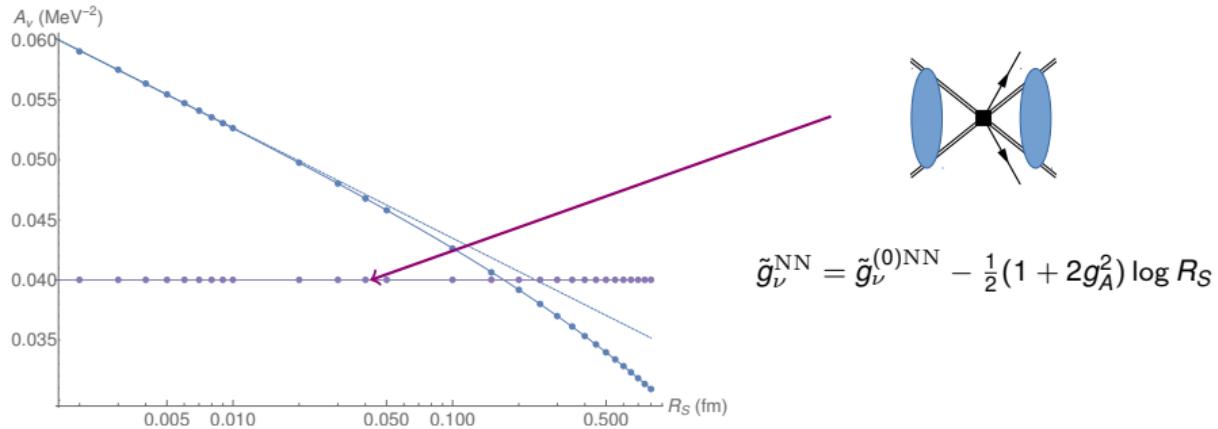
$$V_{\text{strong}}(r) = \tilde{C}_{1S_0} \delta_{R_S}^{(3)}(\vec{r}) + \frac{g_A^2 m_\pi^2}{16\pi F_\pi^2} \frac{e^{-m_\pi r}}{4\pi r}$$

- take the matrix element and check that it is cut-off independent

$$\mathcal{A}_\nu(nn \rightarrow ppe^- e^-) = \langle pp | V_{\nu,0} | nn \rangle$$



$0\nu\beta\beta$ in renormalized chiral EFT

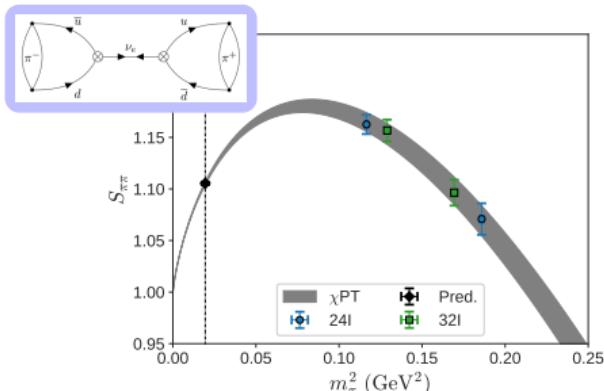


V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, S. Pastore, U. van Kolck, '18

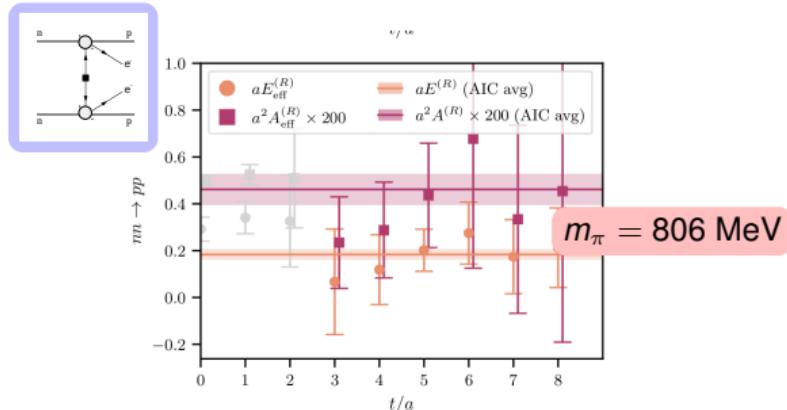
- the matrix element of the long-range neutrino potential is UV divergent!
- need to promote the N²LO counterterm to LO!

unexpected systematic only diagnosed with EFT tools!

Determination of g_ν^{NN} and interplay with Lattice QCD



W. Detmold and D. Murphy, '20



Z. Davoudi et al, '24

0. data driven extraction?

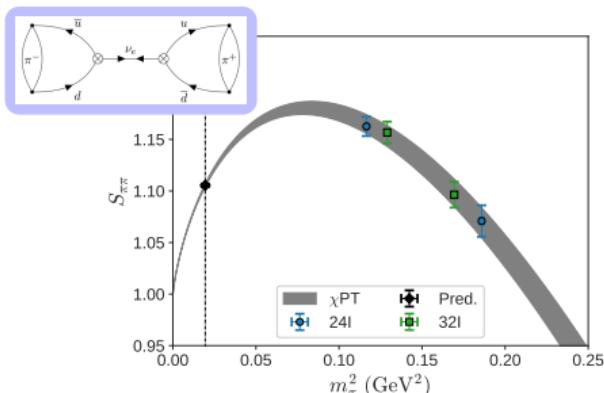
- no LNV data **X**
- chiral symmetry relation to isospin-breaking photon exchange processes

$$g_\nu^{\text{NN}} = g^{\text{CIB}}$$

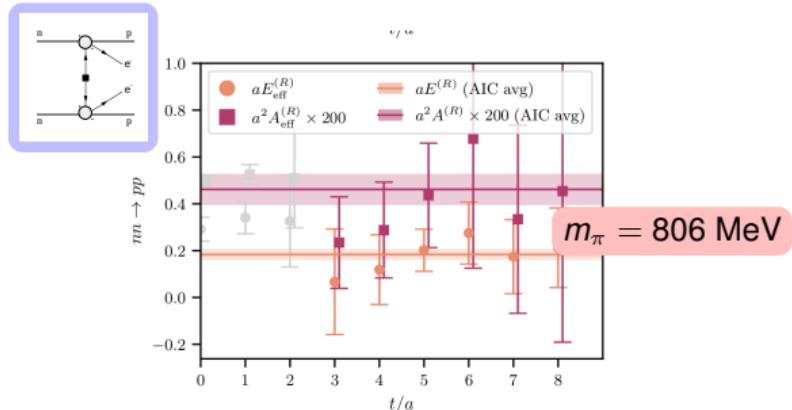
only true at the order-of-magnitude level



Determination of g_ν^{NN} and interplay with Lattice QCD



W. Detmold and D. Murphy, '20



Z. Davoudi et al, '24

1. Lattice QCD offers the most direct avenue

- long distance contributions to $\pi 0\nu\beta\beta$ already computed

X.-Y. Tuo, X. Feng and L.-C. Jin, '19, W. Detmold and D. Murphy, '20

- first calculation of the nn amplitude!

2. model the forward $W^+ nn \rightarrow W^- pp$ amplitude with chiral EFT + OPE

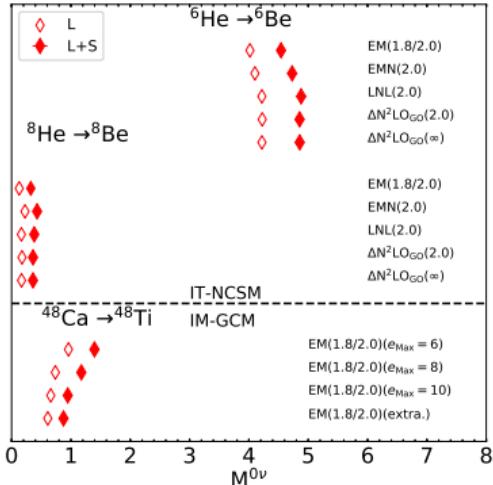
$$\tilde{g}_\nu^{\text{NN}}(\mu = m_\pi) = 1.32(50)_{\text{inel}}(20)_{\text{r}}(5)_{\text{par}} = 1.3(6)$$

compares well with “naive” CIB assumption

V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, EM, '20
7/26/2023 | 37



Impact on $0\nu\beta\beta$ nuclear matrix elements



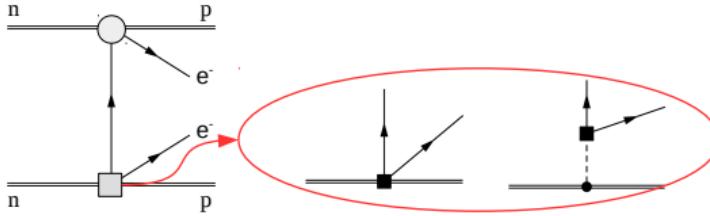
R. Wirth, J. M. Yao, H. Hergert, '21

- fit the “synthetic” amplitude to 3 different chiral potentials
- SRG-evolve strong and weak potential & calculate ${}^{48}\text{Ca}$ NME

43% shift in ${}^{48}\text{Ca}$



$0\nu\beta\beta$ transition operators from dim-7 operators



$$g_A = 1.27$$

$$g_S = 1.02 \pm 0.10$$

$$g_M = 4.7$$

$$g_T = 0.99 \pm 0.03$$

$$g'_T = \mathcal{O}(1)$$

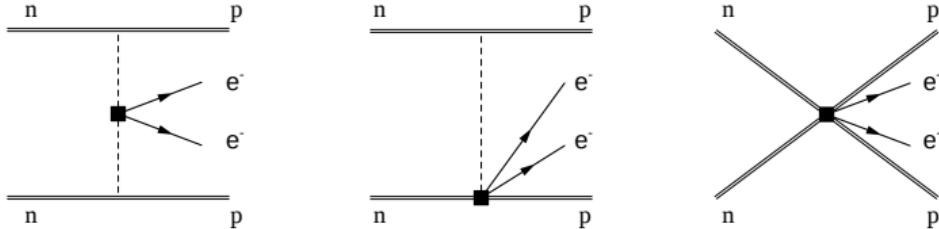
$$B = 2.7 \text{ GeV}$$

$$\begin{aligned} \mathcal{L}^{(6)} = & \frac{2G_F}{\sqrt{2}} \left\{ \bar{u}_L \gamma^\mu d_L \left[\bar{e}_R \gamma_\mu C_{VLR}^{(6)} \nu + \bar{e}_L \gamma_\mu C_{VLL}^{(6)} \nu \right] + \bar{u}_R \gamma^\mu d_R \left[\bar{e}_R \gamma_\mu C_{VRR}^{(6)} \nu + \bar{e}_L \gamma_\mu C_{VRL}^{(6)} \nu \right] \right. \\ & + \bar{u}_L d_R \left[\bar{e}_L C_{SRR}^{(6)} \nu + \bar{e}_R C_{SRL}^{(6)} \nu \right] + \bar{u}_R d_L \left[\bar{e}_L C_{SLR}^{(6)} \nu + \bar{e}_R C_{SLL}^{(6)} \nu \right] + \bar{u}_L \sigma^{\mu\nu} d_R \bar{e}_L \sigma_{\mu\nu} C_{TRR}^{(6)} \nu + \bar{u}_R \sigma^{\mu\nu} d_L \bar{e}_R \sigma_{\mu\nu} C_{TLL}^{(6)} \nu \Big\} + \text{h.c.} \end{aligned}$$

- need axial, vector, scalar, pseudoscalar and tensor one-body currents
- nucleon matrix elements are well determined experimentally or in LQCD (with one exception)
- the diagrams can be computed exactly as in the “standard case”.

The final result is a bit of a mess [V. Cirigliano et al, '18](#); [W. Dekens et al, '20](#)

$0\nu\beta\beta$ from dimension-9 operators



- need to study the hadronization of 4-quark 2-electron operators
- once the chiral analysis is done, very large $\pi\pi$ couplings for most operators

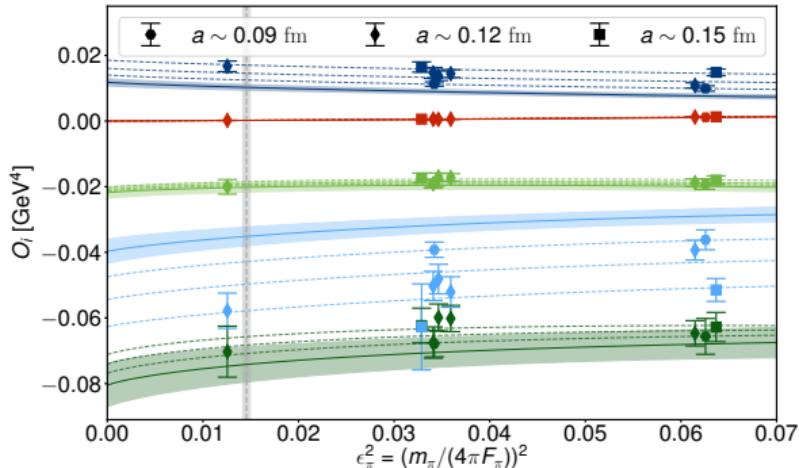
G. Prezeau, M. Ramsey-Musolf, P. Vogel, '03;
A. Faessler, S. Kovalenko, F. Simkovic, J. Schwieger, '97

- renormalization then requires NN couplings @ LO
- factorization is a bad approximation!
e.g \mathcal{O}_4

$$\langle pp | \bar{u}_L \gamma^\mu d_L \bar{u}_R \gamma_\mu d_R | nn \rangle \neq \langle p | \bar{u}_L \gamma^\mu d_L | n \rangle \langle p | \bar{u}_R \gamma_\mu d_R | n \rangle$$

error from neglecting $\pi\pi$ couplings \gg than from NME

$\pi\pi$ matrix elements



A. Nicholson *et al.*, CalLat collaboration, '18

- $\pi\pi$ matrix elements well determined in LQCD
- NME differ dramatically from factorization

good agreement with naive chiral counting

$$M_{\pi\pi} = -\frac{g_4^{\pi\pi} C_4^{(9)}}{2m_N^2} \left(\frac{1}{2} M_{AP,sd}^{GT} + M_{PP,sd}^{GT} \right) \sim -0.60 C_4^{(9)} \quad \text{vs} \quad M_{\text{fact}} = -\frac{3g_A^2 - 1}{2g_A^2} \frac{m_\pi^2}{m_N^2} C_4^{(9)} M_{F,sd} \sim -0.04 C_4^{(9)}$$

- which becomes a factor of 225 in the rate!



$$g_1^{\pi\pi} = +0.4$$

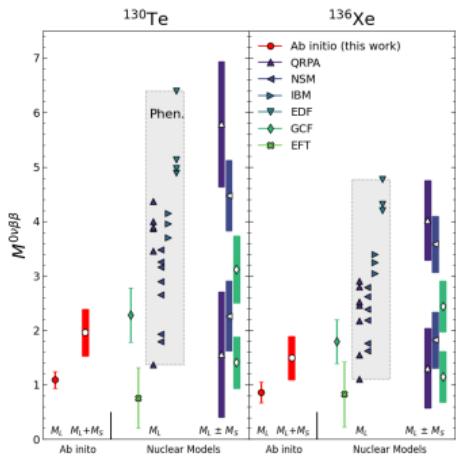
$$g_2^{\pi\pi} = -(1.8 \text{ GeV})^2$$

$$g_3^{\pi\pi} = +(1.0 \text{ GeV})^2$$

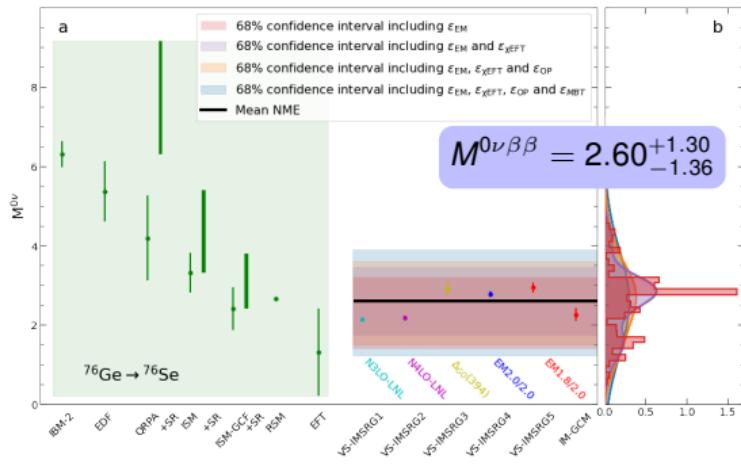
$$g_4^{\pi\pi} = -(1.7 \text{ GeV})^2$$

$$g_5^{\pi\pi} = -(3.6 \text{ GeV})^2$$

Ab initio calculations of $0\nu\beta\beta$ ME



A. Belley, S. Stroberg, J. Holt, '23



A. Belley, J. M. Yao et al, '23

- first *ab initio* calculations in ^{48}Ca , ^{76}Ge , ^{82}Se , ^{130}Te and ^{136}Xe
promising step towards controlled calculations with solid estimate of theory systematics
- nuclear matrix elements for BSM mechanisms can be evaluated in the same way



Light new physics

Nuclear physics with light new particles

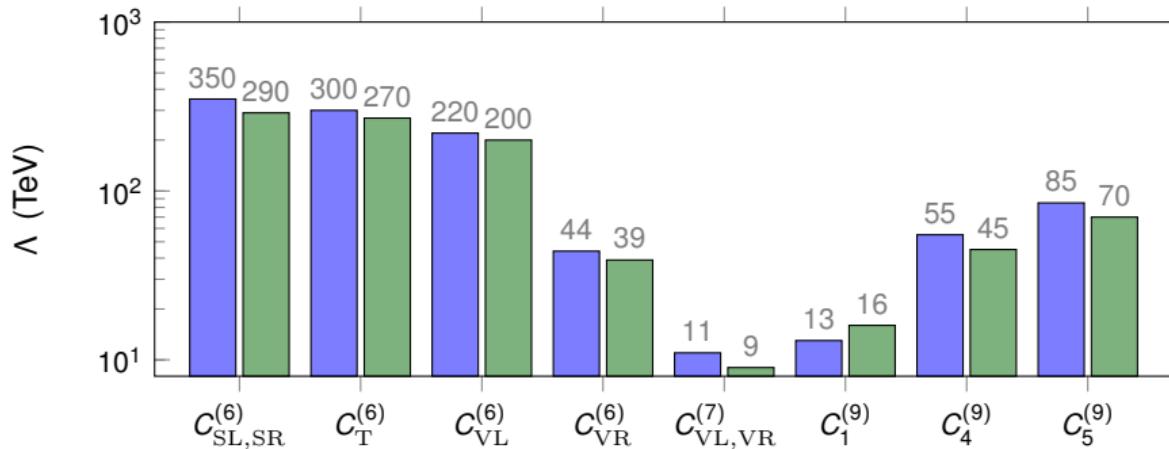


Phenomenology

Neutrinoless double beta decay

Bounds on effective operators

Hyvärinen et al. '15 Menéndez et al. '17



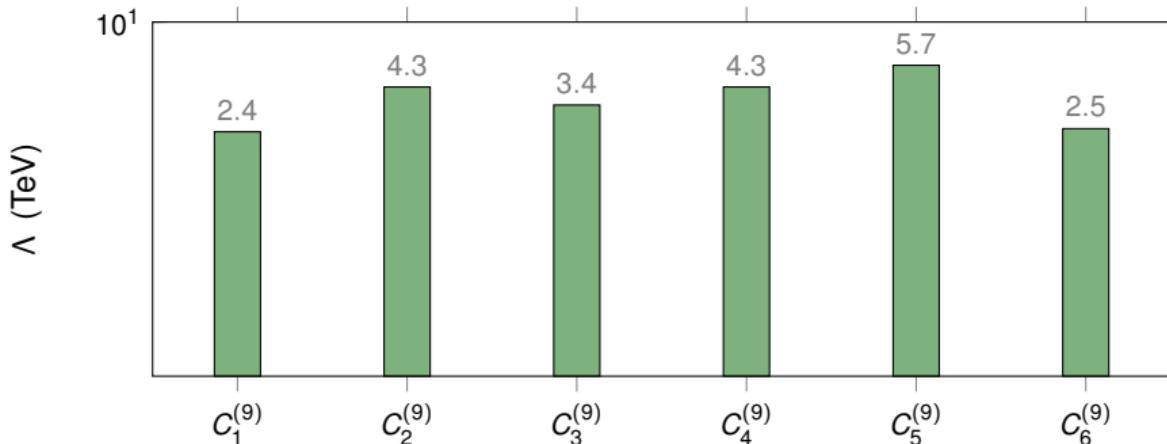
- $0\nu\beta\beta$ puts strong limits on dim. 7 operators
- dim. 9 in the TeV range

no way to probe at LHC

pattern can be understood from effective dimension & chiral properties of $0\nu\beta\beta$ operator

Bounds on effective operators

Menéndez et al. '17

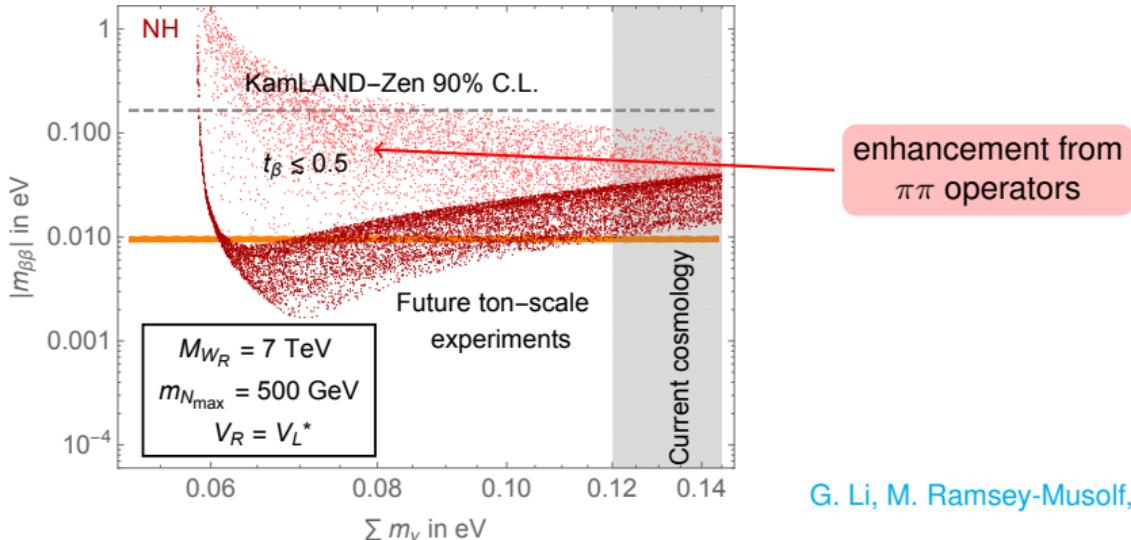


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$0\nu\beta\beta$ in the left-right symmetric models



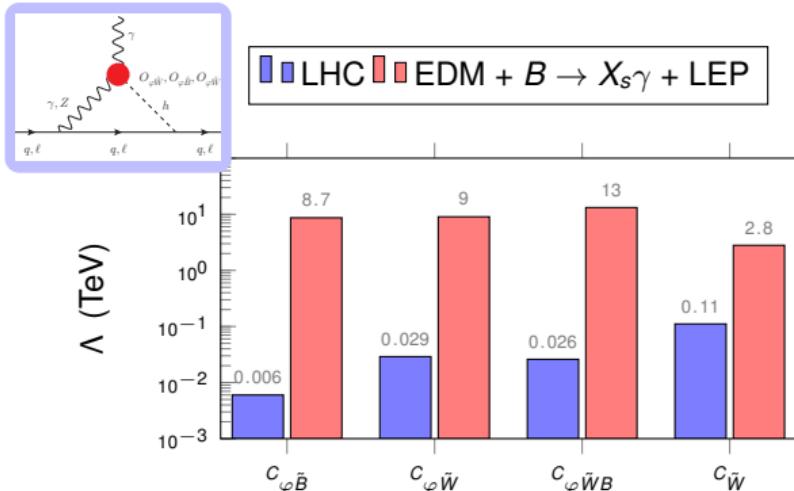
G. Li, M. Ramsey-Musolf, J. C. Vasquez, '20

Having the right hadronic and nuclear physics is important for pheno! E.g. LR symmetric model

- W_L - W_R mixing contribution enhanced by $g_4^{\pi\pi}, g_5^{\pi\pi}$
- possible signal in tonne-scale experiments even with NH

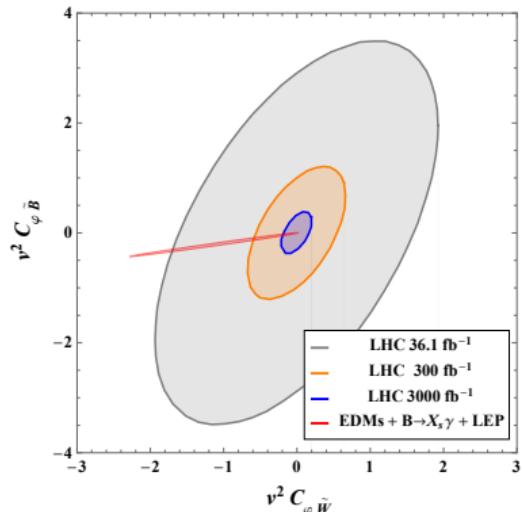
Electric dipole moments

Constraints on weak gauge-Higgs operators



V. Cirigliano, A. Crivellin, W. Dekens, J. de Vries, M. Hoferichter, EM, '19

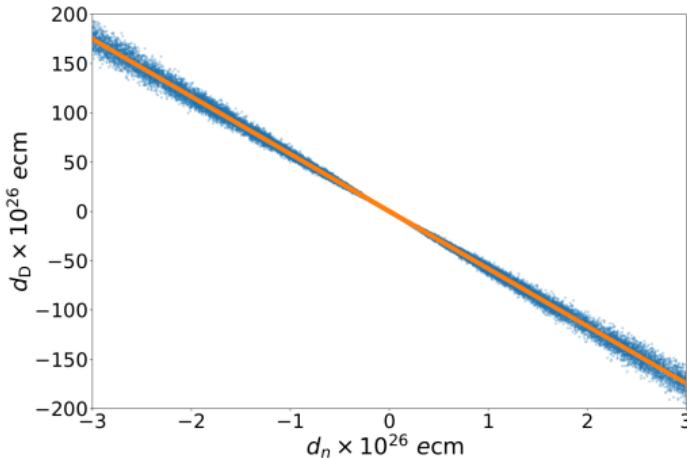
- eEDM dominates single coupling analysis
- hadronic EDMs constrain 2 directions
 d_n , d_{Hg} and d_{Ra} largely degenerate
- need LEP, $B \rightarrow X_s \gamma$ or LHC to close free directions



LHC projections of Bernlochner et al., '18

strong correlations to avoid EDMs

Identifying right-handed charged currents

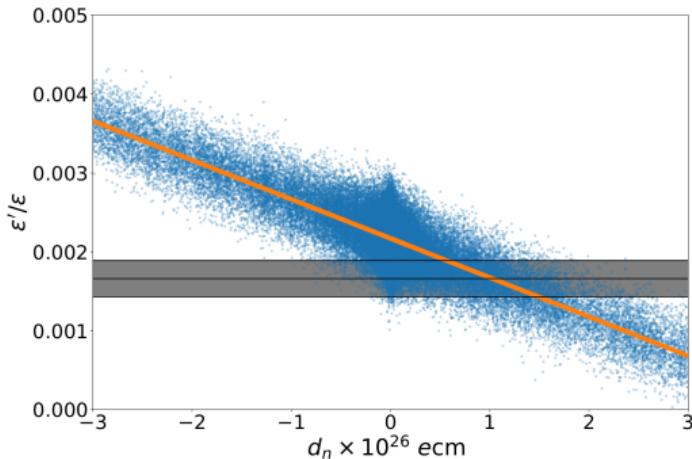


$[C_{Hud}]_{ud}$

assuming factor of 3 improvement
on theory errors

- $u - d$ RHCC can explain Cabibbo anomaly
- eEDM is very small (two loop and light quark mass suppression)
- π -N contributions to nuclear and atomic EDMs enhanced
- an observation of d_n should lead to large expected deuteron EDM (or Hg and Ra)

Identifying right-handed charged currents



$[C_{Hud}]_{ud}$

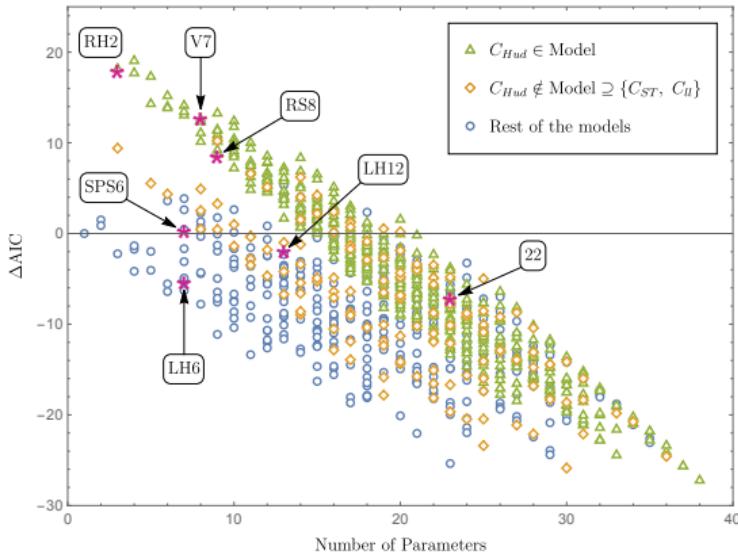
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- π -N contributions to nuclear and atomic EDMs enhanced
- an observation of d_n should lead to large expected deuteron EDM (or Hg and Ra)
- but could lead to too large corrections to ϵ'/ϵ !

once again, important to reduce errors!

CKM unitarity

SMEFT interpretations of the Cabibbo anomaly



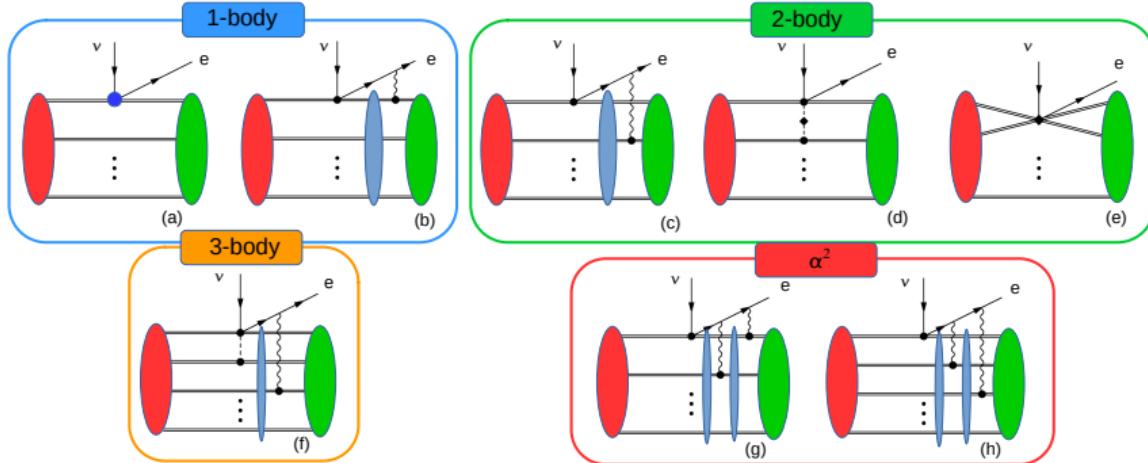
$\bar{u}_R \gamma^\mu d_R W_\mu$ couplings win!

V. Cirigliano, W. Dekens, T. Tong *et al.*, '23

- β decays + EWPO constraints + Drell-Yan crucial for interpretation of CAA
- consider 1024 choices of combinations of SMEFT couplings
- perform a simultaneous fit to low-energy, Z-pole and collider data
- and organize the “models” according to their AIC score

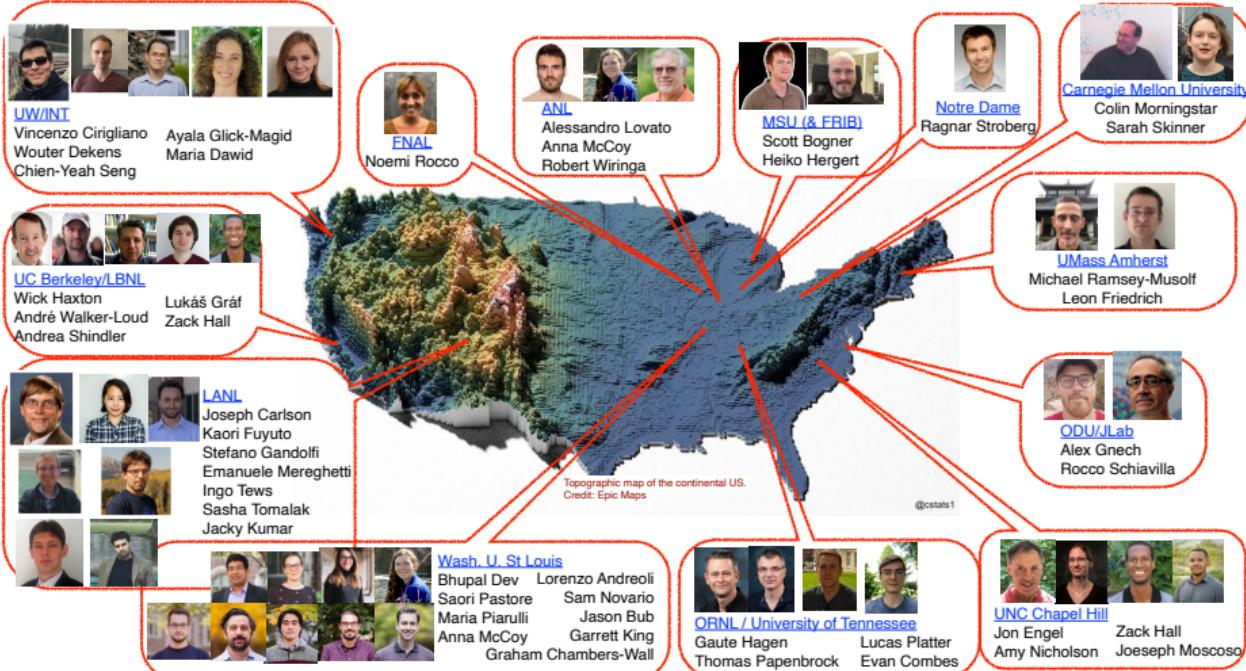


Assessing the error on $0^+ \rightarrow 0^+$ decays

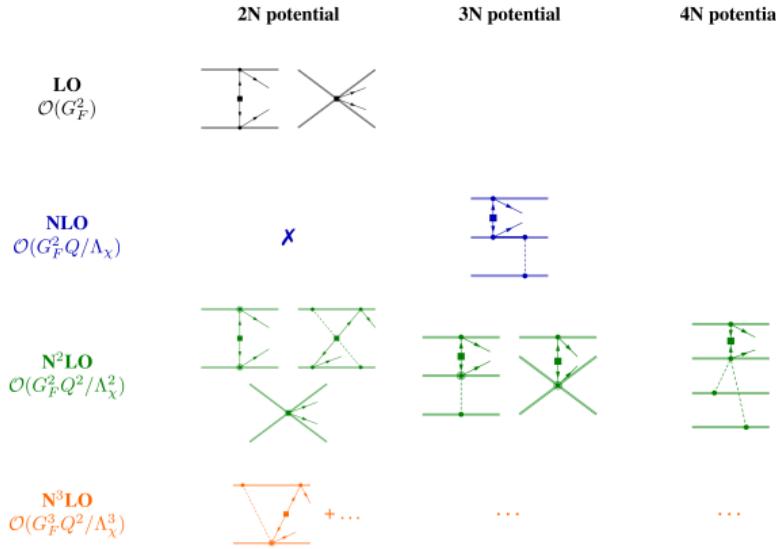


- but even more important is to develop EFT for radiative corrections and validate estimates of the theory error
- “Nuclear Theory for New Physics” topical collaboration, to address all aspects of the problem, from LQCD, to EFT, to nuclear structure

Faces of NTNP



Conclusion



- EFTs + LQCD + *ab initio* methods promise to deliver predictions of nuclear processes from QCD particularly important for the interpretation of “fundamental symmetry” experiments
- plenty of work to do on all three fronts!

Backup



LA-UR-24-27605

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Nuclear physics with light new particles

1. $m_W \gtrsim m_X \gtrsim m_N$: integrate out X in perturbation theory, match onto LEFT
2. $500 \text{ MeV} \gtrsim m_X \gtrsim m_N$: trickiest region, as no perturbative tools available
3. $m_X \sim m_\pi$: treat X as a pion, typically only soft and potential regions matter
3. $m_X \ll m_\pi$: soft, potential and ultrasoft contributions to be expected

One example: sterile neutrinos and $0\nu\beta\beta$

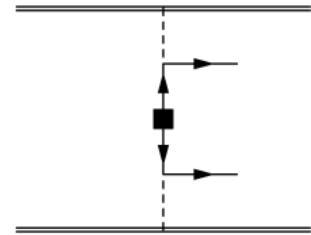
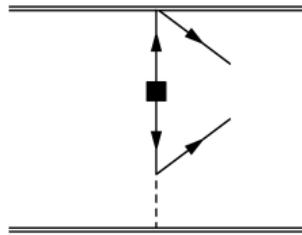
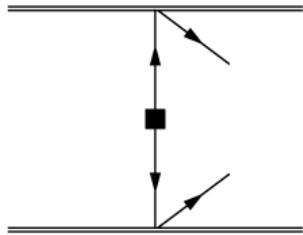
- singlet under $SU(2)_L$, can be treated in ν SMEFT
- retaining only dim-4 interactions, and after EWSB

$$\mathcal{L} = - \left[\frac{M_R}{2} \nu_R^T C \nu_R + \frac{v}{\sqrt{2}} \nu_L \nu_R + \text{h.c.} \right]$$

- $3 + n$ massive neutrinos, interactions parameterized by a $(3 + n) \times (3 + n)$ unitary matrix U
- which satisfy

$$\sum_{i=1}^{3+n} m_i U_{\ell i}^2 = 0 \quad \ell = e, \mu, \tau$$

$0\nu\beta\beta$ with sterile neutrinos



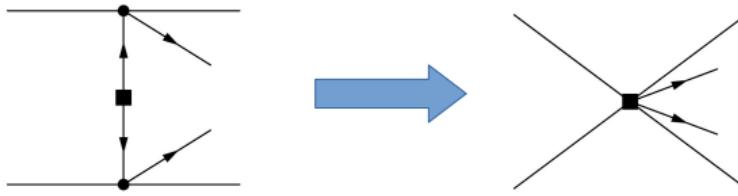
- naively

$$V_{\nu,0} = \sum_{i=1}^3 m_i U_{ei}^2 \tau^{(a)+} \tau^{(b)+} \left\{ \frac{1}{\vec{q}^2} \left(1 - g_A^2 \boldsymbol{\sigma}^{(a)} \cdot \boldsymbol{\sigma}^{(b)} \right) - 2g_\nu^{\text{NN}} \right\}$$

$$\Rightarrow \sum_{i=1}^{3+n} m_i U_{ei}^2 \tau^{(a)+} \tau^{(b)+} \left\{ \frac{1}{\vec{q}^2 + \cancel{m_i^2}} \left(1 - g_A^2 \boldsymbol{\sigma}^{(a)} \cdot \boldsymbol{\sigma}^{(b)} \right) - 2g_\nu^{\text{NN}} \right\} .$$

only true if $m_i \sim m_\pi$!

$0\nu\beta\beta$ with sterile neutrinos



- if $m_i >$ few GeVs, integrate out at the quark level

$$\mathcal{L} - \frac{4G_F^2}{m_i} U_{ei}^2 \bar{u}_L \gamma^\mu d_L \bar{u}_L \gamma_\mu d_L \bar{e}_L C \bar{e}_L$$

- which then hadronizes like a 4-fermion operator

$$\bar{u}_L \gamma^\mu d_L \bar{u}_L \gamma_\mu d_L \bar{e}_L C \bar{e}_L \Rightarrow g_1^{NN} \bar{p} n \bar{p} n \bar{e}_L C \bar{e}_L$$

- and the double beta operator becomes

$$\tau^{(a)+} \tau^{(b)+} \left\{ \sum_{i=1}^3 m_i U_{ei}^2 \left[\frac{1}{\vec{q}^2} \left(1 - g_A^2 \boldsymbol{\sigma}^{(a)} \cdot \boldsymbol{\sigma}^{(b)} \right) - 2g_\nu^{NN} \right] + \sum_{h=4}^{3+n} \frac{1}{m_h} U_{eh}^2 4g_1^{NN} \right\}$$

- In the factorization assumption $4g_1^{NN} = 1 + 3g_A^2 \Rightarrow$ large m_i limit of the naive expressions
... but we know factorization can be deeply wrong

$0\nu\beta\beta$ with sterile neutrinos

- if $m_i \ll m_\pi$, we can drop it from the neutrino potential
- but if $m_i \sim E_f - E_i$, will affect the usoft integrals

$$A_\nu^{(\text{usoft})}(m_i) = 8 \frac{\pi R_A}{g_A^2} \sum_n \langle 0_f^+ | \mathcal{J}_\mu | 1_n^+ \rangle \langle 1_n^+ | \mathcal{J}^\mu | 0_i^+ \rangle \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{E_\nu [E_\nu + \Delta E_1 - i\epsilon]} + (\Delta E_1 \rightarrow \Delta E_2),$$

- which gives

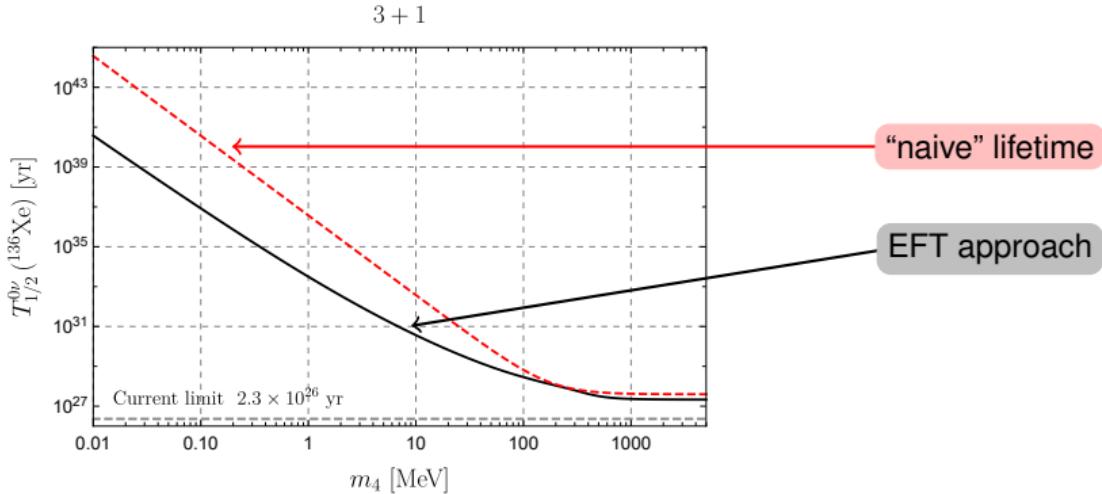
$$A_\nu^{(\text{usoft})} = 2 \frac{R_A}{\pi g_A^2} \sum_n \langle 0_f^+ | \mathcal{J}^\mu | 1_n^+ \rangle \langle 1_n^+ | \mathcal{J}_\mu | 0_i^+ \rangle \left(f(m_i, \Delta E_1) + f(m_i, \Delta E_2) \right),$$

with

$$f(m, E) = \begin{cases} -\pi m & \text{if } m \gg E, \\ \frac{m^2}{E} \log \frac{m}{2E}, & \text{if } m \ll E. \end{cases}$$

- in both regions, these are larger than what one would get taking the limit of the naive expression

$0\nu\beta\beta$ with sterile neutrinos



- so the $0\nu\beta\beta$ amplitude

$$\left[T_{1/2}^{0\nu} \right]^{-1} = g_A^4 G_{01} V_{ud}^4 \left| \sum_{i=1}^{3+n} m_i U_{ei}^2 (A_{\text{pot}}^\nu(0) + A_{\text{usoft}}^\nu(m_i)) \right|^2$$

- but remember $\sum_{i=1}^{3+n} m_i U_{ei}^2 = 0!$

usoft is the leading contribution surviving for sterile neutrino!